Wage Price Spirals*

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When firms and workers disagree on the relative price of labor and goods, they try to outpace each other in setting nominal wages and prices, and inflation follows. This mechanism is at work in a standard new Keynesian model, where the degree of disagreement is tied to the distance of aggregate output from its natural level. We look at how different shocks translate into different degrees of inflationary pressure on the good market and on the labor market side of the model. Depending on the relative force of these pressures, real wages can increase or fall. The direction in which the real wage moves is not indicative of how powerful the wage price spiral is. If the economy features a scarce non-labor input, inelastically supplied, with a relatively flexible price, episodes of excess demand are characterized by an initial spike in the input price, followed by persistent price inflation, and by a smaller but more persistent increase in wage inflation. The real wage falls early on and recovers later. In response to a supply shock optimal policy may involve choosing a positive output gap, if it helps relieve negative pressure on nominal wages.

1 Introduction

What is meant by a wage-price spiral? While there may not be universal agreement, in this paper we use the expression to describe a feedback mechanism where wages and prices compete adjusting upwards; wage earners try to keep up with rising prices; price setters try to keep up with rising wages. This mechanism amplifies and perpetuate the effects of certain inflationary shocks.

Our perspective is that this feedback mechanism is present in virtually all models—including standard New Keynesian varieties. The purpose of this paper is to elucidate and explore this feedback mechanism in detail in some simple variants of the New Keynesian model, and focus on the path for real wages in response to both supply and demand shocks.

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At heart, the economic logic of the wage-price spiral mechanism is that workers and firms disagree on the relative price of goods and labor, that is, on the real wage W/P. When firms adjust nominal prices they do so with some goal for the ratio P/W, or equivalently for W/P. As workers adjust nominal wages they may aspire to a different, higher ratio for W/P. If they do, then the outcome of this disagreement is nominal escalation, with inflation in both prices and wages.

Our interpretation of the concept of a wage-price spiral, highlighting disagreement or conflict as a proximate cause of inflation, is an idea that we explore in much greater generality in Lorenzoni and Werning (2022). The present paper, in contrast, studies how this conflict plays out in particular variants of the New Keynesian model and places attention on the path for real wages in response to both demand and supply shocks.

Our model is relatively close to standard models, but with two essential features that are not always present in the most basic New Keynesian setups. One important feature of our analysis is the inclusion of a scarce non-labor input with low substitutability in production (lower than Cobb-Douglas). We do not have in mind general forms of capital but rather, inputs like energy, other primary commodities, or intermediate inputs that may be subject to shortages or in relatively fixed supply in the short run, e.g. lumber or microchips. These non-labor inputs provide both a potential supply shock or a supply constraint for demand shocks. This feature of our modeling is motivated by the 2020-23 Covid crises and post-Covid recovery.

The other important feature of our model is that we include both nominal price and wage rigidities, as in many medium-scale models, but unlike the simplest New Keynesian which often include only of the two forms of rigidity.

With our model laid out, we ask various questions. First, we ask whether the direction in which real wages move following a shock tells us something about the strength of the wage-price spiral mechanism. We argue that this is not the case. The total force of a wage price spiral, that is, its power to translate a given shock into higher (price and wage) inflation, is different from its relative force on price versus wages. The direction of the real wage adjustment depends on the spiral's relative force, not on its total force.

Second, we ask whether the direction in which real wages move tells us something about the nature of the shock hitting the economy. In particular, can a pure aggregate demand shock cause a decline in real wages? We show that this depends on the situation the economy finds itself in—determined in our model by various parameters—when the disturbance arrives. In particular, if the supply of the scarce non-labor input is relatively inelastic and there is limited substitutability, then a demand shock can push prices up faster than wages and cause a real wage decline.

Indeed, we show that the response in our model to a pure demand shock described above can be quite similar to the response to a pure supply shock. To generate a pure supply shock, we consider a temporary reduction in the non-labor input with monetary policy not adjusting interest rates to counterbalance and reduce demand.

Both the demand and supply shocks create a situation of excess demand. In the demand shock case, natural output is unchanged, but demand temporally expands. In the supply shock case, the "natural" level of output is lower, but demand is unchanged. The excess demand leads to a tension between the level of the real wage that firms and workers aspire to, resulting in a wage-price spiral that produces inflation in both wages and prices. Excess demand is not a sufficient statistic, however. In the supply shock case, real wages always fall, whereas in the demand shock case the real wage may fall depending on parameters. Indeed, under some conditions the effects on wages and prices may be similar for both shocks.

Both demand and supply shocks can display a similar three-phase pattern of adjustment in nominal prices. First, there is a bout of very high price inflation in the price of the inelastic non-labor inputs, followed by a gradual reduction in the nominal price of these inputs. Second, there is a more persistent period of high good price inflation. Third, there is a smaller, but even more persistent increase in wage inflation. This pattern follows from our assumptions on the relative degree of price stickiness, with the input price being perfectly flexible, and with good prices being more flexible than wages. This pattern implies that at some point wage inflation crosses price inflation, so a period in which real wages fall is followed by a period in which they recover.

We then turn to normative questions and ask what is the optimal policy response to a supply shock coming from the scarce input. In particular, we ask two questions. First, could it be part of optimal policy to "run the economy hot", that is, allow for a positive output gap despite high inflation? Second, could it be part of optimal policy to go further and allow for inflation in both prices and wages?

Our answer to the first question is affirmative: if the economy needs a lower real wage, it may be more efficient to reach the adjustment with the help of higher price inflation and moderate wage deflation, rather than though lower price inflation and deeper wage deflation. A positive output gap helps shift the adjustment in the direction of price inflation, so is socially beneficial in this manner.

The answer to the second question is also affirmative. We construct examples in which, at some point, along the adjustment path, the output gap is positive and price and wage inflation are both positive. The economic intuition is that this aspect of policy is a form of "forward guidance": by promising to heat up the economy in the future, we speed up the adjustment of the real wage today. Underlying this result is the assumption of forward-looking price- and wage-setting behavior and the commitment of policy. In contrast, when policy has full discretion the equilibrium outcome never features both price and wage inflation.

1.1 Related literature

Our paper builds on the idea of inflation as the result of distributional conflict, something we explore in more detail in Lorenzoni and Werning (2022). A seminal contribution on this conflict perspective of inflation is Rowthorn (1977). That paper provides a model where, each period, wages are first set by workers and then prices are set by firms. Inflation is shown to be increasing in the conflict or "aspirational gap". Because of the assumed sequential timing of price and wage setting, conflict and inflation must not be fully anticipated by workers. Indeed, no rational expectations equilibrium exists with conflict. In contrast our model features staggered wages and prices that ensure that there is an equilibrium with finite conflict and inflation, even under rational expectations.

The idea of the wage price spiral as an important element of inflation dynamics has a long history. Blanchard (1986) is the seminal paper connecting that idea to New Keynesian models of staggered price setting. The model has nominal prices and wages that are fixed for two periods, with prices reset in even periods and wages in odd periods. The main result in the paper is that the alternating wage and price setting leads to a slow adjustment of the price level in response to a permanent money supply shock and that the adjustment features dampening oscillations in the real wage. Our paper instead builds on the (by now) canonical New Keynesian setting with sticky-price and sticky-wages of the Calvo variety as developed by Erceg et al. (2000). Relative to Blanchard, price and wage setting occur in a staggered fashion without the predictable alternation between wages and prices, so our model is not prone to the same type of oscillations. We also do not focus on a permanent money shock or study monetary policy in terms of money supply. Instead, we focus on supply and demand shocks under different policy responses. Finally, we investigate optimal monetary policy.

Our contribution also focuses on the role of non-labor inputs and to characterize the relative size and persistence of the adjustment of the input price, nominal prices, and wages. Our emphasis on the non-labor input connects our analysis to the analysis of oil shocks in Blanchard and Gali (2007a).[1] An important modeling difference is that we focus

[1] In turn, this connects us to the enormous literature on the effects of oil shocks, going back to Bruno and Sachs (1985).

on nominal wage rigidities, while they study a form of real-wage rigidity.

On the normative side, our paper is connected to the welfare analysis of alternative policy rules in models where both prices and wages are rigid, going back to the original paper of Erceg et al. (2000) and to the real-rigidity model of Blanchard and Gali (2007b). The starting observation in the literature is that the presence of both price and wage rigidities breaks "divine coincidence" and introduces potentially interesting trade-offs in the response of monetary policy to supply shocks. We offer a complete characterization of optimal policy and explore conditions for the optimum to have a positive output gap in combination with high inflation, as well as cases where it is optimal to have both wage and price inflation.

2 Model

We build our arguments in a standard New Keynesian model with nominal price and wage rigidities. To capture supply shocks, an important ingredient we include is a scarce non-labor input X, which is used alongside labor for production. We assume this input has a flexible price, and we allow the production function to have elasticity of substitution different from one.[2] An important example is energy inputs, but we interpret X more broadly to also capture shortages, bottlenecks and capacity constraints in the supply of intermediates like microchips or lumber, which have been in the spotlight during the post-pandemic recovery.

We focus on a closed economy in which the supply of X is given while the price of X adjusts endogenously in equilibrium. The analysis could be easily expanded to the case of an open economy in which the good X is imported, and, in particular, to the limit case of a small open economy that takes the world price of X as given. In that case, a supply shock would take the form of a shock to the world price instead of a shock to the endowment.

2.1 Setup

Time is continuous and infinite. The representative household has preferences

$$\int_0^\infty e^{-\rho t} \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{\Phi_t}{1+\eta} {N_t}^{1+\eta} \right) dt,$$

[2] This is formally equivalent to having labor and capital, with capital rented at a flexible price, although the interpretation is different. Erceg et al. (2000) have labor and capital. Closer to the interpretation here, Blanchard and Gali (2007a) have an energy input.

where C_t is an aggregate of a continuum of varieties of goods $C_t = \left(\int_0^1 C_{jt}^{1-1/\varepsilon_C} dj\right)^{\frac{1}{1-1/\varepsilon_C}}$, N_t is labor supply, and Φ_t is a labor supply shock. Each good variety j is supplied by a monopolistic firm with production function

$$Y_{jt} = F\left(L_{jt}, X_{jt}\right) \equiv \left(a_L L_{jt}^{\frac{\epsilon-1}{\epsilon}} + a_X X_{jt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

where L_{jt} is the labor input and X_{jt} is the non-labor input. The labor input L_{jt} of each firm j is an aggregate of a continuum of labor varieties $L_{jt} = \left(\int_0^1 L_{jkt}^{1-1/\varepsilon_L} dk\right)^{\frac{1}{1-1/\varepsilon_L}}$. Each labor variety k is supplied by a monopolistic union that employs labor from households and turns it, one for one, into specialized labor services of type k. Integrating over firms, total employment of labor variety k is $N_{kt} = \int_0^1 L_{jkt} dj$. Integrating over unions, total labor supply is $N_t = \int_0^1 N_{kt} dk$. The representative household owns an exogenous endowment X_t of the non-labor input X and sells it to the monopolistic goods producers on a competitive market, at the price P_{Xt}.

Monopolistic firms set the nominal price at which they are willing to sell their variety and then supply the amount chosen by consumers. Similarly, monopolistic unions set the nominal wage and supply the amount chosen by firms. Firms and unions are only allowed to reset their price and their wage rate occasionally. Namely, at each point in time firms are selected randomly to reset their price with Poisson arrival λ_p, and unions are selected with arrival λ_w.

When the exogenous variables X_t, Φ_t are constant, the model has a steady state in which quantities are constant, nominal prices are constant (zero inflation), all good varieties have the same price, and all labor varieties have the same wage. We will consider an economy in steady state and analyze its response to one time, unexpected shocks, either due to changes (transitory or permanent) to X_t or Φ_t or to changes in monetary policy leading to transitory deviations of C_t and N_t from the path consistent with zero inflation.

2.2 Price and wage setting

Let P_t^* and W_t^* denote the price and wage set by the firms and unions that can reset at time t, while P_t and W_t denote the price indexes for the good and labor aggregates.

The nominal marginal cost of producing good j is

$$\frac{W_t}{F_L\left(L_{jt}, X_{jt}\right)} = \frac{W_t}{a_L Y_{jt}^{\frac{1}{\epsilon}} L_{jt}^{-\frac{1}{\epsilon}}}.$$

Using lowercase variables to denote log-linear deviations from steady state and taking a first-order approximation, nominal marginal costs can then be expressed as

$$w_t - mpl_{jt}, \tag{1}$$

where

$$mpl_{jt} = \frac{1}{\epsilon}\left(y_{jt} - l_{jt}\right)$$

is the marginal product of labor. The production function of firm j in log-linear approximation is

$$y_{jt} = s_L l_{jt} + s_X x_{jt}, \tag{2}$$

where s_L and s_X are the steady state shares of the labor and non-labor inputs, with $s_L + s_X = 1$. All firms being price takers in the input market, they all employ inputs in the same ratio L_{jt}/X_{jt}, so in log-linear approximation

$$l_{jt} - x_{jt} = n_t - x_t$$

where n_t and x_t are the aggregate supplies of the two inputs. Combining these results, the marginal product of labor is

$$mpl_t = \frac{s_X}{\epsilon}\left(x_t - n_t\right). \tag{3}$$

Following standard steps, optimal price setting requires that firms set their price at time t equal to an average of future nominal marginal costs, conditional on not resetting. This gives the following optimality condition for P_t^* in log-linear approximation

$$p_t^* = \left(\rho + \lambda_p\right) \int_t^\infty e^{-\left(\rho + \lambda_p\right)(\tau - t)} \left(w_\tau - mpl_\tau\right) d\tau. \tag{4}$$

Following similar steps, we can derive the wage setting equation

$$w_t^* = \left(\rho + \lambda_w\right) \int_t^\infty e^{-\left(\rho + \lambda_w\right)(\tau - t)} \left(p_\tau + mrs_\tau\right) d\tau \tag{5}$$

where

$$mrs_t = \phi_t + \sigma y_t + \eta n_t \tag{6}$$

is the marginal rate of substitution between consumption and leisure of the representative consumer.

The presence of the w_τ's on the right-hand side of equation (4) and of the p_τ's on the right-hand side of equation (5) capture the logic of a wage price spiral in our model. Firms aim to get prices to be a constant markup over nominal marginal costs, and since marginal costs depend on nominal wages, they set nominal prices to catch up with current and anticipated future nominal wages. Symmetrically, wage setters aim to achieve a real wage that reflects their willingness to substitute leisure with consumption goods, so, they set nominal wages to catch up with current and anticipated future nominal good prices.

The optimality condition for the input-ratio of firms can be written as follows

$$p_{Xt} = w_t - \frac{1}{\epsilon}\left(x_t - n_t\right).$$

This condition will be used to derive the equilibrium input price p_{Xt}.

2.3 Inflation equations

To go from equations (4) and (5) to wage and price inflation, combine them with the differential equations for p_t and w_t:

$$\dot{p}_t = \lambda_p\left(p_t^* - p_t\right), \tag{7}$$

$$\dot{w}_t = \lambda_w\left(w_t^* - w_t\right). \tag{8}$$

As shown in the appendix, this leads to the following expressions

$$\rho\pi_t = \Lambda_p\left(\omega_t - mpl_t\right) + \dot{\pi}_t, \tag{9}$$

$$\rho\pi_t^w = \Lambda_w\left(mrs_t - \omega_t\right) + \dot{\pi}_t^w, \tag{10}$$

where we use the notation $\pi_t \equiv \dot{p}_t$ and $\pi_t^w \equiv \dot{w}_t$ for price and wage inflation, we denote by $\omega_t \equiv w_t - p_t$ the real wage, and where the coefficients Λ_p and Λ_w are

$$\Lambda_p = \lambda_p\left(\rho + \lambda_p\right), \quad \Lambda_w = \lambda_w\left(\rho + \lambda_w\right).$$

The real wage dynamics are given by

$$\dot{\omega}_t = \pi_t^w - \pi_t. \tag{11}$$

As in Lorenzoni and Werning (2022), equations (9) and (10) can be interpreted in terms of a conflict between the real wage aspirations of workers and firms. In the context of the New Keynesian model, the workers' aspiration is given by the marginal rate of substitution mrs_t at which the representative worker is willing to exchange labor for goods, while the firms' aspiration is the marginal product of labor mpl_t.[3] As in Lorenzoni and Wern-

[3]That is, the variable ϕ_t in the notation of Lorenzoni and Werning (2022) corrresponds to mpl_t here and the variable γ_t is corresponds to mrs_t here.

ing (2022), a discrepancy between the aspirations mpl_t and mrs_t is the proximate cause of inflation.

Given an initial condition ω_0 and given paths for mpl_t and mrs_t for $t \geq 0$, the three equations (9)-(11) give unique paths for price and wage inflation. In the next section we analyze the implications of these three equations, conditional on given paths of mpl_t and mrs_t. We then go back to the full general equilibrium analysis in which mpl_t and mrs_t are derived endogenously.

3 Inflation and Real Wage Dynamics

This section characterizes the dynamics of inflation and real wages.

3.1 Conflict and adjustment inflation

The analysis in Lorenzoni and Werning (2022) shows that price and wage dynamics can be decomposed in the following way

$$\pi_t = \Pi_t^C - \frac{\Lambda_p}{\Lambda_p + \Lambda_w} \Pi_t^A, \tag{12}$$

$$\pi_t^w = \Pi_t^C + \frac{\Lambda_w}{\Lambda_p + \Lambda_w} \Pi_t^A, \tag{13}$$

where Π_t^C is defined as

$$\Pi_t^C = \frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \int_t^\infty e^{-\rho(\tau - t)} (mrs_\tau - mpl_\tau)\, d\tau \tag{14}$$

and represents the "conflict" component of inflation, driven by the distance between the real wage aspirations of workers and firms, while Π_t^A represents the "adjustment" component of inflation, that is, the inflation needed to move the relative price ω_t, which is simply equal to

$$\Pi_t^A = \dot{\omega}_t.$$

Conflict inflation is already directly expressed in terms of the underlying paths of mpl_t and mrs_t. To express the adjustment component in terms of mpl_t and mrs_t, we need to combine equations (9)-(11) to obtain a second order ODE for ω_t

$$\ddot{\omega}_t = \rho \dot{\omega}_t + \left(\Lambda_p + \Lambda_w\right) \left(\omega_t - \tilde{\omega}_t\right), \tag{15}$$

where

$$\tilde{\omega}_t = \alpha mpl_t + (1-\alpha)\, mrs_t,$$

is the average of the aspirations of workers and firms, weighted by the relative degree of price rigidity which is given by the coefficient

$$\alpha \equiv \frac{\Lambda_p}{\Lambda_p + \Lambda_w}.$$

The next proposition provides the saddle-path stable solution of (15). In the proof, in the Appendix, we also provide an explicit solution for ω_t as a function of past and future values of $\tilde{\omega}_t$.

Proposition 1. *The real wage satisfies the first order ODE*

$$\dot{\omega}_t = r_1 \omega_t + \left(\Lambda_p + \Lambda_w\right) \int_t^\infty e^{-r_2(\tau - t)} \tilde{\omega}_\tau d\tau, \qquad (16)$$

where $r_1$and r_2 are the roots of the quadratic equation

$$r\,(r-\rho) = \Lambda_p + \Lambda_w,$$

and satisfy $r_1 < 0 < \rho < r_2$.

The second term in (15) shows that real wage dynamics are driven by a forward-looking expression, capturing the anticipated levels of the average aspiration $\tilde{\omega}_t$.

The first term in (16) shows that the real wage tends to mean revert, since $r_1 < 0$. The intuition for the mean-reversion is that a higher ω_t increases $\omega_t - mpl_t$, i.e., the distance between the real wage and the firms' aspiration mpl_t, pushing up price inflation. It also reduces $mrs_t - \omega_t$, i.e., the distance between the workers' aspiration mrs_t and the real wage, which pushes down wage inflation. Higher price inflation and lower wage inflation reduce the real wage.

Labor market pressures and mean-reversion shape the real wage response to different shocks and thus the adjustment component Π_t^A.

Let us turn to two simple examples to see these forces at work.

3.2 A permanent change in *mpl*

Suppose the economy is in steady state with all variables equal to 0. At date 0, unexpectedly, there is a one time, permanent reduction in mpl, which goes to $\overline{mpl} < 0$. The level of mrs remains unchanged at 0.

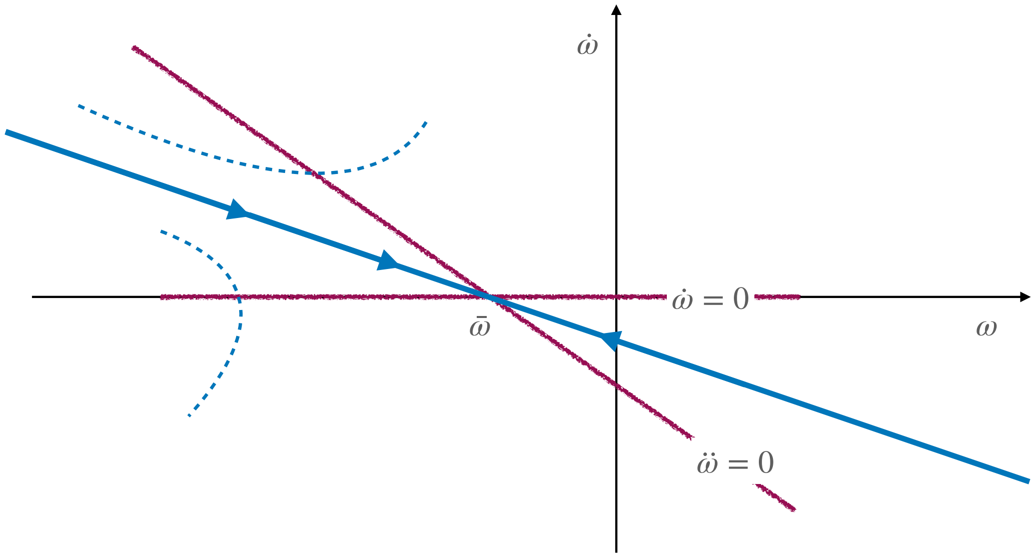

Figure 1: A permanent shock

The conflict component of inflation, from (14), is now permanently higher, constant and equal to

$$\Pi^C = -\frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \frac{1}{\rho} \overline{mpl}.$$

The adjustment component can be deduced from the phase diagram in Figure 1 that represents the second order ODE (15). The stationary locus $\dot{\omega} = 0$ coincides with the horizonatal axis. The stationary locus $\ddot{\omega} = 0$ is downward sloping. Both are drawn in purple. The saddle path, in blue, is given by the equation

$$\dot{\omega}_t = r_1 \left(\omega_t - \overline{\omega} \right),$$

where

$$\overline{\omega} = \frac{\Lambda_p}{\Lambda_p + \Lambda_w} \overline{mpl}$$

is the constant value of $\tilde{\omega}_t$ after the shock and is also the long-run level of the real wage.[4]

The diagram shows that starting at $\omega_0 = 0$, we initially have $\dot{\omega}_t < 0$, so a negative value for Π_t^A. Gradually, as the real wage reaches its new long-run level $\overline{\omega}$, this effect goes away.

Going back to equations (12) and (13), we can then see that there are initially two forces pushing up price inflation: a permantly higher conflict component, plus a temporarily positive adjustment component, reflecting the inital fall in the real wage. On the

[4]The expression for the saddle path comes from 16, using the condition $-r_1 r_2 = \Lambda_p + \Lambda_w$ (see the proof of Proposition 1).

wage inflation side, adjustment inflation has the opposite effect and initally keeps wage inflation lower than Π^C.[5]

In the long run, the adjustment component goes away, and wage and price inflation converge to the same level, equal to the conflict component.

An example featuring a permanent gap between mpl and mrs is useful but extreme. If calibrated with a realistically low value of ρ, such an example yields very large levels of wage and price inflation for a given shock $\overline{mpl}$. This is just a reflection of the fact that the long-run new Keynesian Phillips curve is very steep. Let's turn to a transitory shock.

3.3 A transitory change in mpl

Consider an economy in steady state with all variables at 0. At $t = 0$, unexpectedly, firms realize that for a finite time interval $[0, T]$ they will face $\overline{mpl} < 0$. At T, mpl goes back to zero. The value of mrs remains at zero throughout.

The conflict component is now

$$\Pi_t^C = -\frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \frac{1 - e^{-\rho(T-t)}}{\rho} \overline{mpl}$$

for $t \leq T$ and zero afterwards.

The dynamics of ω following the shock are illustrated in Figure 2. First, the economy follows the red solid line, until that line meets the blue solid line at time T, then the economy follows the blue saddle path asymptoting back to the origin. The real wage first falls towards $\overline{\omega}$ (defined above for a permanent shock). At some point, before time T, the real wage starts growing again, due to the increased strength of the mean-reverting force; finally, after the impulse to mpl is gone, the real wage converges back to zero.

The intuition for the forces at work on impact, at $t = 0$, is very close to the permanent shock example: the adjustment component adds to the conflict component for price inflation, while it dampens wage inflation.[6]

Over time the adjustment component gets weaker, until, at some finite time prior to T, when the red curve meets the horizontal axis, we have $\dot{\omega} = 0$. At that point, price and wage inflation are identical and equal to the conflict component.

[5] It is easy to prove that despite the presence of the adjustment component, wage inflation is always positive in this experiment. From (10) we get

$$\pi_t^w = \int_t^\infty e^{-\rho(\tau - t)} \left(mrs_\tau - \omega_\tau \right) d\tau,$$

and notice that $mrs_t = 0$ and $\omega_t < 0$ for all $t > 0$, from the phase diagram.

[6] A similar argument as in footnote 5 shows that $\pi_0^w > 0$.

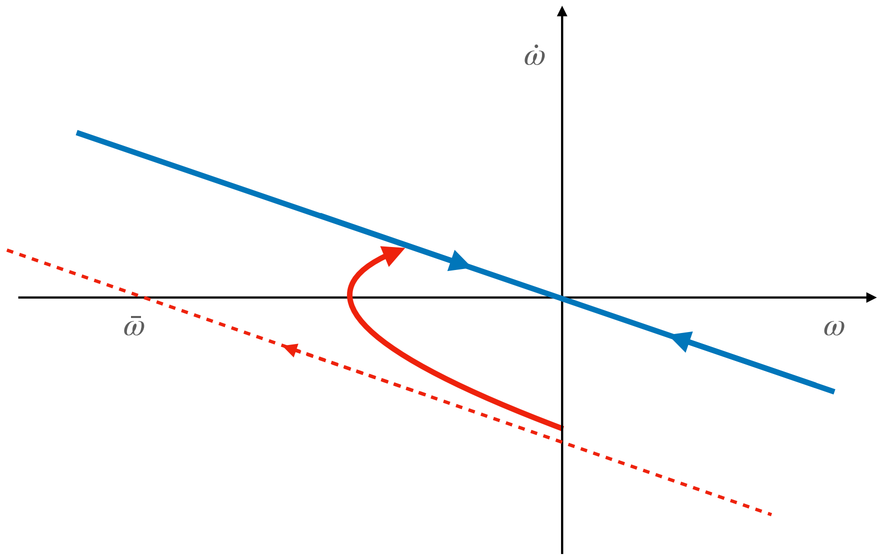

Figure 2: A transitory shock

After that point, the adjustment component switches sign as we have $\dot{\omega} > 0$, while, at the same time the conflict component is converging to zero. This implies that, at some point price, inflation becomes negative. From T onward, conflict inflation is zero and we only have adjustment inflation, which gives negative price inflation and positive wage inflation, to bring the real wage back to its original steady state value.

Figure 3 illustrates these qualitative patterns in a numerical example.

Proposition 5 in the appendix provides formal derivations for a general class of experiments like the two just analyzed, in which only one side of the labor market is affected, that is, where only mpl or only mrs deviate from zero.

However, in most relevant cases, as we shall see, the underlying economic shocks change *both* mpl and mrs at the same time. In that case, the shape of the responses on the two sides can produce a variety of behaviors. We now provide a characterization in the case in which mpl and mrs decay exponentially over time at the same rate.

3.4 AR(1) shocks to Aspirations

To study the combined effect of changes to both mpl and mrs, we now focus on a simple AR(1) shock to both variables, with persistence δ. The economy starts at a steady state with all variables equal to zero and, at $t = 0$, there is a joint unexpected shock with $mpl_0 \neq 0$ and $mrs_0 \neq 0$. From then on the paths of mpl_t and mrs_t decay exponentially

$$mpl_t = mpl_0 e^{-\delta t}, \quad mrs_t = mrs_0 e^{-\delta t}.$$

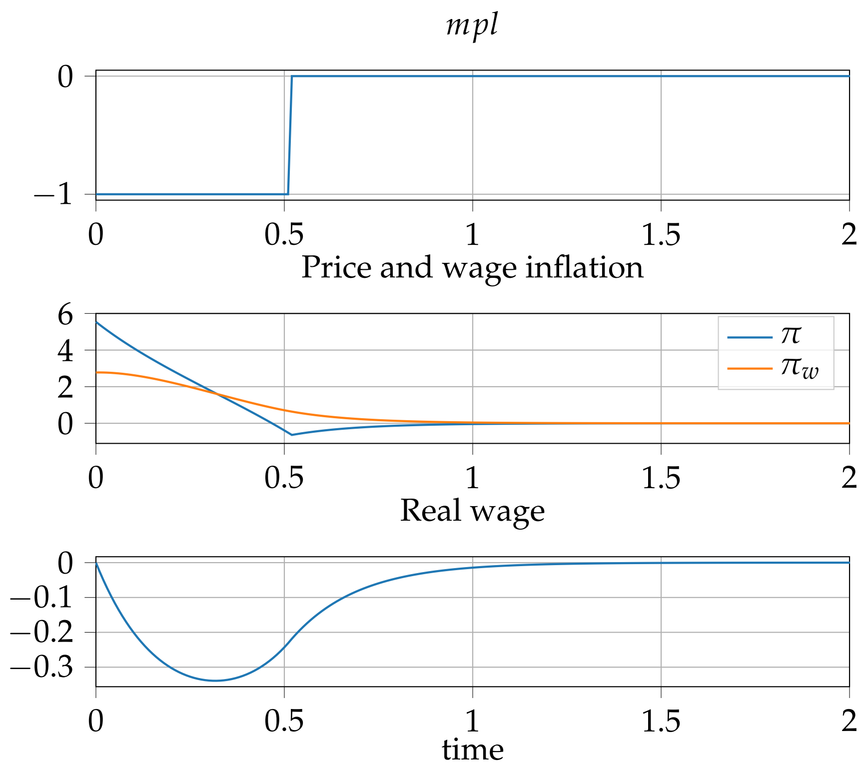

Figure 3: A temporary shift in mpl

The following proposition gives a characterization of the responses of price and wage inflation at $t = 0$. It uses the degree of relative stickiness defined above as $\alpha = \Lambda_p / \left(\Lambda_p + \Lambda_w\right)$ and the coefficient

$$\psi = \frac{r_2}{\delta + r_2} \frac{-r_1}{\rho - r_1}.$$

Proposition 2. *Given exponentially decaying paths for mpl and mrs, the effects on price and wage inflation at $t = 0$ are*

$$\pi_0 > 0 \textit{ iff } mrs_0 > \frac{1 - \alpha\psi}{(1 - \alpha)\,\psi} mpl_0,$$
$$\pi_0^w > 0 \textit{ iff } mrs_0 > \frac{\alpha\psi}{1 - (1 - \alpha)\,\psi} mpl_0,$$

and the effect on the real wage is

$$\dot{\omega}_0 = \pi_0^w - \pi_0 < 0 \textit{ iff } \alpha mpl_0 + (1 - \alpha)\, mrs_0 < 0.$$

The regions identified in the proposition are illustrated in Figure 4. The slope of the

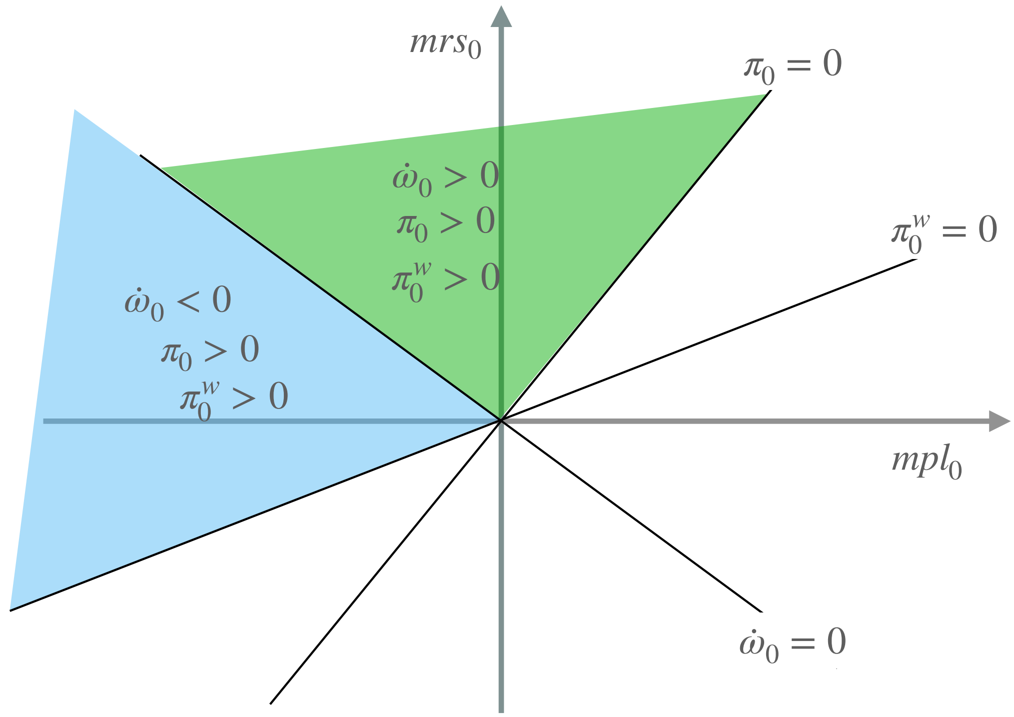

Figure 4: Regions for mpl_0 and mrs_0

boundary of the $\pi_0 > 0$ region is always steeper than that of the $\pi_0^w > 0$ region, because

$$\frac{1-\alpha\psi}{(1-\alpha)\,\psi} > \frac{\alpha\psi}{1-(1-\alpha)\,\psi}.$$

The green and blue regions are those in which the economy features both price and wage inflation. Both $mrs_0 > 0$ and $mpl_0 < 0$ are inflationary forces, and produce inflation as long as one of them is present and strong enough.

In particular, $mrs_0 > 0$ acts directly on workers' wage demands, $mpl_0 < 0$ acts directly on firms' price demands. Both also act indirectly. A high mrs_0, by pushing future real wages up tends to increase expected marginal costs and push up price inflation at $t = 0$. A low mpl_0, by pushing future real wages down, tends to increase wage demands and wage inflation at $t = 0$. The fact that mrs acts directly on wages, while mpl acts directly on prices gives the intuition for why the slope of the $\pi_0 = 0$ line is steeper than that of the $\pi_0^w = 0$ line.

The difference between the green region and the blue region is that in the blue region the real wage declines at $t = 0$ while it increases in the green region. The reason for the difference is the relative strength of the pressure on price setters and wage setters.

We can re-interpret the result above in terms of our decomposition between conflict

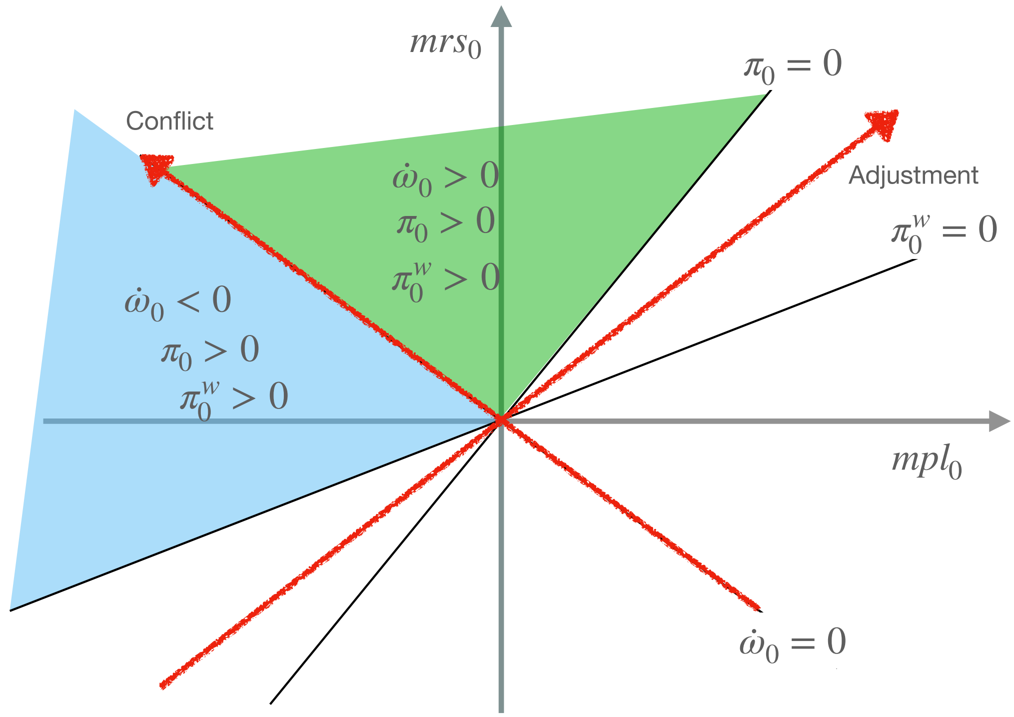

Figure 5: Regions for mpl_0 and mrs_0

and adjustment inflation. Equation (14) immediately implies that with the shocks considered here

$$\Pi_0^C = \frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w} \frac{1}{\rho + \delta} \left(mrs_0 - mpl_0 \right),$$

while Equation (16) implies

$$\Pi_0^A = \dot{\omega}_0 = \frac{\Lambda_p + \Lambda_w}{r_2 + \delta} \left(\alpha mpl_0 + (1 - \alpha) \, mrs_0 \right).$$

Figure 5 reproduces the diagram in Figure 4, adding two axes that represent the conflict and adjustment components. The adjustment axis is simply given by the 45 degree line, given that along that line conflict inflation is zero. Similarly, the conflict axis is given by the pairs that satisfy $\alpha mpl_0 + (1 - \alpha) \, mrs_0 = 0$, so adjustment inflation is zero. Projecting any point (mpl_0, mrs_0) on the two axes, the conflict coordinate gives a value proportional to $mrs_0 - mpl_0$, hence proportional to conflict inflation, while the adjustment coordinate gives a value proportional to $\alpha mpl_0 + (1 - \alpha) \, mrs_0$, hence proportional to adjustment inflation.[7]

[7] The two coordinates are exactly equal to adjustment and conflict inflation if we scale the axes as fol-

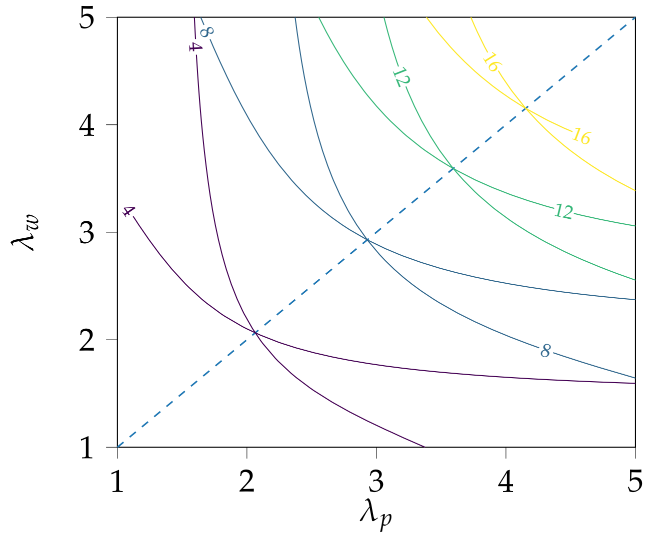

Figure 6: Price and wage inflation contours for different degrees of stickiness

3.5 Stickiness and Amplification

Using the same shocks with exponentially decaying paths, let us turn to a different exercise: fix the size of the initial shocks $mrs_0 > 0$ and $mpl_0 < 0$ and change the economy's parameters to vary the degree by which the shocks get amplified through the wage-price responses. In particular, we change the frequency of adjustment parameters λ_p and λ_w. This exercise speaks directly about the strength of the wage price spiral mechanism.

As we increase the speed at which either prices or wages are reset, the wage price spiral mechanism gets stronger. This is shown in Figure 6, where we plot level curves for π and π_w. The relatively steeper curves (in absolute value) correspond to π, the flatter ones to π_w. A higher frequency of price adjustment λ_p increases both π and π_w, but has a stronger effect on the former. The reverse holds for λ_w. For ease of illustration, we consider an economy hit by a symmetric shock $mrs_0 = -mpl_0$. This implies that when $\lambda_p = \lambda_w$ Proposition 2 gives $\dot{\omega}_0 = 0$ and $\pi_0 = \pi_0^w$. In the figure, the contour levels corresponding to equal price and wage inflation meet on the 45 degree line.

Increasing either price or wage flexibility increases *both* price and wage inflation. This

lows: on the adjustment axis use the unit vector

$$\begin{pmatrix} mpl_0 \\ mrs_0 \end{pmatrix} = \frac{r_2 + \delta}{\Lambda_p + \Lambda_w} \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

and on the conflict axis use the unit vector

$$\begin{pmatrix} mpl_0 \\ mrs_0 \end{pmatrix} = \frac{\Lambda_p + \Lambda_w}{\Lambda_p \Lambda_w} (\rho + \delta) \begin{pmatrix} -(1-\alpha) \\ \alpha \end{pmatrix}.$$

is the total force of the wage price mechanism. At the same time, what happens to the real wage depends on the relative force on the two sides. Increasing λ_p tends to move us to the region below the 45 degree line, where real wages fall. Increasing λ_w has the opposite effect. This is the relative power of the mechanism.

4 Demand and Supply Shocks

We now go back to the full model and trace back price and wage inflation to the general equilibrium effect of underlying shocks. We focus on two shocks, demand and supply.

4.1 A Demand Shock

We start with an expansionary demand shock, driven by easy monetary policy (fiscal policy would have similar implications).

A commonly-held view is that excessive demand works its way from a tight labor market, to higher wages, to higher prices. A demand shock then increases real wages. As we shall show, this is not generally the case. In our model, price and wage rigidities interact with general equilibrium forces on both sides of the labor market and that the direction of adjustment of the real wage is, in general, ambiguous. At a general level, the notion that real wages can potentially fall is obvious and commonly noted in the extreme case where wages are fully rigid: the real wage must fall whenever inflation is positive.[8] Our analysis, in contrast, develops conditions for real wages to fall or rise away from such extreme cases, clarifying the economic forces at play. As we argue, the possibility of a real wage drop is especially relevant when the input X has a high share in costs or a low elasticity of substitution.

Consider a monetary shock that leads to a temporary increase in employment $n_0 > 0$ on impact, the shock decays exponentially at rate δ, so

$$n_t = n_0 e^{-\delta t}.$$

The responses of mpl_t and mrs_t are easily derived from (3) and (6):

$$mpl_t = -\frac{s_X}{\epsilon} e^{-\delta t} n_0 < 0, \quad mrs_t = (\sigma s_L + \eta)\, e^{-\delta t} n_0 > 0.$$

Giving the sign of these responses, the conditions of Proposition 2 immediately show that both price and wage inflation are positive following the shock. What happens to the

[8] See, for example, Figure 6.3 in Gali (2015).

real wage, though, is in general ambiguous. The following is an immediate corollary of Proposition 2.

Proposition 3. *In response to a monetary shock that leads to a transitory increase in employment, real wages fall on impact if and only if*

$$\Lambda_p \frac{s_X}{\epsilon} > \Lambda_w \left(\sigma s_L + \eta \right).$$

The left-hand side of the inequality captures the direct effect on price inflation. This term depends on the effect of higher employment on marginal costs and on stickiness in price setting, captured by Λ_p. The effect of employment on marginal costs is larger when the scarce input X is more important in the production of the final good (higher share s_X) and when the elasticity of substitution between labor and X is lower. The term on the right-hand side captures direct effects on wage inflation. This term depends on the effect on the marginal rate of substitution and on stickiness in wage setting, captured by Λ_w. The effect on the marginal rate of substitution, in turn, depends on an income effect, captured by the term σs_L, since s_L is the elasticity of output to the labor input, and on the inverse Frisch elasticity η.

Overall, if the effect on firms' marginal costs is relatively stronger than the effects on workers' marginal rate of subsitution and if prices are relatively more flexible than wages, we get a reduction in real wages.

In Figure 7 we plot the response to a temporary expansionary shock that increases n above its potential level by 2%, with a decay $\delta = 1$ in a simple numerical example.[9] The parameters used are in the Table 1.

The first panel shows the shock to n. The remaining panels show the responses of different prices.

The input price is flexible, so it jumps on impact and then gradually goes back to its initial level, as the shock goes away. This is shown in the second panel of the figure. Notice that this panel shows the level of the input price, not its rate of inflation. Due to perfect flexibility P_X jumps by 20% at $t = 0$. This large increase is due to our assumption of a low elasticity of substitution between labor and the input X ($\epsilon = 0.1$), so when the employment is growing too fast relative to the supply of X, the price of X reacts strongly.

The effect of the increase in the input price is to increase firm's marginal costs. The impact effect on the nominal marginal cost $w_0 - mpl_0$ is 2%, as the input represents 10% of the cost in steady state ($s_X = 0.1$). This impulse translates into fast inflation on impact,

[9]All plots show log deviations from steady state times 100, or, approximately, percentage deviations from steady state.

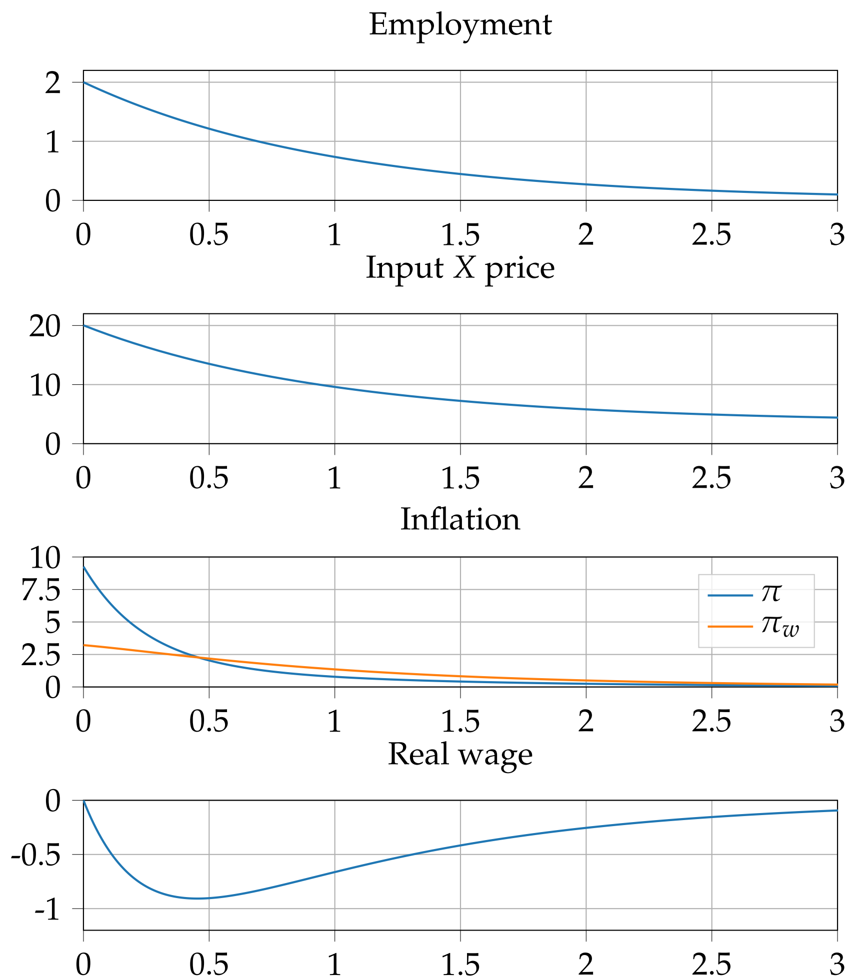

Figure 7: A supply-constrained demand shock

due to our assumption of relatively flexible prices ($\lambda_p = 4$, i.e, prices reset every quarter). This is plotted in the third panel.

Wages respond because high employment translates into high real wage demands. In our simple model with $\eta = 0$, this is only due only to an income effect: as consumption grows, workers need higher wages to be induced to work. For illustration we have chosen parameters such that the impact effect on the nominal marginal cost of labor $p_0 + mrs_0$ is identical to the effect on the marginal cost of goods, both are 2%. However, wages are more sticky ($\lambda_w = 1$), so the effect on wage inflation is weaker. Wage inflation is also plotted in the third panel. The conditions for Proposition 3 are satisfied and the real wage falls on impact, as shown in the fourth panel.

To be clear, this is just a numerical example with numbers chosen for clarity of illustration. Nonetheless, there is clear qualitative feature that we want to highlight: the adjustment happens in three phases.

Preferences	$\sigma = 1$	$\eta = 0$	$\rho = 0.05$
Technology	$s_X = 0.1$	$\epsilon = 0.1,$	
Stickiness	$\lambda_p = 4$	$\lambda_w = 1$	

Table 1: Parameters

1. First, there is a bout of very fast inflation in the sector where the supply constraints are binding, here the market for input X.

2. Second, there is a phase in which price inflation is faster than wage inflation, as price setters react relatively quickly to the increase in input costs.

3. At some point (near $t = 0.5$ in our example) wage inflation crosses price inflation and we enter the third phase in which real wages recover. The input scarcity is going away, so the pressure on firms' marginal costs is weaker, while workers are still trying to catch up to the higher cost of living, given their real wage aspirations.

4.2 A supply shock

Consider now the same economy's response to a supply shock due to a temporary reduction in the supply of the input X. Suppose for now that the central bank responds in such a way as to keep employment constant at its initial steady state level, $n_t = 0$. The responses of mpl and mrs are now

$$mpl_t = \frac{s_X}{\epsilon} e^{-\delta t} x_0 < 0, \quad mrs_t = \sigma s_X e^{-\delta t} x_0 < 0.$$

The main difference is that now the reduction in output reduces workers' mrs, via an income effect. This weakens real wage demands. Given the parameter choices in Table (1), the inflationary forces on the firms' side are still strong enough that we obtain positive wage and price inflation. In the representation of Figure 4 we are in the portion of the blue region that intersects the lower left quadrant. From Proposition 2, we also know that $mpl_0 < 0$ and $mrs_0 < 0$ implies that the real wage falls on impact for any parameter configuration.

The responses are illustrated in Figure 8. For ease of comparison, we pick a negative shock to x_0 that produces the same increase in the input price as the positive n_0 shock in the demand shock exercise of Figure 7.

While nominal wages are growing less and the real wage drop is larger than in Figure 7, there is a common element to the demand and supply shocks just analyzed: the three-phase adjustment discussed above is qualitatively the same.

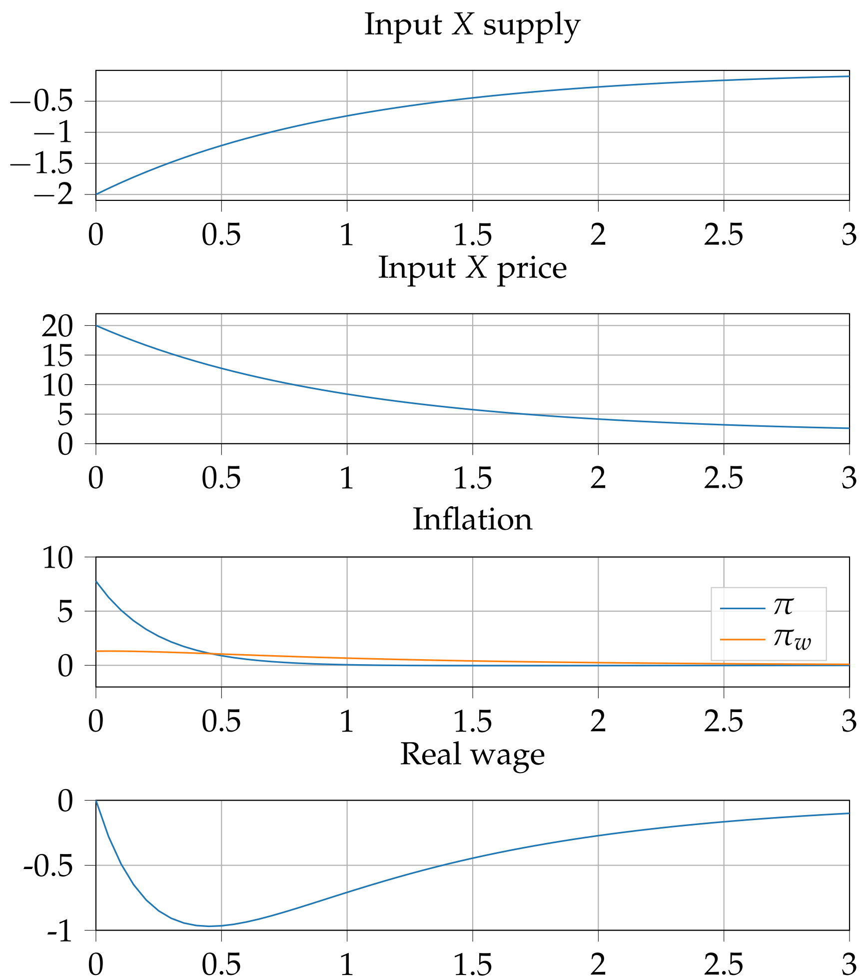

Figure 8: A supply shock

The response to the supply shock depend on how monetary policy adjusts. So far, we assumed a policy that keeps the employment path unchanged. However, the natural level of employment depends in general on x_t. In particular, keeping employment and output at their the natural level requires $mrs_t = mpl_t$, and so n_t^* can be derived from the condition

$$\sigma \left(s_N n_t^* + s_X x_t \right) + \eta n_t^* = \frac{s_X}{\epsilon} \left(x_t - n_t^* \right).$$

The responses of price and wage inflation when

$$n_t = n_t^* = \frac{\frac{1}{\epsilon} - \sigma}{\sigma \left(s_N + \frac{s_X}{\epsilon} \right) + \eta} s_X x_t$$

are plotted in Figure 9. Since our parametrization features a low degree of substitutability between labor and the input X, we have $\frac{1}{\epsilon} - \sigma > 0$ and a reduction in x_t lowers the natural level of employment, as shown in the first panel. The natural level of output

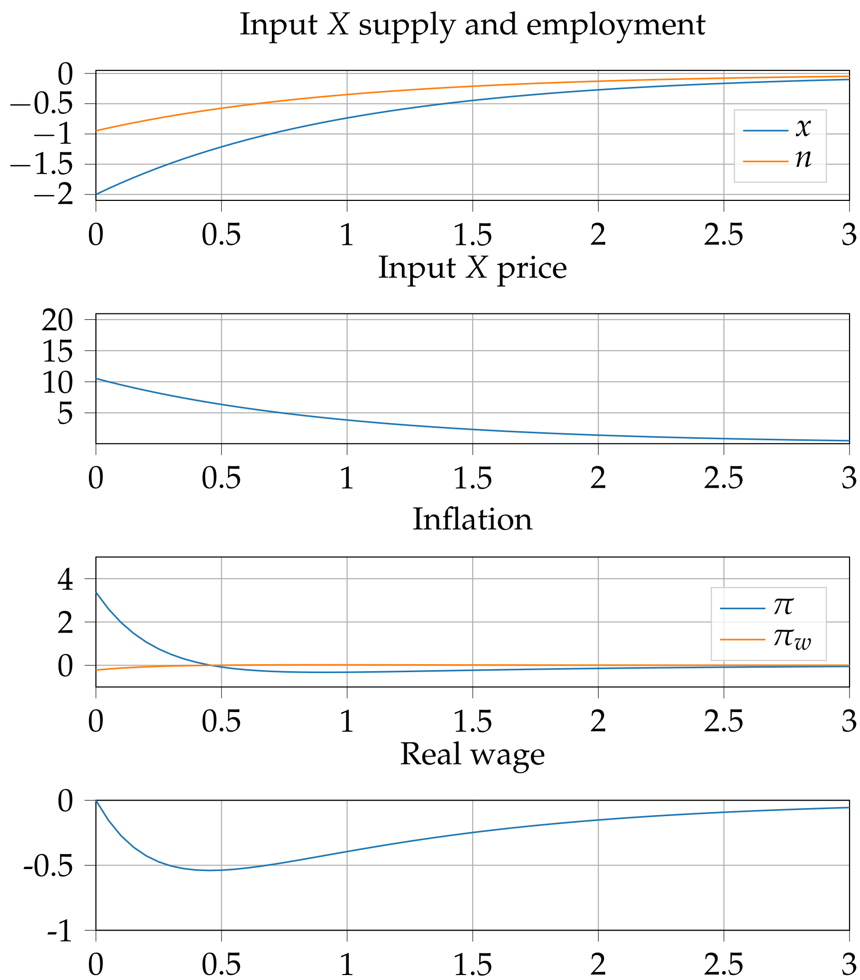

Figure 9: A supply shock with quantities on their natural path

$y_t^* = s_X x_t + s_N n_t^*$ is then lower for two reason, the direct effect of a lower x_t and for the lower level of natural employment. There is a clear difference in the inflation paths when quantities are at their natural levels: we see positive price inflation, but negative wage inflation. This goes on as long as the real wage falls, once the real wage starts growing again, the signs of price and wage inflation flip. In other words, real wage adjustments always take place with nominal prices and wages moving in opposite directions.

This is not just an outcome of our choice of parameters. When quantities are at their natural level we have $mrs_t = mpl_t$ and both are equal, by definition, to the natural real wage ω_t^*. The inflation equations then become

$$\pi_t = \Lambda_p \int_t^\infty e^{-\rho(s-t)} \left(\omega_s - \omega_s^*\right) ds,$$
$$\pi_t^w = \Lambda_w \int_t^\infty e^{-\rho(s-t)} \left(\omega_s^* - \omega_s\right) ds.$$

The following general result follows immediately.

Proposition 4. *If quantities are at their natural level, price and wage inflation π_t and π_t^w are either both zero or have opposite sign.*

This result can be visualized in the diagram of Figure 4, by noticing that the regions where π and π^w have the same sign are either entirely above or entirely below the 45 degree line, where $mrs = mpl$.

Comparing Figures 8 and 9 also shows that while employment falls more at the natural allocation, real wages fall less. This may seem surprising, but it is due to the fact the dynamics of the real wage are more strongly affected by mpl than by mrs, and mpl is higher along the path with lower employment. A different intuition for the same phenomenon is that lower employment reduces the pressure on the market for the scarce input, as seen in the second panel, weakening good inflation due to the high X price and increasing the real wage. Yet another intuition is that due to the fact that prices of goods and non-labor inputs are relatively more flexible than wages, the relation between real wages and employment is dominated by the labor demand side, so higher employment levels require lower real wages.

To summarize the findings of this section, there is a common adjustment pattern, illustrated in Figures 7, 8 and 8, that may be caused either by a positive demand shock or by an insufficient demand contraction in response to a negative supply shock. This adjustment pattern shows both price and wage inflation, with price inflation stronger early on and wage inflation catching up later. If the central bank keeps always the economy at its flexible price allocation this pattern is not present, as price and wage inflation have opposite signs.

However, as it's well known, an economy with both price and wage rigidities does not feature "divine coincidence," so a policy of keeping quantities at their flexible price levels is not necessarily optimal in our environment. In the next section, we turn to optimal policy.

5 Optimal Policy

In the previous section, we looked at economies in which the central bank unnecessarily stimulates the economy (demand shock) or in which the central bank responds weakly to a supply shock, so as to allow for both price and wage inflation (the supply shock with $n_t = 0$). The first example is a policy mistake, by construction. Of course, due to imperfect information and lags in the effects of monetary policy, similar mistakes can happen.

However, in this section, we focus on the second shock, a supply shock, and ask what is the optimal response. Throughout, we assume monetary policy has perfect information on the underlying shocks and instantaneous control on the level of real activity.

The questions we address in this section are two: is it possible that following a supply shock the optimal response is to let the economy overheat, that is, to choose a positive output gap $y_t - y_t^* > 0$? Is it possible that the optimal response entails both positive price and wage inflation?

It is well known that divine coincidence fails in our environment. But that is really just a statement about feasibility: an outcome with no inflationary distortions, $\pi_t = \pi_t^w = 0$, and a zero output gap, $y_t = y_t^*$, are simply not feasible in our economy. The real wage needs to move in the flexible price equilibrium and that is incompatible with zero nominal inflation in p_t and w_t. Our contribution here is to characterize the signs of the deviations of π_t, π_t^w and $y_t - y_t^*$ from zero, under optimal policy.

In particular, Proposition 5 in the previous section tells us that if the central bank chooses $y_t = y_t^*$, then the signs of π_t and π_t^w will always be opposite. In other words, with a zero output gap the adjustment in the real wage never requires *both* price and wage inflation. Therefore, one could conjecture that generalized inflation, that is, inflation in both prices and wages is never optimal. However, a zero output gap is not necessarily optimal so that conjecture is not generally correct.

5.1 Optimal policy problem

Following standard steps, the objective function of the central bank can be derived as a quadratic approximation to the social welfare function:

$$\int_0^\infty e^{-\rho t} \frac{1}{2} \left[-\left(y_t - y_t^*\right)^2 - \Phi_p \pi_t^2 - \Phi_w \left(\pi_t^w\right)^2 \right] dt. \tag{17}$$

Deviations from first-best welfare come from two type of distortions: output deviations from its natural level, that is, from the level that equalizes the marginal benefit of producing goods with its marginal cost in terms of labor effort; and inflation in prices and wages that causes inefficient dispersion in relative prices of different varieties. The terms in (17) reflect these distortions. The value of the coefficients Φ_p and Φ_w depend on the model parameters and are derived and reported in the appendix.

The natural real wage following an X supply shock is

$$\omega_t^* = \frac{s_X}{\epsilon} \frac{\sigma + \eta + (\sigma - 1)\frac{s_X}{\epsilon}}{\sigma\left(s_L + \frac{s_X}{\epsilon}\right) + \eta} x_t.$$

We can then express mpl and mrs in terms of the natural real wage and deviations of employment from its natural path

$$mpl_t = \omega_t^* - \frac{s_X}{\epsilon} \left(n_t - n_t^* \right), \tag{18}$$

$$mrs_t = \omega_t^* + \left(\sigma s_L + \eta \right) \left(n_t - n_t^* \right). \tag{19}$$

The optimal policy problem is to maximize (17), subject to the constraints coming from price setting (9) and (10), condition

$$\dot{\omega}_t = \pi_t^w - \pi_t,$$

and the aggregate production function

$$y_t = s_L n_t + s_X x_t.$$

The optimality conditions that characterize an optimal policy are derived in the appendix.

5.2 Examples

We now consider examples that illustrate a variety of possible outcomes.

It helps the interpretation of the policy trade-offs to focus on the simple case of a permanent shock to x_t. With this shock, in all our examples, in the long run, the real wage is permanently lower and so are mpl and mrs, so that the economy eventually reaches a new steady state with zero inflation and zero output gap. To reach that new steady state requires ω_t to fall. This can be achieved by many combinations of price and wage inflation or deflation, as long as price inflation is larger than wage inflation. The question is what is the optimal way to get there.

Example 1: a symmetric case

Our first example is an economy with parameters that have the following properties:[10]

- the welfare costs of wage and price inflation enter symmetrically the objective function, $\Phi_p = \Phi_w$;

[10]The parameters are as follows:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 1/2$	$\epsilon = 1,$	$\varepsilon_C = 1.5$	$\varepsilon_L = 3$
$\lambda_p = 4$	$\lambda_w = 4$		

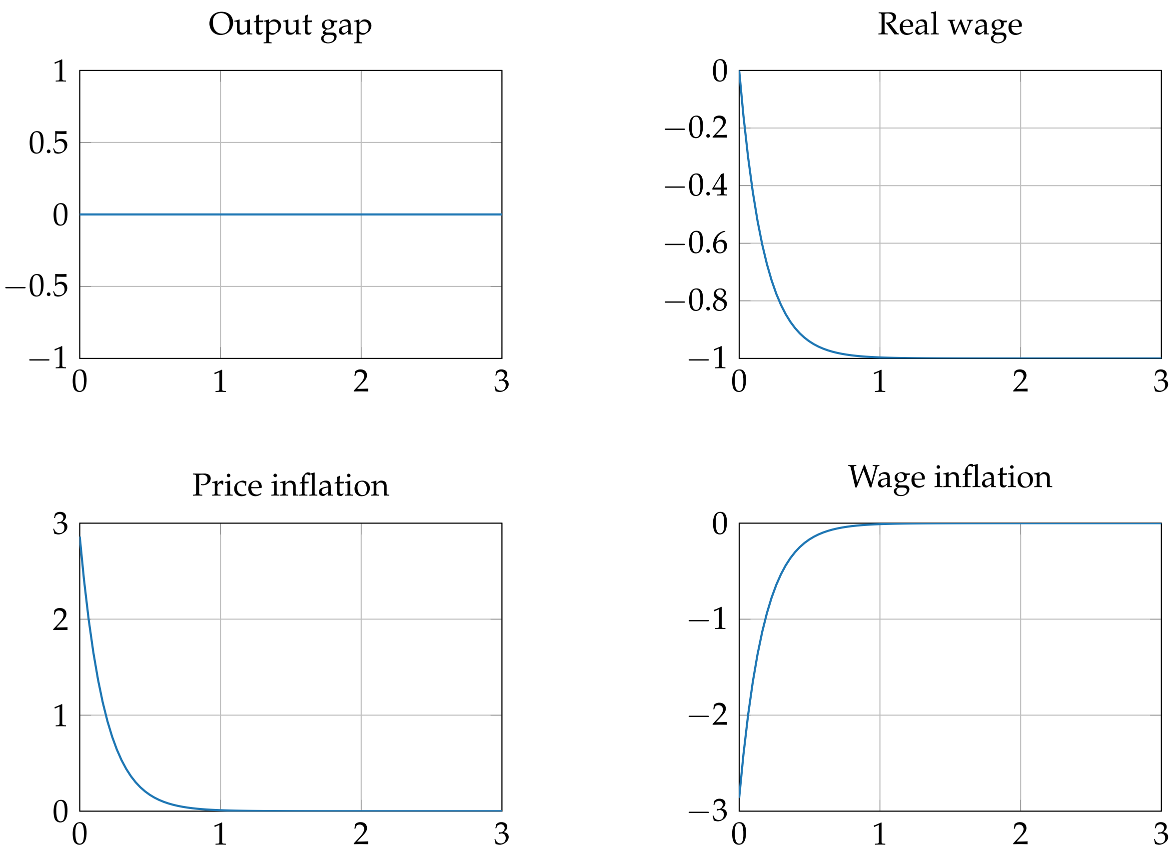

Figure 10: A symmetric example

- wages and prices are equally sticky, $\Lambda_p = \Lambda_w$;
- the output gap has symmetric effects on mpl and mrs.[11]

Figure 10 illustrates optimal policy outcomes in this example. Given the symmetry of the problem, the reduction in real wages is achieved by spreading the adjustment equally between nominal wage deflation and nominal price inflation. The output gap is kept exactly at zero. This example is clearly a knife edge case and relies on the symmetry of the parameters. As soon as we abandon this symmetry things get more interesting.

Example 2: a hot economy

In the second example, the parameters chosen imply that:[12]

[11]Given the expressions above this requires $\frac{s_X}{\epsilon} = \sigma s_L + \eta$.
[12]The parameters are as follows:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 0.1$	$\epsilon = 1,$	$\varepsilon_C = 1.5$	$\varepsilon_L = 4$
$\lambda_p = 4$	$\lambda_w = 2$		

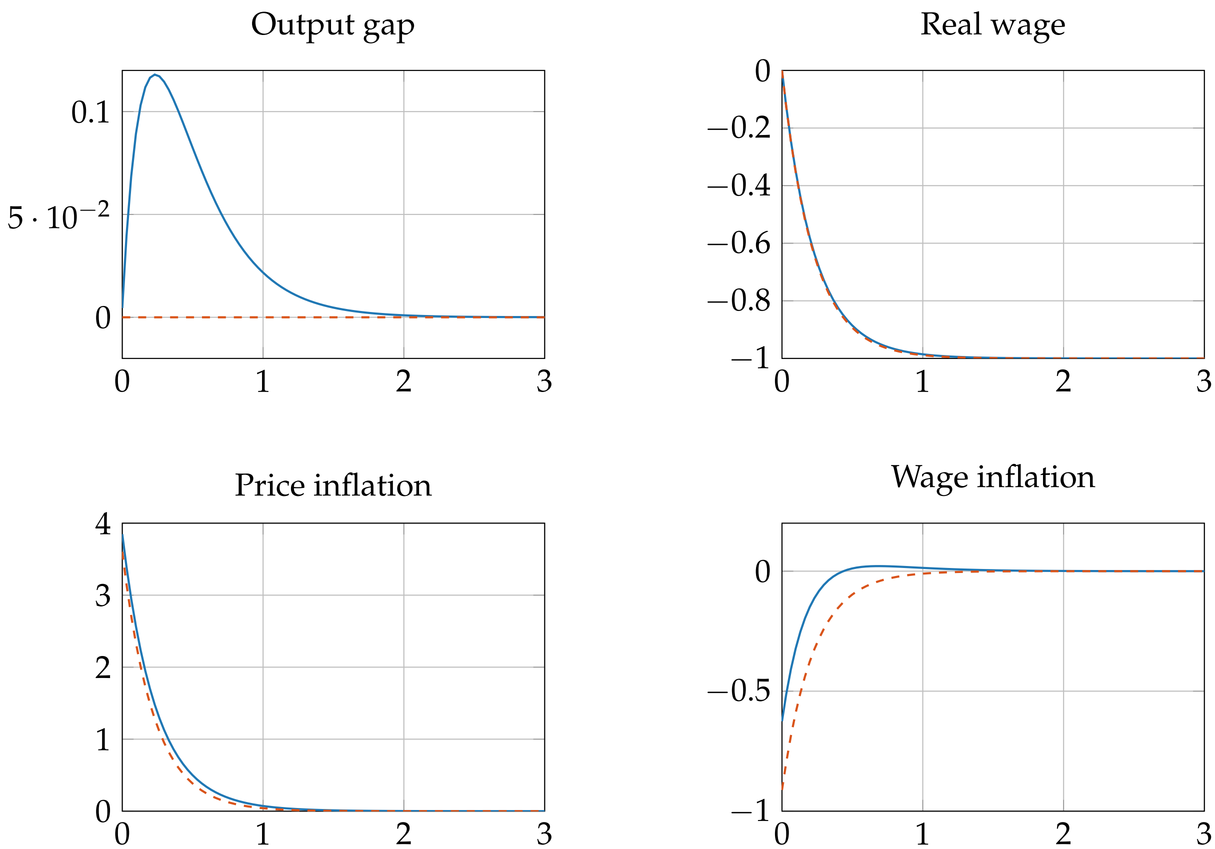

Figure 11: An optimal hot economy

- the welfare cost of wage inflation is larger than that of price inflation, $\Phi_p < \Phi_w$;
- wages are more sticky than prices, $\Lambda_p > \Lambda_w$.

We still have a set of parameters that imply roughly symmetric effects of the output gap on *mpl* and *mrs*, but the differences above are sufficient to obtain a quite different result. Figure 11 illustrates optimal policy outcomes in this case. For comparison, in the figure we also plot outcomes under a zero output gap policy (red dashed lines).

In this second example, it is optimal to have a positive output gap throughout the transition. To get some intuition for this result it is useful to recall from equations (9)-(10) and (18)-(19) that increasing the output gap has two direct effects. By decreasing *mpl* it leads to higher price inflation, by increasing *mrs* it leads to higher wage inflation. If we start at a zero-output-gap policy, with positive price inflation and negative wage inflation, the effect can be welfare improving because the welfare cost of price inflation is smaller than the welfare cost of wage deflation.

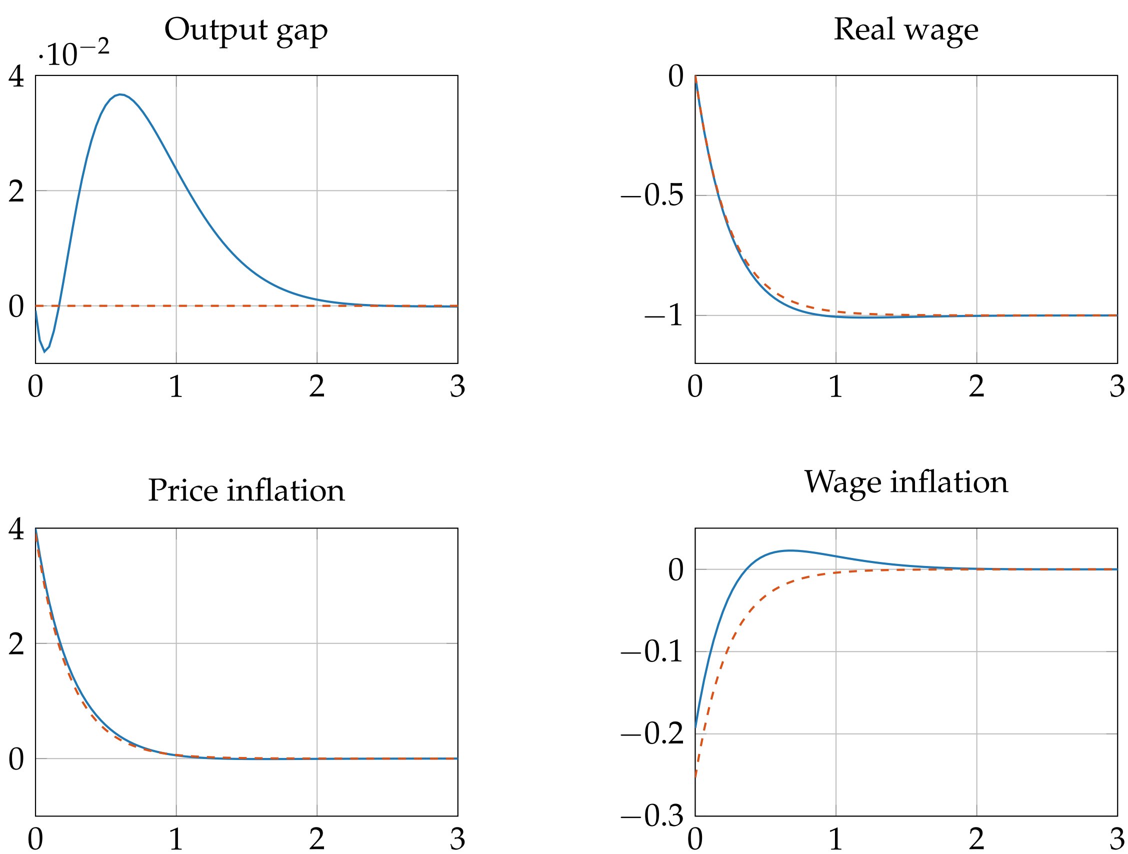

Figure 12: An example with generalized inflation and a hot economy

The role of $\Lambda_p > \Lambda_w$ is subtler and has to do with dynamics. With $\Lambda_p > \Lambda_w$ and $\xi_p \approx \xi_w$ a higher output gap also implies a faster declining real wage. Since a lower real wage in the future requires less adjustment, lowering the real wage today is welfare improving from a dynamic point of view. Therefore, a parametrization with $\Lambda_p > \Lambda_w$ makes it easier to obain examples with a welfare improving positive output gap.[13]

By choosing parameters that yield the opposite inequality, $\Phi_p > \Phi_w$, in the welfare coefficients it is possible to construct examples of the opposite: economies in which it is optimal to run a negative output gap in the transition.

Example 3: Generalized inflation and a hot economy

Our third example is a variant on the second example, with an even larger welfare cost associated to wage dispersion (a larger Φ_w), a larger distance between price and wage stickiness, and with a smaller value of the elasticity of substitution between labor and the X input, ϵ, which implies that running a hot economy has larger benefits in terms

[13] The discussion of Figure X in the Appendix expands on this argument.

of lowering the real wage by having a larger effect on firms' marginal costs and thus on price inflation.[14]

The parametric choices above amplify the forces we saw in example 2 and they imply that there is an interval during the transition in which the optimal policy yields both a hot economy ($y_t > y_t^*$) and generalized price and wage inflation ($\pi_t > 0$ and $\pi_t^w > 0$).[15]

This result is surprising from a static point of view. Given the welfare function (17), at any point in time in which $y_t > y_t^*, \pi_t > 0$ and $\pi_t^w > 0$ it is welfare improving, from a static point of view, to reduce y_t, as it unambiguously lowers π_t and π_t^w and leads to an increase in the current payoff. However, from a dynamic perspectives there is an additional argument. Increasing y_t at time t has the effect of increasing π_s and π_s^w in all previous periods, due to the forward looking element in price setting. This entails welfare gains in early periods in the transition in which $\pi_s^w < 0$. Through this forward looking force a positive output gap later in the transition can be beneficial even if, at that point $\pi_t^w > 0$.

Now, while this example is theoretically interesting, it does have the flavor of a overly sophisticated form of forward guidance. Therefore, we do not think it provides a strong argument in favor of policies that deliver $y_t > y_t^*, \pi_t > 0$ and $\pi_t^w > 0$ at the same time. In the context of the present model, given the distortions it captures, it is hard to make a compelling practical case that the combination of a hot economy with positive wage and price inflation are a desirable outcome, even in response to a supply shock and even in presence of inelastic supply constraints.[16]

6 Adaptive Expectations and Real Rigidities

The model with rational expectations analyzed so far has two embedded features: the effect of any shock tends to be front-loaded, as agents perfectly anticipate its future effects on prices, and there is no room for persistent deviations of inflation expectations from target, as agents anticipate the economy will go back to its initial steady state. We now explore variants of the model that deviate from rational expectations and allow for more

[14]The parameters are as follows:

$\sigma = 1$	$\eta = 0$	$\rho = 0.05$	
$s_X = 0.1$	$\epsilon = 0.1,$	$\varepsilon_C = 1.5$	$\varepsilon_L = 8$
$\lambda_p = 4$	$\lambda_w = 1$		

[15]Notice, that these qualitative features can actually be seen in example 2 too, but it is useful to choose an example where they are more clearly visible.

[16]This does not mean that such a case could not maybe be made in richer models, which capture, just to make an example, the benefits of labor reallocation. But that is clearly outside the scope of this paper.

inertial responses by introducing two ingredients: adaptive expectations on expected inflation and a gradual adjustment of price-setters' and wage-setters' relative price objectives. For this second ingredient we use the label "real rigidities."

The objective of this sections is twofold. First, by allowing for inertial responses we allow the feedback between price and wages to play out more explicitly over time: shocks that produce high prices in the goods market only gradually lead to higher wage demands in the labor market. In other words, the wage-price spiral instead of playing out in the "virtual time" of best responses, plays out in the observed dynamics of prices and wages. Second, by allowing for deviations of inflation expectations from target we capture the common concern of central bankers that prolonged episodes of high inflation may lead to de-anchoring of inflation expectations.

From an empirical perspective, we show that adaptive expectations and inertia reinforce the main prediction of the baseline model in Section 4: there is a lagged and persistent increase in wage inflation following a large increase in price inflation. However, the medium term implications are different depending on the sources of inertia: if inertia is mostly due to de-anchoring, inflation can take a long time to go back to target, absent a recession, if instead inertia is mostly due to real rigidities, then a path of immaculate disinflation is possible.

Let us begin by rewriting the price setting conditions making explicit agents' expectations. Letting E_t^f and E_t^w denote firms' and workers' expectations, we can write

$$\begin{aligned} p_t^* &= \left(\rho+\lambda_p\right) E_t^f \int_t^\infty e^{-\left(\rho+\lambda_p\right)(\tau-t)} \left(w_\tau + s_X \left(p_{X\tau} - w_\tau\right)\right) d\tau = \\ &= w_t + \left(\rho+\lambda_p\right) E_t^f \int_t^\infty e^{-\left(\rho+\lambda_p\right)(\tau-t)} s_X \left(p_{X\tau} - w_\tau\right) d\tau + E_t^f \int_t^\infty e^{-\left(\rho+\lambda_p\right)(\tau-t)} \dot{w}_\tau d\tau. \end{aligned}$$

Reset prices are decomposed in three components: the current nominal wage, the expected path of the relative price of input X vs labor, the expected path of future wage inflation.

We assume that agents expect a constant inflation rate over the future horizon

$$E_t^f \dot{w}_t = \pi_t^{w,e},$$

and expected inflation is driven by the simple adaptive, constant-gain rule

$$\dot{\pi}_t^{w,e} = \gamma \left(\dot{w}_t - \pi_t^{w,e}\right). \tag{20}$$

Moreover, we assume that agents perfectly anticipate the path of real variables n_t, x_t, y_t

Inflation

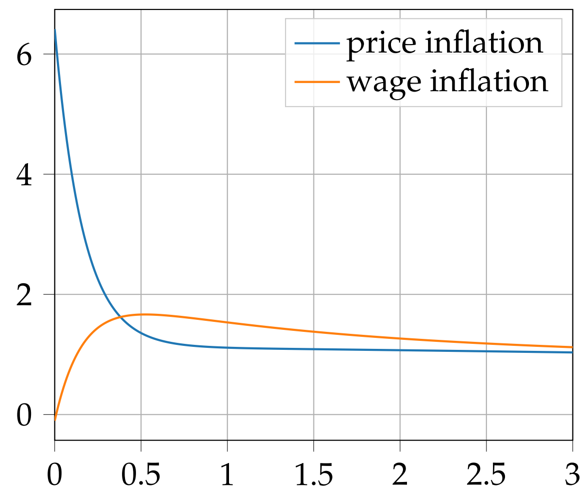

Figure 13: A supply shock with adaptive expectations

and can deduce the path of the relative price $p_{Xt} - w_t$ from the equilibrium condition in factor markets

$$x_t - n_t = -\epsilon \left(p_{Xt} - w_t \right).$$

Combining these assumptions with exponentially decaying, one time shocks at date 0, as in Section 4, we can substitute in the expression above for p_t^*, substitute in the inflation equation (7), and obtain the following

$$\dot{p}_t = \lambda_p \left[\frac{s_X}{\epsilon} \frac{\rho + \lambda_p}{\rho + \lambda_p + \delta} \left(n_t - x_t \right) - \left(p_t - w_t \right) \right] + \frac{\lambda_p}{\rho + \lambda_p} \pi_t^{w,e}. \tag{21}$$

Similar steps on the wage setting side of the model lead to

$$\dot{w}_t = \lambda_w \left[\frac{\rho + \lambda_w}{\rho + \lambda_w + \delta} \left(\sigma y_t + \eta n_t \right) - \left(w_t - p_t \right) \right] + \frac{\lambda_w}{\rho + \lambda_w} \pi_t^e, \tag{22}$$

where price inflation follows the adaptive rule

$$\dot{\pi}_t^e = \gamma \left(\dot{p}_t - \pi_t^e \right). \tag{23}$$

Equations (20)-(23) can be solved forward for any given initial condition w_0, p_0.

An example of de-anchoring

Figure 13 shows the response of inflation to a supply shock in a numerical example analogous to the one shown in Figure 8, except for the assumption of adaptive expectations. The parameters are the same as in Table 1 and we set $\gamma = 1$. There are two main differences from the case of rational expectations. First, wage inflation is weaker on impact and only picks up gradually, as initally workers do not anticipate higher prices and so do not start trying to catch up until their purchasing power has actually been eroded by past inflation.[17] Second, there is a very persistent effect on inflation, due to the learning dynamics. Since ρ is small, the coefficients on the expected inflation terms on the right-hand side of equations (21)-(22) are close to 1. This implies that even though all quantities and all relative price targets for workers and firms have gone back to steady state, we can have a prolonger period of self-sustaining inflation. This is a case of de-anchoring, in which the only way to go back to target inflation faster is for the central bank to keep activity low for some time.

The wage-price spiral is active in the self-sustaining phase of prolonged inflation, but it is exactly balanced on the two sides, so real wages remain constant.

An example with real rigidities

We now consider a different source of inertia, due to a gradual adjustment of the relative price targets of price and wage setters. In particular, we assume that changes in real marginal costs and in the marginal rate of substitution between consumption and leisure only gradually change the behavior of price and wage setters. We replace the inflation dynamics above, (21)-(22) with the following equations

$$\dot{p}_t = \lambda_p \left[a_t^p - (p_t - w_t)\right] + \frac{\lambda_p}{\rho + \lambda_p} \pi_t^{w,e},$$

$$\dot{w}_t = \lambda_w \left[a_t^w - (w_t - p_t)\right] + \frac{\lambda_w}{\rho + \lambda_w} \pi_t^{e}.$$

The real aspirations of price setters and wage setters, a_t^p and a_t^w, follow the adjustment equations

$$\dot{a}_t^p = \xi_p \left[\frac{s_X}{\epsilon} \frac{\rho + \lambda_p}{\rho + \lambda_p + \delta} (n_t - x_t) - a_t^p\right],$$

[17]Notice that given that n is kept on its pre-shock path ($n = 0$) and that output falls due to the supply shock ($y_0 = s_X x_0 < 0$), there is an income effect that depresses the real wage demands of workers on impact, causing a very small initial nominal wage deflation, which is barely visible in the figure.

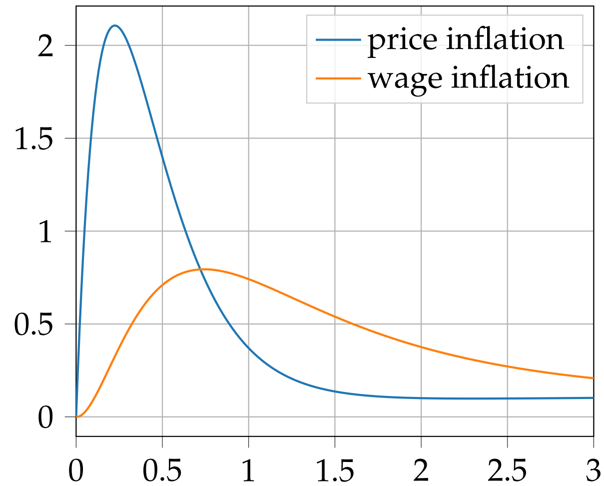

Figure 14: A supply shock with adaptive expectations

and

$$\dot{a}_t^w = \xi_w \left[\frac{\rho + \lambda_w}{\rho + \lambda_w + \delta} \left(\sigma y_t + \eta n_t \right) - a_t^w \right].$$

Aspirations are driven by the same forces that drive them in the baseline model, which, in the case of firms are anticipated real input prices, captured by the term $\frac{s_X}{\epsilon} \frac{\rho+\lambda_p}{\rho+\lambda_p+\delta} \left(n_t - x_t \right)$, and in the case of workers are anticipated marginal rates of substitution between consumption and leisure, captured by $\frac{\rho+\lambda_w}{\rho+\lambda_w+\delta} \left(\sigma y_t + \eta n_t \right)$. However, these forces only gradually modify the aspirations of firms in terms of the desired margins ($p_t - w_t$ for the firms and $w_t - p_t$ for the workers).

We assume that the inflation expectations $\pi_t^{w,e}$ and π_t^e still follow the learning processes (20) and (23), so this version of the model includes both inertia caused by slow adjustment of inflation expectations and inertia caused by real rigidities. The choice to combine the two is because an interpretation of the real rigidities here is also some form of bounded rationality in processing observed changes in input prices and changes in labor market conditions, and combining that with perfect foresight on future price paths seems less natural. However, to focus on the role of real rigidites we choose a parametrization with a lower $\gamma = 0.1$, relative to the parametrization used for Figure 13, so inflation expectations play a more limited role. For the parameters ξ_p and ξ_w we experiment with values equal to 4 and 1, so the degree of real rigidity in the goods and labor market mirror the degree of nominal rigidity (capture by λ_p and λ_w). The inflation responses to the same supply shock used above are reported in Figure 14.

In this economy, both price and wage inflation display hump-shaped responses and the wage response is more delayed and more persistent than in the rational expectations baseline. The delay in the wage response is essentially due to the same reason as in model with only adaptive inflation expectations: wage setters only start to demand higher nominal wages when price inflation has been going on for a while and has moved real wages away from their aspirations. The additional delay here is due to the fact that prices also take longer to respond, due to the real rigidity in price setting.[18]

The example in Figure 14 comes closest to capture an immaculate inflation-disinflation scenario. The shock causes peristent responses of prices and wages. The persistence is purely due to the fact that price setters takes some time to respond and wage inflation follows with further delay because wage setters only start responding after price setters have increased the price level enough to lower $w - p$. The persistence of wage inflation in this scenario is not a symptom of persistent overheating in the labor market, but of a gradual return to pre-shock trends for the real wage.

7 Conclusions

We explored the wage price spiral in a canonical model of price and wage setting.

Interpreting inflation as the outcome of inconsistent aspirations for the real wage (or other relative prices) opens the door to many theoretical and empirical questions. We are especially interested in extending our work to explore potential sources of inertia in the inflation process, expanding the models explored in Section 6.

In the model analyzed here there is an instantaneous connection between the output gap and the real wage aspirations of workers' and firms. However, it is plausible that workers' real wage aspirations respond gradually to changes in labor market conditions. Similarly, changes in goods market conditions could affect slowly firms' expected profit margins. These are sources of inertia in inflation that come from agents' views on relative prices, and so are different from sources of inertia tied to future inflation expectations, on which most research has focused on. Even if inflation expectations are well anchored it is possible for inflation to persist if the disagreement between firms and workers is inertial. On the empirical front, while there is a large literature measuring inflation expectations, there has been limited effort so far at measuring workers' and firms' aspirations for real pay and for real profit margins.

[18] The real rigidity in wage setting does not really play an important role in this simulation, because with a pure supply shock to x the effect on $\sigma y + \eta n$ is very small, so workers' aspirations are essentially constant at 0. In line with this observation, simulations with larger and smaller values of ζ_w produce responses very similar to those in Figure 14. Of course, in the case of other shocks this is no longer the case.

Appendix

A Derivation of equations (9) and (10)

Differentiate both sides of (4) and (7) with respect to time to get

$$\dot{p}_t^* = -\left(\rho + \lambda_p\right)\left(w_t - mpl_t\right) + \left(\rho + \lambda_p\right) p_t^*,$$

and

$$\ddot{p}_t = \lambda_p \left(\dot{p}_t^* - \dot{p}_t\right).$$

Substituting $\dot{p}_t^*$ from the first equation on the right-hand side of the second equation and changing notation for inflation, yields

$$\dot{\pi}_t = \lambda_p \left(-\left(\rho + \lambda_p\right)\left(w_t - p_t - mpl_t\right) + \left(\rho + \lambda_p\right)\left(p_t^* - p_t\right) - \pi_t\right).$$

Using $\lambda_p \left(p_t^* - p_t\right) = \pi_t$ and rearranging gives

$$\dot{\pi}_t = -\lambda_p \left(\rho + \lambda_p\right)\left(w_t - p_t - mpl_t\right) + \rho \pi_t,$$

which correponds to (9). Equation (10) is derived in a similar way.

B Proof of Proposition 1

Consider the second order non-autonomous ODE

$$\ddot{\omega}_t - \rho\dot{\omega}_t - \Lambda\omega_t + \Lambda\tilde{\omega}_t = 0,$$

where

$$\Lambda = \Lambda_p + \Lambda_w.$$

Since $\Lambda > 0$ there are two real eigenvalues r_1, r_2 that solve

$$r^2 - \rho r - \Lambda = 0,$$

or, equivalently, that satisfy $r_1 + r_2 = \rho$ and $r_1 r_2 = -\Lambda$. Then the ODE can be written as

$$\left(\partial - r_1\right)\left(\partial - r_2\right)\omega_t = -\Lambda\tilde{\omega}_t$$

where ∂ is the time-derivative operator. Integrating forward gives

$$\left(\partial - r_1\right)\omega_t = -\frac{1}{\partial - r_2}\Lambda\tilde{\omega}_t = \Lambda \int_t^\infty e^{-r_2(\tau - t)}\tilde{\omega}_\tau d\tau,$$

which gives (16). Integrating backward gives

$$\omega_t = e^{r_1 t}\omega_0 + \Lambda \int_0^t e^{r_1(t-s)} \int_s^\infty e^{-r_2(\tau - s)}\tilde{\omega}_\tau d\tau ds. \tag{24}$$

Changing the order of integration, the double integral on the right-hand side becomes

$$\int_0^t \int_0^\tau e^{r_1(t-s)} e^{-r_2(\tau-s)} \tilde{\omega}_\tau ds d\tau + \int_t^\infty \int_0^t e^{r_1(t-s)} e^{-r_2(\tau-s)} \tilde{\omega}_\tau ds d\tau$$

which gives

$$\omega_t = e^{r_1 t}\omega_0 + \Lambda \int_0^t \frac{e^{r_1(t-\tau)} - e^{r_1 t - r_2\tau}}{r_2 - r_1} \tilde{\omega}_\tau d\tau + \Lambda \int_t^\infty \frac{e^{-r_2(\tau-t)} - e^{r_1 t} e^{-r_2\tau}}{r_2 - r_1} \tilde{\omega}_\tau d\tau.$$

For computations, this can also be written compactly as

$$\omega_t = e^{r_1 t}\omega_0 + \int_0^\infty H_{\tau,t} \tilde{\omega}_\tau d\tau,$$

where $H_{\tau,t}$ is defined as

$$H_{\tau,t} = \frac{\Lambda}{r_2 - r_1} \left(e^{\min\{r_1(t-\tau), -r_2(\tau-t)\}} - e^{r_1 t - r_2 \tau} \right).$$

C General Result for One-side Changes in *mrs* and *mpl* 2

The following result focuses on the effects of shocks that exclusively affect the labor demand side or the labor supply side of the model, in the sense that they perturb mpl_t without affecting mrs_t, or, vice versa.

Proposition 5. *Suppose there is no change in $mrs_t = 0$ and the path for mpl_t is negative for all $t \in [0, \infty)$. Then the impact responses at $t = 0$ are*

$$\pi_0 > \pi_0^w > 0.$$

Suppose there is no change in $mpl_t = 0$ and the path for mrs_t is positive for all $t \in [0, \infty)$. Then the impact responses at $t = 0$ are

$$\pi_0^w > \pi_0 > 0.$$

D Proof of Proposition 2

We first derive the real wage path using (24) in the proof of Proposition 1. Solving the integrals gives

$$\begin{aligned}\omega_t =& \Lambda \int_0^t e^{r_1(t-s)} \int_s^\infty e^{-r_2(\tau-s)} e^{-\delta\tau} d\tau ds = \Lambda \frac{1}{r_2+\delta} \int_0^t e^{r_1(t-s)-\delta s} ds = \\ =& \frac{e^{r_1 t} - e^{-\delta t}}{(r_2+\delta)(r_1+\delta)} \left(\Lambda_p mpl_0 + \Lambda_w mrs_0 \right).\end{aligned}$$

Write price inflation as

$$\pi_t = \int_t^\infty e^{-\rho(\tau-t)} \left(\omega_\tau - mpl_\tau\right) d\tau,$$

substituting ω_t and integrating gives

$$\pi_t = \frac{1}{r_1+\delta}\frac{1}{r_2+\delta}\left(\frac{e^{r_1 t}}{\rho - r_1} - \frac{e^{-\delta t}}{\delta+\rho}\right)\left[\Lambda_p mpl_0 + \Lambda_w mrs_0\right] - \frac{e^{-\delta t}}{\rho+\delta} mpl_0.$$

We then get that $\pi_t > 0$ if and only if

$$\frac{1}{r_1+\delta}\frac{1}{r_2+\delta}\left(\frac{e^{r_1 t}}{\rho - r_1} - \frac{e^{-\delta t}}{\delta+\rho}\right)\left[\Lambda_p mpl_0 + \Lambda_w mrs_0\right] > \frac{e^{-\delta t}}{\rho+\delta} mpl_0,$$

which can be rewritten using $-r_1 r_2 = \Lambda_p + \Lambda_w$ (from the proof of Proposition (1)), to get

$$\frac{r_2}{r_2+\delta}\frac{-r_1}{r_1+\delta}\left(\frac{e^{r_1 t}}{\rho - r_1} - \frac{e^{-\delta t}}{\delta+\rho}\right)\frac{\Lambda_p mpl_0 + \Lambda_w mrs_0}{\Lambda_p + \Lambda_w} > \frac{e^{-\delta t}}{\rho+\delta} mpl_0.$$

Setting $t = 0$ and rearranging gives the condition for $\pi_0 > 0$ in the statement of the proposition.

Write wage inflation as

$$\pi_t^w = \int_t^\infty e^{-\rho(\tau-t)} \left(mrs_\tau - \omega_\tau\right) d\tau.$$

Similar steps as those above yield the following condition for $\pi_t^w > 0$

$$\frac{r_2}{r_2+\delta}\frac{-r_1}{r_1+\delta}\left(\frac{e^{r_1 t}}{\rho - r_1} - \frac{e^{-\delta t}}{\delta+\rho}\right)\frac{\Lambda_p mpl_0 + \Lambda_w mrs_0}{\Lambda_p + \Lambda_w} < \frac{e^{-\delta t}}{\rho+\delta} mrs_0.$$

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