Chapter 4. Public Debt Sustainability[1]

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1. Introduction

The question as to whether public debt is sustainable (or not) is a central consideration in any macroeconomic analysis of fiscal policy. And that question may be even more relevant today than ever before. With central banks more likely to test the limits of the effective lower bound on nominal interest rates, the existence of "fiscal space" (and the corresponding ability to use fiscal policy without jeopardizing access to financing) has taken center stage in policy discussions. At the same time, rising public debts in many advanced and emerging economies, and repeated bouts of stress in certain sovereign markets previously thought to be safe have been powerful reminders that sustainability can never be taken for granted. Of course, it bears keeping in mind that because the intertemporal budget constraint always holds ex-post, there are many ways to ultimately ensure it, and they do not always involve clear-cut approaches such as defaults or high inflation. Hence, the notion of (ex-ante) sustainability is not uniquely defined because it depends on which strategy we deem acceptable to satisfy the intertemporal budget constraint.[2]

While theory generally equates debt sustainability with public sector solvency, practitioners have been struggling to give an operational meaning to the concept. There are many obvious difficulties. First, while solvency is precisely defined—existing obligations cannot exceed the present value of all future primary balances—it escapes any easy assessment because it is inherently forward looking; it is a prediction, an informed judgment on a known unknown. Second, standard macroeconomic analysis operates under the presumption that the government is committed to always repay its obligation in full. It seems clear, however, that the benefits of default may in some cases exceed the costs, at least ex-ante, putting into question the credibility of such commitments. Third, the very risk of default brings market beliefs into the picture, and

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[2] For example: is financial repression acceptable? And if so, up to what point?

with it, the issue of multiple equilibria and self-fulfilling prophecies. Hence, mere liquidity crises triggered by senseless panic can lead otherwise solvent governments to default.

Such complexities can easily explain the apparent puzzle of seeing countries like Japan defy gravity with gross public debt levels above 200 percent of GDP, while others default on a considerably smaller stock of obligations (30 percent of GDP in Ukraine). The aim of this chapter is to provide a selective overview of the knowns and unknowns of debt sustainability, including the range of tools we have at our disposal to understand vulnerabilities and inform what will always remain a difficult judgment call. Important considerations, such as the motivations and effects of borrowing in foreign currency or the need for a balance sheet view of public sector solvency, are out of scope and will only be mentioned in passing. Instead, the chapter focuses on the classic nexus between fiscal policy and gross public debt dynamics.

The chapter is structured as follows. Section 2 will clarify the basic concepts, including the deterministic arithmetic of the government's budget constraint, the notion of solvency, and the usual definitions of debt sustainability. The main practical tools used to inform sustainability assessments will be discussed in Section 3, including the logic behind the IMF's DSA framework, and market information and indicators (bond and CDS spreads and sovereign ratings). As the risk of default is not zero, the fourth section will review models of debt limits explaining why a government may find itself to be either unable or unwilling to face its obligations when debt exceeds certain levels. The chapter will focus on the analytics and practicalities for countries with market access.

2. Concepts and Theory

This section reviews the conceptual foundations of public debt sustainability analysis. It first describes the government's budget constraint and defines solvency. Because the latter boils down to a pure prediction about future fiscal policy over an indefinite horizon, solvency has no immediate operational implication, notably in terms of well-defined and intuitive patterns in fiscal data. The concrete approaches used in the literature to assess solvency from hard data generally capture properties or conditions that are largely sufficient (but not necessary) to ensure solvency. These conditions fall under the label "debt sustainability."

2.1. The Government Budget Constraint and Solvency

Because resource scarcity lies at the core any meaningful economic analysis, budget constraints concern all economic agents, including the government. The idea is simply that nobody can have the cake and eat it. Of course, this does not have to be the case every period. In modern economies, financial markets and banks allow some to spend more than their income, but only if others, in the domestic economy or elsewhere, spend less than theirs. The level of interest rates is expected to balance the demand and supply of funds.

For such a system to work, any debt contracted by an agent in deficit must be considered as an asset (wealth) by the agent in surplus. Hence debt contracts must ultimately be honored.

Expectations of default dry up funds available for borrowers and borrowing costs explode to offset the risk taken by the lender. Again, this applies to all consumers, firms and to the government as well.

Of course, the government is a special borrower on several counts. First, it is usually not expected to die or disappear any time soon so that there is no obvious end-period where all debts should be repaid. Second, default by the government is a particularly scary prospect, if only because the size of the entity typically entails a considerable destruction of wealth, a collapse in national income, and guaranteed misery for those who cannot insure against such risks, usually the less affluent in society (Borensztein and Panizza, 2009). Think only of the pension funds holding government securities on their books or the financial institutions using these bonds as "safe" collateral. Third, a government is sovereign. Concretely, this means that (i) there is no well-defined bankruptcy procedure giving lenders any claim on its assets, (ii) that it can create fiat money to face its obligations, at least those denominated in domestic currency, and (iii) that it can also raise vast revenues at discretion by hiking taxes—at least up to the point when tax rates become so toxic for the economy that revenues ultimately fall (the dreaded Laffer curve).

Because governments are such special borrowers, their budget constraint is not immediately obvious and certainly less easily understandable than for household Lambda or firm Kappa. In particular, the budget constraint applies over an infinite horizon, asset disposal is usually not a viable option, and expedients (default or inflationary money printing) are often too costly to contemplate ex-ante, although they remain options ex-post. Given this, it is worth asking what makes a government solvent, i.e. able to honor its financial obligations in full.[3] Some minimal arithmetic is required to fix ideas.

In any given period t, total government spending must be covered by revenues and bond issuance. To keep the notation as simple as possible, we assume, as is commonly done in the literature, that public debt takes the form of a one-period bond. The entire stock of inherited debt (D_{t-1}) must be repaid at the end of the period plus interest due (applying a rate r_t). The period-t government budget constraint thus writes as follows:

$$G_t + (1 + r_t)D_{t-1} = T_t + D_t \,, \tag{1}$$

where G_t is the non-interest (or primary expenditure) and T_t, the total tax revenues.[4] At the end of period t, public debt D_t is the stock of past obligations D_{t-1} to which we add the

[3] The expectation that governments will honor their debt in all states of the world is ultimately what makes their bonds safe. This characteristic has economy-wide benefits, if only for the stability of the financial sector, the viability of pension funds, and the conduct of monetary policy.

[4] Non-tax revenues (including interest-sensitive ones, and those related to monetary policy operations) are ignored here for convenience.

interest due $r_t D_{t-1}$, and the difference between primary expenditure and total revenues, known as the *primary deficit*: $P_t \equiv G_t - T_t$.

$$D_t = (1 + r_t) D_{t-1} + P_t \;. \tag{2}$$

Because the economy's taxable income roughly grows with nominal GDP, it is common to scale the nominal amounts in identity (2) in terms of ratios to nominal GDP (denoted by Y_t). The idea is simply that if government's revenues can grow indefinitely, so could expenditure and debt. Assuming that Y_t grows at an annual rate θ_t, we can transform equation (2) as follows:

$$\frac{D_t}{Y_t} = (1 + r_t) \frac{D_{t-1}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} + \frac{P_t}{Y_t},$$

$$d_t = \left(\frac{1 + r_t}{1 + \theta_t}\right) d_{t-1} + p_t \qquad (3).$$

At any given time t, the public debt-to-GDP ratio results from the interest burden of past debt, which is only indirectly dependent on government's policies, and the present primary deficit, which directly reflects fiscal policy decisions.

Note that the impact of the interest bill on debt-ratio dynamics depends on nominal growth. Under the conventional assumption that the interest rate exceeds growth ($r_t > \theta_t$),[5] the debt-to-GDP ratio tends to increase automatically because the rise in GDP—higher denominator—cannot counterbalance the additional debt—higher numerator—that would be required to pay the interest bill with new debt. In the end, the evolution of the public debt ratio boils down the government's choice to pay the interest bill either with own revenues or with new debt. If at least part of the interest bill is funded with revenues, the budget exhibits a primary surplus, which drags the debt ratio down ($p_t < 0$). If instead newly borrowed funds in period t exceed the interest bill, the primary deficit ($p_t > 0$) further adds to debt in that period.

To fully understand the hydraulics of the government budget constraint, we need to acknowledge the possibility to roll over public debt indefinitely. To do so, we first use equation (3) to see how the current debt level is linked to the *future*. Denoting $R_t = \frac{1+r_t}{1+\theta_t}$, and observing that $d_{t+1} = R_{t+1} d_t + p_{t+1}$, we can write:

$$d_t = \frac{1}{R_{t+1}} d_{t+1} - \frac{1}{R_{t+1}} p_{t+1} \qquad (4).$$

[5] In macroeconomic theory, this assumption is known as dynamic efficiency. It ensures that budget constraints are well defined by making it impossible for Ponzi schemes to be sustained.

The period budget constraint thus imposes a relationship between current debt and the *present value* of *future* debt and the primary deficit. To account for the possibility of refinancing existing debt indefinitely, we must solve equation (4) "forward." This is done by repeated substitutions of future debt in equation (4) as follows:

1. Note that $d_{t+1} = \frac{1}{R_{t+2}} d_{t+2} - \frac{1}{R_{t+2}} p_{t+2}$.

2. Substitute for d_{t+1} in (4) to find $d_t = \frac{1}{R_{t+1}} \frac{1}{R_{t+2}} d_{t+2} - \frac{1}{R_{t+1}} p_{t+1} - \frac{1}{R_{t+1}} \frac{1}{R_{t+2}} p_{t+2}$.

3. At some period T, we can write: $d_t = \prod_{j=1}^{T} \frac{1}{R_{t+j}} d_{t+T} - \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} p_{t+j}$.

4. And as T approaches infinity, we have:

$$d_t = \lim_{T \to \infty} \prod_{j=1}^{T} \frac{1}{R_{t+j}} d_{t+T} - \sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R_{t+k}} p_{t+j} \tag{5}$$

Considering the a priori infinite time horizon of a government, its budget constraint, now written in a fully "intertemporal" fashion, requires that any current amount of debt d_t be covered by the net present value of all future primary balances (second term in (5)) adjusted down for any remaining "terminal" value of the debt stock (first term in (5)).

That said, it is intuitively clear that there cannot be any terminal debt stock the government could conveniently dispose of at the end of times. Nobody in the economy would ever accept holding a government bond that could not be realized to finance some future spending (e.g. O'Connell and Zeldes, 1988). Another way to see this argument is that a Ponzi scheme in which a government could always find an agent willing to hold a bond to finance interest payment is not feasible.

Government solvency thus requires[6] that $\lim_{T \to \infty} \prod_{j=1}^{T} \frac{1}{R_{t+j}} d_{t+T} = 0$. Under normal conditions for growth and interest rates, solvency imposes public debt d_t to be at most equal to the present value of all *future* primary balances. Equivalently, primary deficits must at some point be fully offset by surpluses.

$$d_t = -\sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R_{t+k}} p_{t+k}. \tag{6}$$

[6] The terms no-Ponzi or "transversality" condition are most commonly used in the literature.

The concrete challenge of assessing solvency is immediately clear from equation (6): it amounts to predict future fiscal policy (primary balances) over an infinite horizon. As if that was not hard enough, the simple deterministic arithmetic above ignores that such prediction is subject to considerable uncertainty surrounding (nominal) economic growth, borrowing costs, and the primary balance itself.

The bottom line of this discussion could be that government solvency is a genuine "known unknown," and that assessing it is "mission impossible" (Wyplosz, 2011). However, regardless of the immense practical challenge, knowing whether (6) holds or not (without resorting to strategies such as monetary financing, default or restructuring) is vital for policymakers, taxpayers and market participants. Debt trajectories inconsistent with it—such as an ever increasing or explosive debt ratio—cannot be sustained indefinitely, and at some point, default, restructuring and acute inflation are the only strategies left to re-establish (6). Such events entail a significant destruction of wealth and income loss, particularly for the poor, who have no or only limited access to financial instruments allowing them to insure against such risks.

As usual, trying to know the unknowable demands pragmatism and acceptance that the result will be imperfect, debatable and subject to judgment. Practitioners and empirical economists have elaborated a variety of approaches to analyze the "sustainability" of public debt. These approaches have in common specific patterns of primary balance behavior sufficient to guarantee solvency. Thus, debt sustainability is a higher bar to pass than solvency, and the question becomes how high the bar should be.

2.2. From Solvency to Sustainability

The fact that solvency is purely forward-looking poses two related challenges for practitioners. The first is that econometric tests of the solvency condition using time-series data are of limited value. These tests can tell us whether public debt and primary balance behavior have historically been consistent with solvency, but any forward-looking assessment hinges on the assumption that the future will look like the past. The problem is that real concerns about public debt, and the corresponding need for sound debt sustainability assessment tools, often arise in times of doubt about making such an assumption (or of a perceived need to break from the past).

The second challenge is that if we completely ignore the past, then virtually any government could be deemed solvent regardless of its existing debt level. All it takes is to consider as *credible* any public commitment to generate sufficiently high primary surpluses at some point in (and possibly very far into) the future. Hence, the final call on solvency is a mere judgment on a government's credibility.[7] Thus giving an operational definition to debt sustainability—

[7] Judgment is an integral part of any forecasting exercise, and as such, it is inevitable in debt sustainability analysis.

instead of solvency strictly speaking—is about designing a procedure to adequately discipline and inform our judgment with "hard" data and "objective" criteria and indicators.

Common definitions of sustainability squarely place the onus on fiscal policy behavior (i.e. the right-hand side of (6)). This makes sense from a practical point of view because the government controls policy levers (tax and expenditure policies) determining the primary balance, whereas public debt responds to a range of shocks hitting the public sector balance sheet independently of policy choices. The IMF (2002) considers that public debt is sustainable "if it satisfies the present value budget [i.e. solvency] constraint *without a major correction in the balance of income and expenditure* given the *costs of financing* it faces in the market." (italics added). IMF (2014) is even more explicit in acknowledging the centrality of fiscal policy sustainability, which is characterized as the ability of a government to service its debt without unrealistic fiscal adjustment. As such, fiscal policy sustainability is indistinguishable from debt sustainability.

2.2.1. Testing for Sustainability

Chalk and Hemming (2000) review early government solvency tests based only on historical data. They note that these tests capture sufficient conditions for solvency, and as such, effectively examine the stricter notion of debt sustainability. That line of research revolves around the so-called *stationarity* of the two relevant time series in equation (6), namely public debt and the primary balance.

By definition, the unconditional distribution of a stationary time series does not change over time. In concrete terms, a stationary variable does not exhibit any trend in its mean.[8] In a seminal study, Hamilton and Flavin (1986) argue that if the solvency condition holds, stationarity in the primary balance series implies that public debt is also stationary. Once again, this test is a sufficient condition for solvency. Trehan and Walsh (1988) indeed show that even if debt and the primary balance are non-stationary (or integrated), solvency is satisfied if both series move together (are "cointegrated"), with higher debt systematically associated with higher primary balances.[9]

In a celebrated article, Bohn (1998) goes one step further, arguing that tests based purely on time-series properties of debt and the primary balance miss the general equilibrium conditions linking fiscal policy to the rest of the economy. Bohn's "model-based-sustainability" framework suggests estimating econometrically the conditional relationship between public debt and the primary balance. This is done with a single-equation empirical model explaining

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[8] A stationary series has neither a deterministic trend nor a "unit root" (that would imply the absence of mean-reversion).

[9] Bohn (2007) shows that stationarity and cointegration are not even necessary to satisfy the intertemporal budget constraint.

the primary balance ($pb_t = -p_t$ in the notation above) by public debt and transitory variations in government expenditure ($\tilde{g}_t$) and output ($\tilde{y}_t$):

$$pb_t = \beta_0 + \beta_1 \tilde{g}_t + \beta_2 \tilde{y}_t + \rho d_{t-1} + \varepsilon_t \quad (7)$$

Bohn shows that a positive conditional response of the primary balance to public debt (i.e. $\hat{\rho} > 0$) is sufficient to fulfill the solvency condition in a general equilibrium model under reasonable assumptions. As discussed in Section 3, this test has been widely used in the literature to assess whether fiscal policy was "responsible" in the sense of being broadly consistent with solvency.

As informative as econometric analyses can be, they cannot be the sole guide to the work of practitioners. The reason is that they cannot convincingly answer the basic question: is country X's current public debt likely to be serviced now and in the foreseeable future? As always, one chief concern is that even if econometrics can capture the past quite well, it may fail miserably to inform the future. A more serious concern, however, has to do with the design of the tests themselves. None of them provides directly relevant guidance on the debt paths and levels that we could safely consider as sustainable. For instance, knowing that the primary balance and the debt level should tend to move together over time is certainly useful, but it does not rule out steadily rising debt for a long time and to levels most observers, including market participants, would, for right or wrong reasons, deem dangerous and somehow "unsustainable."[10] As discussed in section 3, practitioners have moved to more sensible but also more restrictive debt sustainability requirements.

2.2.2. *Stable Debt Dynamics as a Condition for Price Stability*

So far, we have learned that even when tested in the form of largely sufficient conditions, solvency imposes too little constraint on fiscal variables to usefully guide sustainability assessments. However, one class of theoretical macroeconomic models adopts a notion of sustainability that may be more appealing to practitioners. These models suggest that stable public debt dynamics around a well-defined steady-state is a precondition to price stability. As illustrated in Section 3, imposing mean-reversion in public debt is usually stronger than Bohn's (1998) solvency condition on primary balance behavior.

That argument takes roots in a seminal article by Sargent and Wallace (1981), who assume that monetary and fiscal policies are decided by different authorities, none of which is *a priori* constrained by government's solvency. Sargent and Wallace link monetary and fiscal instruments by adding seigniorage revenue (i.e. profits from money creation transferred to the budget) to an intertemporal budget constraint defined in real terms. In their setup, the ability by one policymaker to always deliver on its promised policies independently of government solvency automatically shifts to the other authority the responsibility for fulfilling the

[10] The Bohn test does not imply any boundary on the primary balance, which makes many scary debt paths and levels consistent with solvency. Section 3 reviews the Bohn test in details.

intertemporal budget constraint. So if the government sticks to a predetermined path for the primary balance, the central bank ultimately has to create enough money to meet the budget constraint; this is the famous "unpleasant monetarist arithmetic."

Leeper (1991), Sims (1994) and Woodford (1994), among others, add realism to this view of monetary-fiscal interactions by acknowledging that government liabilities are defined in nominal terms. Monetary policy can thus be used to keep the government solvent by setting a price level in line with the intertemporal budget constraint (in real terms). In equilibrium, monetary and fiscal policies *jointly* ensure stable dynamics for both the price level (inflation) and the real public debt, but no specific "policy assignment" (i.e. an answer to the question of which instrument targets which objective?) is imposed. The theory nevertheless suggests that assigning monetary policy to achieve price stability can only work (i.e. be an equilibrium) if the primary fiscal balance responds to stabilize real public debt around its steady-state. Failure to do so either drives the economy off equilibrium (e.g. explosive debt or price dynamics) or requires monetary policy to do the job (a situation known as fiscal dominance).[11]

2.2.3. *Solvency and Liquidity*

Solvency, both in its pure and stricter forms (i.e. various definitions of sustainability), is by essence a medium-to-long-run concept. As such, it largely ignores constraints that may bind only in the short-term and that may jeopardize a debtor's ability to honor financial obligations. Liquidity problems, as they are known, have the same effects as the sudden realization of insolvency: default, restructuring, or other expedients.

As noted by Wyplosz (2011), however, the IMF (2002) definition of sustainability goes beyond pure solvency issues and covers circumstances typical of illiquidity. First, by considering "major corrections" in fiscal policy as inconsistent with debt sustainability, the definition implicitly captures what happens in a liquidity crisis, when in the absence of new financing at reasonable conditions, public spending should instantly match revenues to stick the period budget constraint. Second, the explicit reference to the "costs of financing" acknowledges the role of market expectations and risk aversion, as reflected in sovereign risk premiums. By referring to sustainability as the ability to service debt, IMF (2014) effectively lumps together solvency and liquidity.

It remains that a perfectly solvent government can face liquidity crises, and an insolvent one can go on for a long time before hitting the wall of illiquidity. This is the ugly face of the so-called multiple equilibria. As long as lenders' expectations converge on good outcomes (solvency/sustainability), borrowing costs can remain low enough for public debt to stay on a

[11] This view of the fiscal preconditions for price stability remains highly controversial. For instance, Buiter (2002) argues that the intertemporal budget constraint is a permanent constraint on fiscal policy and cannot be treated as a mere equilibrium condition. For instance, what happens off equilibrium in these models is questionable because it results from a basic misspecification (i.e. a non-binding budget constraint).

sustainable trajectory. However, if for some reason, views about the riskiness of a country's public debt change, liquidity stress can suddenly arise, borrowing costs explode, and solvency, instantly become a problem. The self-fulfilling nature of sovereign debt crises complicates sustainability assessments.

Clearly, liquidity must be an important consideration in any comprehensive debt sustainability assessment. Here too, judgment is central and it concerns lenders' willingness to cover the government's gross financing needs (that is the sum of the deficit and rollover needs) without sharp increases in risk premiums. To inform that judgement, indicators of the risks surrounding the debt trajectory (e.g. will markets finance a large commercial bank bailout?) will prove useful (Section 3). In addition, detailed information about the debt structure in terms of maturity (long vs. short) and currency composition (domestic vs foreign), and about bondholders' profile (domestic vs. foreign), the repayment schedule (smooth vs. lumpy), and the overall quality of debt management will help obtain a more reliable forecast of gross financing needs and a better understanding of refinancing risks.

2.2.4. *Summary*

This Section has discussed the theoretical foundations of public debt sustainability. Sustainability is anchored in government's solvency, which is a theoretically clear concept. However, the solvency condition defines an intertemporal budget constraint imposing existing public debt levels to be at most equal to the present value of all primary balances over the indefinite future. Hence, solvency is a pure prediction and has no operational meaning.

In practice, the focus is on debt sustainability, which is a stricter requirement than solvency and is not uniquely defined. Therefore, any debt sustainability assessment based on these definitions is tool-dependent and bound to be partial.

3. Practical Approaches

This section develops the main practical approaches to assess public debt sustainability. We first review in greater details the empirical evidence about primary balance behavior as a key determinant of debt sustainability. We see that those econometric tests rarely put sustainability into question. We then flesh out the analytics underlying the most common debt-sustainability assessment tools, such as those developed at the IMF. These tools focus on medium-term projections for public debt and on an evaluation of the risks surrounding the predicted trajectories. The Section ends with a brief presentation of other practical approaches, and a discussion of solvency considerations beyond gross public debt.

3.1. Is Fiscal Policy Responsible?

As introduced in Section 2, Bohn's (1998, 2005) seminal contributions framed the question of fiscal sustainability in terms of fiscal "reaction functions:" how do governments respond to the accumulation of debt? Bohn looked for evidence that governments take corrective action whenever the public debt-to-GDP ratio rises—corrective action in the form of a rising primary balance ratio.

Bohn showed theoretically that in a regression of the primary surplus against public debt, the cyclical position of the economy, and the transitory component of government spending, a positive regression coefficient on debt ($\hat{\rho} > 0$ in equation (7)) is sufficient to establish that the government is satisfying the intertemporal budget constraint, i.e., that fiscal policy is *responsible.* Bohn documented a number of advantages of his test of sustainability over other alternatives that had been proposed and used in the literature; he also showed how to take account of the whole host of heterogeneous, often transitory, influences on government behavior that make it difficult to disentangle the systematic effects of the government's efforts to adhere to an intertemporal budget constraint.[12] He showed that his test is valid regardless of whether debt and the primary balance are measured at constant prices, in levels or in relation to GDP, and also does not depend on knowledge about any fiscal rules that may have been adopted or the portfolio of public debt instruments. He found strong evidence that fiscal policy in the United States was sustainable in the long period since 1916, in the sense that the data are consistent with a robust positive response of primary fiscal balances to rising public debt ratios.

The insights from Bohn's contribution have stimulated other researchers to ask a number of follow-up questions, some of which were already implicit or partially addressed in Bohn's own work. First, though, is the issue of perspective that Bohn has in his work, which is very much a long-run perspective. In other words, the response must be sufficiently systematic and stable over time. Hence testing for a countervailing response of fiscal policy to debt shocks is something that should be done over very long periods in order to draw appropriate inferences about the consistency of fiscal policy with intertemporal solvency. If the response of primary balances to rising debt is positive for a decade, but this falls away in the ensuing 5-10 years, it is far from obvious what inference can be drawn in terms of whether fiscal behavior (i.e., evolves in a manner that is consistent with satisfaction of the intertemporal solvency constraint) or not. The issue of possible time-variation in the responsiveness of different governments may be of independent interest, but it does not shed light on the long-run sustainability question, which refers instead to the totality of behavior of an entire sequence of governments, likely over many decades. That said, the econometrician, faced with structural breaks in the responsiveness of policy over time, may ask what inference can be drawn from extrapolating the most recent behavior (provided sufficient data exist to identify that behavior—more on this below).

[12] Bohn embeds his test within the framework of Barros's (1986) tax-smoothing model, under which temporary increases in government spending (for example, due to wars) are financed by a higher deficit, and business-cycle fluctuations are accommodated in a similar fashion. Bohn is thus jointly testing the implications of the intertemporal constraint and the tax-smoothing framework. His test is designed to gauge whether movements in fiscal balances that occur alongside these various shocks are nevertheless operating in a systematic way to offset the impact of transitory factors, and thereby preventing public debt from straying onto a divergent path.

What are some of the questions that researchers have pondered in the light of Bohn's work? A first issue is whether the response of the primary balance to debt is linear or nonlinear, and if nonlinear, how responsiveness changes with the level of debt: does higher debt induce governments to adjust faster, or does fiscal effort instead peter out above some debt threshold? Bohn himself tested for such nonlinearities and found an increasing marginal response of surpluses to changes in debt for the United States, rising from insignificantly negative values at very low debt ratios to significantly positive responses for debt in a moderate range (around 50 percent of GDP), and rising even further as debt ratios rose above 100 percent. The tenor of the results, moreover, was remarkably uniform across different sample periods.

Motivated by the greater systemic importance of a number of industrial and emerging market countries over the past couple decades, other researchers have looked into these questions using cross-country data. Mendoza and Ostry (2008) use a data set comprising 34 emerging market and 22 industrial countries over a 25-year period beginning in the early 1990s. They find a robust positive conditional relationship between primary balances and debt across a range of specifications and find that for 42 out of the 56 countries in the sample, the key homogeneity assumption of the panel regressions imposing a common response of primary balances to debt in all countries cannot be rejected. The authors look at a common panel, as well as at emerging markets and advanced economies separately; in all three samples, the tenor of the results is similar, and presents an encouraging message from a policy perspective since it suggests that, far from being prone to default, government policy is consistent with fiscal solvency in many countries (not just the United States, as investigated by Bohn).

Focusing on the differences between the emerging-market and advanced-economy samples, Mendoza and Ostry provide evidence that the response of the primary balance to debt is *stronger* in emerging markets than in industrial countries. The implication is that, owing to this stronger response, emerging-market countries would tend to see their public debt ratios converge to lower mean levels than those in advanced economies.[13] The authors compute estimates of the mean debt ratios predicted by the model, and find them to be in line with the data, in particular with the finding that emerging-market public debt ratios are in fact lower than those in the advanced economies.

Mendoza and Ostry also shed light on the nonlinearity issue, and in particular on whether the cross-country data favor the hypothesis of stronger primary balance responsiveness at high debt levels (as suggested by Bohn for the United States) or weaker responsiveness. They find that the positive response of the primary balance to debt is much weaker in emerging market

[13] One interpretation of the finding of a stronger conditional response of the primary balance to debt in emerging market countries than in advanced economies is that the riskier fiscal and financial environment in the former requires a stronger response to maintain fiscal solvency. The stronger primary balance response is therefore not an indicator of more fiscal discipline.

countries with debt ratios above 50 percent of GDP. More generally, sorting the three samples (advanced, emerging, and combined) into high- and low-debt countries (relative to sample means or medians), the authors find that the sustainability criterion test generally fails in high-debt countries. In some nonlinear specifications where the formal Bohn test does not fail, there is nevertheless a weaker primary balance response as the debt ratio rises above 50 percent of GDP. The tenor of Mendoza and Ostry's panel results, therefore, is quite different from those of Bohn for the United States (which suggest that responsiveness actually strengthens with the debt ratio). Mendoza and Ostry conclude that countries should be wary of allowing their public debt ratios to rise above the 50-60 percent of GDP range, because the ability of policy makers to maintain fiscal solvency through higher primary balances in countries with mean and median debt ratios above this range appears to wane.

The idea that the responsiveness of fiscal policy to the level of public debt is sensitive to the country's indebtedness level is intuitive. Indeed, it cannot literally be true that the primary balance would always increase enough to offset a rising interest bill associated with higher debt because, for sufficiently high debt levels, primary balances that exceed GDP would eventually be required (whereas in reality such balances rarely exceed a few percentage points of GDP). A more plausible notion, consistent with the above observation, is that that the primary balance beyond some threshold will no longer be able to keep up with rising interest payments as debt increases. An implication is that there will be a debt level or limit above which the debt dynamics become explosive and the government will necessarily default. [14] This debt limit internalizes the fact that the default premium will rise as debt approaches its limit. Eventually, the "fixed point" problem of a higher default probability leading to a larger risk premium, in turn leading to a higher probability of default, will have no solution at a finite interest rate. Formally, the *debt limit* is the largest debt ratio for which the fixed-point problem has an interior solution at a finite interest rate. We illustrate the basic hydraulics of the debt limit later in this section.

To follow up on this issue, Ostry et al. (2010) and Ghosh et al. (2013) develop a stochastic model of sovereign default in which risk-neutral investors lend to a government that displays *fiscal fatigue*, whereby its ability to increase primary balances cannot keep pace with rising debt. As a result, the government faces an endogenous debt limit beyond which debt cannot be rolled over. Using a cross-country dataset covering 23 advanced economies over the period 1970-2007, these papers find evidence to support a fiscal reaction function with these

[14] Bi and Leeper (2012) develops a similar notion of debt limit in a dynamic and stochastic general equilibrium model. In their model, the primary balance is bounded upwards because of a Laffer-curve effect on tax revenues (high tax rates ultimately shrinks the tax base too much) and incompressible primary outlays.

features.[15] They then use the empirical results (estimated fiscal reaction functions) to compute *fiscal space*, defined as the difference between current debt ratios and estimated debt limits.[16]

Ostry et al. (2010) and Ghosh et al. (2013) find strong empirical support for the existence of a non-linear reduced-form relationship between the primary balance and public debt that exhibits the fiscal-fatigue characteristic. The relationship seems to be well-approximated by a cubic function. At low levels of debt there is no, or even a slightly negative, relationship between the primary balance and debt. As debt increases, the primary balance also increases but the responsiveness eventually weakens and then actually decreases at very high levels of debt. This relationship is robust to the addition of a multiplicity of conditioning variables and a variety of estimation techniques.

The theoretical framework of the Ostry-Ghosh papers also carries several insights, notably that, as debt approaches the debt limit, the cost of financing will tend to shoot up from the risk-free rate to a prohibitively high interest rate within a very narrow range of debt ratios. In other words, markets will tend to provide little advanced warning of a looming crisis: governments can thus *suddenly* lose financing access and revisions to market estimates about the size of possible shocks to the primary balance can *abruptly* push a country into a situation of unsustainable debt dynamics. Insurance considerations, therefore, strongly suggest that normatively-desirable debt ratios are far below the public debt limits (see Debrun et al, 2018).

The discussion so far has dwelt on whether behavior in individual countries, or in a cross-section of countries, is consistent with the notion that governments obey their intertemporal budget constraint—an inherently long-run concept. But if fiscal behavior is discontinuous, then not accounting for such discontinuities will result in misspecified empirical models and, therefore, misleading inferences. Mauro et al. (2015) exploit historical data for 55 countries over two centuries, and find large variations in $\hat{\rho}$ over time and across countries. They suggest that lower values for $\hat{\rho}$ arise when inflation is high, borrowing costs are low, and potential growth declines unexpectedly. More generally, their long historical time series allows them to document significant variation in the response across countries and within countries across time. By way of example among their findings: for the advanced countries, there is a break in

[15] In an early paper, Abiad and Ostry (2005) explore a range of fiscal reactions and draw implications for how primary surpluses are projected (including in the context of IMF-supported programs) when making assessments of public debt sustainability. They argue that projections should be tied either to the country's historical track record in generating surpluses—if the institutional and other factors accounting for this track record are expected to persist—or to some model that links primary surpluses to their fundamental determinants, either on the basis of constant institutions or a credible reform program. Their message is that fiscal reaction functions with some empirical basis can provide a useful benchmark for judging the realism of fiscal projections underlying debt-sustainability calculations.

[16] The methodology developed by Ostry et al. (2010) and Ghosh et al. (2013) has been used by rating agencies, including Moody's, to compute on a regular basis estimates of fiscal space in the advanced economies, and place such countries into different zones (green, yellow, red), according to the estimated extent of fiscal space.

fiscal policy responsiveness around the time of the global financial crisis in 2008, and for a number of the largest advanced economies, tests that include sample periods beyond 2008 fail to find evidence consistent with countries meeting their intertemporal budget constraint.

Overall, regression-based solvency tests à la Bohn must be interpreted with care when it comes to judge debt sustainability. First, valid inferences require stable primary balance behavior for a long time, a condition that does not seem to be satisfied always and everywhere. Second, non-linearities in fiscal behavior seem pervasive, pointing to upper bounds on the achievable primary balance, and therefore on public debt itself. Third, while estimated fiscal reactions can be useful guides to check the realism of primary balance predictions, one should always remain careful when putting similar countries into “buckets” of similarly behaved governments. The case studies of Germany and Japan neatly illustrates how two comparable economies can exhibit very different fiscal behaviors and debt dynamics (Box 1).

3.2. Pragmatism Rules: Stable Debt Dynamics

The serious caveats associated with model-based sustainability assessments à la Bohn led practitioners to develop more pragmatic approaches where judgment is mainly informed by the predicted public debt trajectory over the medium term. Because the IMF and other international institutions with a policy surveillance function had to cater for equal treatment among their members, they had to settle on simple Debt-Sustainability-Analysis (DSA) templates applicable to widely different economies with sometimes limited data availability. These methodologies have been extensively documented and analyzed elsewhere,[17] and we only provide a summary of the deterministic arithmetic behind them.

DSA tools reflect the consensual principle that debt sustainability rules out explosive debt trajectories under realistic predictions about future economic developments and policies. Although sustainability is a long-term concept, the reliability of forecasts rapidly diminishes with the length of the forecasting horizon. As a result, DSA tools can at best inform a judgment based on medium-term projections for the debt-to-GDP ratio (5-10 years)[18] and the gross financing needs. These two series form the backbone of the analysis, which is usually complemented by various summary indicators, risk assessments, and realism checks.

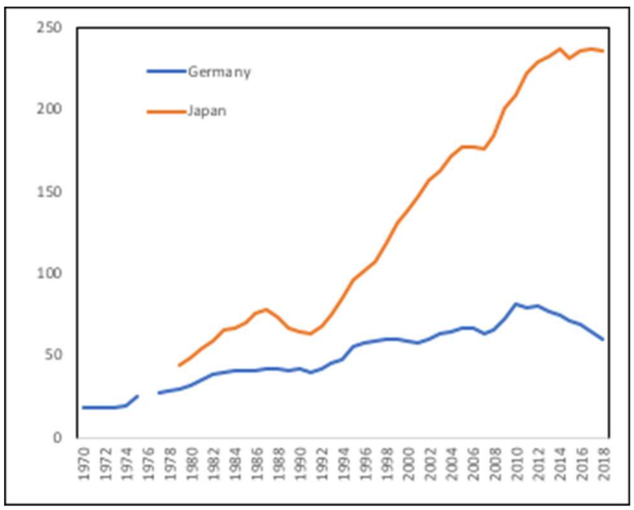
[17] See IMF (2002, 2005, 2014), Escolano (2010), or European Commission (2017).

[18] Some institutions publish very long-term projections for the debt ratio (50 years or more). However, such projections have a pedagogical role to describe a state of public finance “if nothing was ever done about long-term challenges.” Such a hypothetical state is not realistic, and not meant to be a prediction. Another issue with long-term projections has to do with the arithmetic of stock-flow accumulation and discounting, which implies that small changes in the assumptions entail extremely large variations in the end result. In the end, very long-term projections are as helpful as the intertemporal budget constraint itself, it is easy to conclude that pretty much any debt level is sustainable or not sustainable depending on what we want to illustrate.

Box 1—A Tale of Two Advanced Economies: Germany and Japan

In many ways, Germany and Japan are similar. They are sizable economies that rely on a strong industrial base favoring export-led growth. Politically, they are stable parliamentary democracies that involve well-established political parties. And, yet, while broadly similar for a long time, the evolution of their public debts could not be more different (Figure 1). While debt ratios had been creeping upward in both countries until the late 1980s, the situation then changed radically in Japan. By now, the (gross) debt of the Japanese government is by far the highest among advanced economies. The puzzle is that despite studies consistently showing that there was no fiscal space in Japan, debt kept rising at a breathtaking pace until the mid-2010s without causing the slightest concerns among lenders (the Japanese public itself, for the most part). In contrast, Germany successfully contained the debt buildup, reversing it after 2014.

Figure 1. Germany and Japan: Gross public debt (in percent of GDP)



Source: WEO

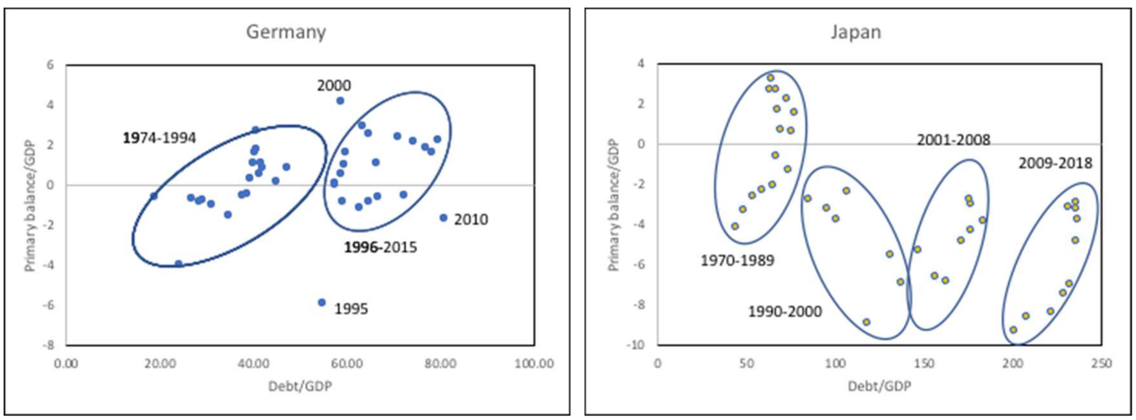
The debt pickup in both countries around 1990 corresponds to a structural slowdown in growth rates, from an average of 2.5 percent over 1970-1989 to an average of 1.7 percent in 1990-2018 in Germany, and from 4.8 percent to 1.2 percent in Japan. In addition, Germany's reunification also weighed of public finances during the early 1990s. In general, explanations for upward trends in public debt include:

- Implicit or explicit strategy of eventually defaulting.
- Confusion between trend and cycle: the authorities observe lower growth and adopt expansionary policies that fail to deliver the expected sustained boost.
- Conflict with the central bank that responds by raising interest rates.
- Lack of domestic support for fiscal discipline, which leads to destabilizing budgetary cycles when fiscal fatigue sets in.

The first explanation can be ruled out in both cases. This is obvious in the case of Germany but it also applies to Japan whose public debt is mostly held by local financial institutions, the central bank and households. It would just be too costly to default. The second explanation is implausible over the long run but may have played a role for a while in both countries. The third explanation can be justified by central bank statements at various junctures, but there is no evidence that central banks systematically raised their policy rates in response to debt buildup and that higher policy rates are a significant deterrent for deficits.

The fourth explanation would imply a wrongly-signed coefficient on debt in the Bohn's fiscal reaction function (i.e. $\hat{\rho} < 0$). This is not the case in Germany, where the estimated coefficient is positive and highly significant (0.0357). For Japan, however, it is negative and highly significant (-0.300). This might reflect the non-linearity discussed in the text but the scatterplot in Figure 2 suggests an alternative interpretation. While visual inspection confirms that the debt coefficient is negative for the overall period, it points to instability over time, with four distinct subperiods. The first one (1970-1989), displays increasing efforts, eventually successful, at reducing the debt. The following period (1990-2000) is characterized by a clearly negative link between the primary budget balance and debt. Then come two periods of positive relationship (2001-2008 and 2009-2018), whith debt-stabilization efforts of diminishing intensity from one period to the next. The opposite seems to characterize Germany. After 1996, the stabilizing response to higher debt appears to have become more vigorous.

Figure 2. Germany and Japan: Primary balance and debt (in percent of GDP)



Source: WEO

In in a deterministic setting, two basic relationships determine debt dynamics. The first is the period public-debt-accumulation equation, which can be obtained from equation (3):

$$\Delta d_t \equiv d_t - d_{t-1} = \Gamma_t d_{t-1} - pb_t\,, \tag{8}$$

where Δ symbolizes the discreet first-time-difference operator, and Γ_t captures the difference between GDP growth and the interest rate paid on public debt $\left(\Gamma_t = \frac{r_t - \theta_t}{1+\theta_t}\right)$. The second key relationship is the fiscal reaction function describing the endogenous (and sluggish) response of fiscal policy to public debt developments:

$$pb_t = \kappa + \lambda pb_{t-1} + \rho d_{t-1}, \tag{9}$$

where λ symbilizes the well-documented persistence in fiscal balances (i.e. $0 < \lambda < 1$). As in (7), ρ is the policy response (in terms of a change in the primary balance) to variations in the public debt. Combining (8) and (9), the requirement for debt dynamics to be stable, or equivalently, for debt to be mean-reverting is:

$$\rho > \Gamma^*(1-\lambda), \tag{10}$$

where a * superscript denotes the hypothetical constant value of a variable observed in the long run, after the effects of all current shocks have dissipated (a situation known as the "steady-state"). Even though (10) describes the relevant stability condition regardless of the sign of Γ^* (Bartolini and Cottarelli, 1994), a conventional assumption is that in steady state, the interest rate is greater than GDP growth (i.e. $\Gamma^* > 0$). Hence equation (10) clearly shows that the mean-reversion criterion is in principle stricter than Bohn's condition ($\rho > 0$): it requires the primary balance's stabilizing response to the debt ratio to more than offset the automatic debt buildup that would arise if interest payments were covered with borrowed funds (or debt "snowball" effect).

In that case, public debt is sustainable because it is expected to always converge to a well-defined level d^* in the long run:[19]

$$d^* = \frac{-\kappa}{\rho - \gamma^*(1-\lambda)} \tag{11}$$

As indicated in Section 2, what matters is fiscal policy behavior, not the long-term debt level per se. Debt could in principle rise much above d^* without raising any concerns about sustainability. All it takes is a sufficiently aggressive fiscal policy response to offset debt shocks. However, as discussed above, the debt level matters if the primary balance cannot rise without bound, which we know is the case. Acknowledging the existence of an upper bound $\overline{pb}$ on the primary balance (call it a "fiscal fatigue" threshold), the fiscal reaction function (9) could be rewritten as:

$$pb_t = min\left(\kappa + \lambda pb_{t-1} + \rho d_t, \overline{pb}\right). \tag{12}$$

In that particular case, *two* long-term debt levels catch attention. Of course, as long as the primary balance remains below its fiscal fatigue level ($pb_t \leq \overline{pb}$), d^* is a dynamically stable situation and any debt level is sustainable because fiscal policy ensures convergence back to d^*.

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[19] Under condition (10), the long-term debt level will be positive if $\kappa < 0$. Equation (11) is obtained by applying (8) and (9) to stead-sate values of the primary balance (p^*) and the public debt (d^*).

What happens if fiscal fatigue sets in and the primary balance is stuck at $\overline{pb}$ while debt keeps rising over time? At some point, the primary surplus will fall short of the snowball effect. The exact debt level at which it happens is $d^{**} = \frac{\overline{pb}}{\Gamma^{*}}$ (the growth-adjusted annuity value of the maximum primary balance). Maintaining the assumption that the interest rate exceeds GDP growth in the long run ($\Gamma^{*} > 0$), the higher debt level d^{**} is dynamically unstable because $\rho = 0$. Hence, d^{**} is literally the edge of a cliff beyond which the government loses control of debt dynamics and debt is not sustainable with certainty. In a sense, d^{**} is the "debt limit" à la Ostry et al. (2010) expressed the deterministic arithmetic framework characteristic of DSA tools. As public debt, financial conditions, fiscal policy, the economy are subject to random shocks, insuring against the risk of unsustainability requires public debt to remain well below d^{**}. (More on this issue later).

From this discussion, we see that in practice, assessing public debt sustainability is to figure out whether the debt-to-GDP ratio is on a dynamically stable debt path. Several operational challenges arise from this approach:

1. Sustainability critically depends on the assumed fiscal policy behavior. Beyond short-to-medium term predictions, knowing about stable patterns in primary balance behavior is useful to have a sense of (i) the steady sate debt level to which all sustainable trajectories can be expected to converge and (ii) the debt limit beyond which trajectories would be at an uncomfortably high risk of becoming explosive.

2. The quality of the underlying economic forecasts, especially for interest rates and growth, is the other critical ingredient of a reliable sustainability assessment.

3. Defining specific debt thresholds beyond which sustainability can be put into question is very difficult, notably given the role of market expectations and the related "fixed point" problem (Ostry et al., 2010). In addition, choosing how far below an eventual debt limit a government should stay depends on assumptions about the economy's steady-state, the risk profile of the existing debt, the state of the public sector balance sheet (e.g. the extent of contingent liabilities), and policymakers' preferences. Any threshold is likely to be highly country-specific and subject to controversy. "Long-run" numbers like d^{*} and d^{**} are by nature extremely sensitive to small variations in the underlying assumptions because these apply over many successive periods.

4. Since a failure to service the debt is the main manifestation of unsustainability, liquidity and solvency issues are equally important. As discussed above, assessing liquidity is particularly challenging because it effectively requires guessing turning points in sovereign debt markets, and a knowledge of risk factors that could trigger a spike in financing needs or mood swings in sovereign bond holders.

To address or mitigate these challenges, DSA tools have been equipped with additional indicators and reporting requirements:

1. The consistency between policy behavior and debt sustainability is usually captured by measures of the gap between the actual primary balance and a hypothetical level required to stabilize public debt at a certain level over a given horizon. The most common and straightforward indicator measures this gap on a period-by-period basis. The primary balance required to stabilize the debt ratio between $t-1$ and t is found by solving equation (8) for $\Delta d_t = 0$ to obtain $pb_t^o = \Gamma_t d_{t-1}$. This results in a gap: $pb_t^o - pb_t$ (Blanchard, 1990).[20] Since closing the gap in one year may not be feasible nor desirable, other metrics take a medium-term perspective. For instance, the European Commission's "S1" indicator calculates the constant yearly adjustment in the *structural* primary balance (i.e. adjusted for temporary influences on the budget, including the economic cycle) needed to reach a given debt level at a predetermined date.[21] While sustainability indicators capture the size of fiscal adjustment (i.e. higher revenues and or lower expenditure) eventually required to remain on a sustainable debt path, they say nothing about the realism of these hypothetical policies. As argued by Abiad and Ostry (2005), there is value in referring to estimated fiscal reaction functions to compare primary balance forecasts (or normative scenarios) against the country-specific context (e.g. history, external and other institutional anchors). Recent work by Mauro et al. (2015) and Debrun and Kinda (2016) also suggest that the average response of the primary balance to public debt varies with the level of interest rates, long-term growth, or inflation. The IMF DSA template has gone one step in that direction by comparing the projected fiscal adjustment for the country under review to the distribution of fiscal adjustments in a large panel of countries.

2. As illustrated in Box 2, large forecast errors—especially optimism in forecasts—can have deleterious effects on debt sustainability assessments and on the policy choices they inform. Optimism feeds overborrowing, which ends in tears when growth disappoints, and investors flee. To encourage the use of realistic forecasts in DSA templates, imposing transparency as to how these forecasts compare to the recent past and to alternative predictions is good practice. For instance, the IMF DSA framework reports past forecast errors for growth, inflation, and the primary balance. More could be done, notably by benchmarking the forecasts to model-based predictions, thereby

[20] For instance, if public debt is at 60 percent of GDP and the differential between the interest rate and growth is 100 basis points, a primary surplus slightly below 0.6 percent of GDP keeps the debt ratio constant year on year.

[21] The algebra is obviously more involved than for the year-on-year gap, but remains straightforward. Escolano (2010) provides detailed and general derivations for this type of indicators.

revealing the extent of judgement present in the DSA forecast. Another significant concern is that the post-2009 world economy has been characterized not only by atypical recoveries that escaped easy predictions, but also by growing uncertainty about new steady-state levels for growth and interest rates. A host of factors, including demographics and fears of a secular stagnation suggest that both GDP growth and interest rates could be permanently lower (e.g. Summers, 2016; and Holston et al., 2017). The fact that interest-growth differentials can remain low or negative for protracted periods of time and then normalize raises questions as to how macroeconomic and fiscal risks can be properly reflected in the standard DSA (Barrett, 2018). Let us keep in mind that the debt arithmetic of even modest changes in steady state values for growth and interest rates can be brutal.[22]

3. Suggesting debt thresholds beyond which sustainability can be declared in jeopardy is valuable, and has a been a fixture in the various incarnations of the IMF DSA framework since the early 2000s (e.g. IMF, 2011, p. 12). These thresholds can be estimated in various ways, including turning points in the debt-primary-balance relationships in a fiscal reaction function, or average debt levels beyond which "distress" is likely to occur based on past events observed in a panel of countries.[23] As such, they reflect country averages and should be interpreted with great caution given the overwhelming importance of country specific factors in shaping fiscal behavior, as emphasized earlier. Looking forward, the concept of debt limit might be more promising and could be developed more. First, as shown above, it can be embedded in the standard DSA arithmetic and its definition lends itself to model-based approaches capturing key relationships among relevant variables (see Ghosh et al. 2013 or Bi, 2012). If plausible debt limits can be estimated, then safety buffers below the limit can be defined for self-insurance purposes. As such, those buffers should reflect the distribution of risks around the predicted debt trajectory (see Fall et al. 2015; Clements et al. 2016; or Debrun et al. 2018).

4. Detecting liquidity issues is as important as anticipating unsustainability. However, the exercise is frustratingly difficult because it requires guessing lenders' behaviors and beliefs, which can be fickle. In practice, preparing reliable medium-term projections of gross financing needs is of the essence. This requires detailed information on the debt

[22] A case in point is the relevance of that the European Union overall deficit cap of 3 percent of GDP to keep public debt at or below 60 percent. Long-term nominal growth must be 5 percent per annum for convergence 60 percent of GDP to occur. With real growth and inflation struggling to reach 2 percent in the region, the deficit limit now suggests a much higher effective debt ceilings ranging from 75 to 100 percent depending nominal growth reaching either 4 or only 3 percent.

[23] The relevance of such an approach for countries without recent experience of genuine distress (i.e. default, restructuring, accumulation of arrears, large fiscal adjustment) is of course debatable.

maturity structure (including redemption schedules to anticipate spikes), and currency composition. Key factors shaping liquidity risk such as the ownership structure of government bonds (domestic vs. foreign bondholders, institutional investors vs. unstable ones) are also relevant. Finally, a close monitoring of market conditions and indicators (such as risk premiums and Credit-Default-Swaps spreads), and developments in similar countries potentially at the origin of contagion is advisable.

Box 2--How a bias towards optimism can muddle debt sustainability analyses?

"*In 2014 Mozambique seemed a good place to host the IMF's "Africa Rising" conference. The economy was buoyant, having grown by about 7% a year for a decade. Offshore gas promised riches. Investors were optimistic, so much so that, in 2013, they snapped up $850m of bonds issued by a state-owned tuna-fishing company, with temptingly high yields. […] But Mozambique's rise has halted. Those "tuna bonds" were the first misstep in a widening scandal that led the government to say on January 16th [2017] that it would default on its debt.*"[24]

As the above quote illustrates, a bullish atmosphere surrounding a country (or region) can fuel capital inflows and might lead to a situation of over-borrowing—particularly if, ex post, the optimism turns out to have been undue. As borrowing decisions are often based upon medium- to long-run projections (especially for loans with longer maturities), the relevant forecast errors can be substantial and the consequences for debt dynamics serious (see Easterly (2001, 2013) for detailed analyses on advanced and developing economies).

In the case of Mozambique, the IMF's October 2012 vintage of the *World Economic Outlook* (*WEO*) forecasted an average real growth rate of 8.0 percent over the period 2013-2016 (see Table 1). In reality, however, average growth over that period fell short, by an average of 1.7 percentage points per year, and amounted to no more than 6.3 percent. The over-estimation of growth was particularly large for 2016 (4.0 percentage points), with the unforeseen growth slump being the trigger for the government's payment difficulties to arise in January 2017.

Table 1: real GDP growth in Mozambique, October 2012 forecast versus realization

	2013	2014	2015	2016
October 2012 forecast	8.4	7.8	7.8	7.8
Realization	7.1	7.4	6.6	3.8
Forecast – realization	*1.3*	*1.4*	*1.2*	*4.0*

Source: IMF WEO database

Recent work by Beaudry and Willems (2018) investigates the link between growth (over-) optimism and (over) borrowing more systematically. Using data from all IMF member states, they show that more optimistic growth projections typically induce countries to accumulate more debt. This is consistent "consumption smoothing," whereby borrowing is used to transfer expected future wealth to the present and make it available for spending. Such a response is not without risk though, as Beaudry and Willems also find that countries for which growth forecasts have been *overly* optimistic in the past, are more likely to develop debt crises in the future. If past borrowing decisions are based upon elevated growth expectations that fail to materialize, servicing the accumulated debt might become problematic.

[24] The Economist, January 19th 2017.

Greece provides another case in point. In October 2008, before the first signs of deep fiscal troubles emerges, the IMF's *WEO* predicted that average growth over the years 2009-2012 would be 2.8 percent per year. The actual outcome was -6.6 percent—implying an average over-estimation of 9.4 percentage points per year (see Table 2).

Optimism seems to have endured. At the time of Greece's first IMF program request (in May 2010, see IMF (2010)), it was expected that Greek government debt would peak at 149 percent of GDP in 2013, subsequently declining to 120 percent of GDP by 2020. In reality (Figure 2), public debt quickly shot up to about 180 percent of GDP before stabilizing. The initially projected target of 120 percent of GDP is out of reach. At the time of writing, the IMF projections suggest that Greek public debt is highly unsustainable. Under the IMF's baseline scenario, the debt ratio is projected to exceed 300 percent of GDP by 2080 (IMF, 2017).

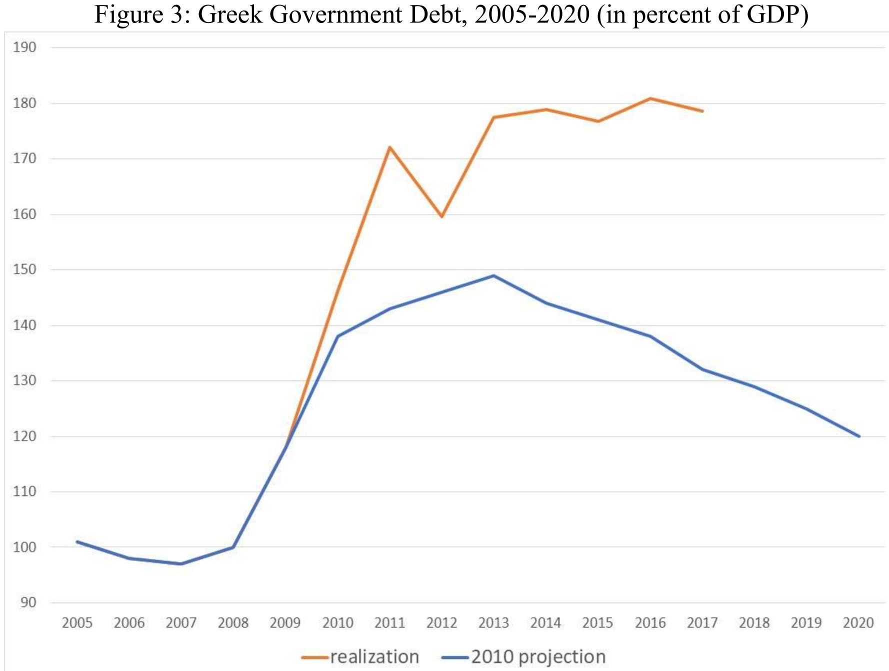
Table 2: real GDP growth in Greece, October 2008 forecast versus realization

	2009	2010	2011	2012
October 2008 forecast	2.0	2.6	3.0	3.5
Realization	-4.3	-5.5	-9.1	-7.3
Forecast – realization	*6.3*	*8.1*	*12.1*	*10.8*

Source: IMF WEO database.

Two main factors seem to have contributed to the divergence of the debt path from initial predictions (Figure 3). First, over-optimism on growth persisted beyond 2008. When comparing actual real GDP growth over the years 2010-2017 with projections in the baseline scenario underlying Greece's program request (IMF, 2010), the shortfall averages 3.4 percentage points per year. Second, and related, the expected fiscal adjustment could not materialize under such depressed conditions, and the average primary deficit exceeded projections by 3.2 percentage points per year during 2010-2017.

Figure 3: Greek Government Debt, 2005-2020 (in percent of GDP)



Source: Eurostat and IMF (2010)

3.3. Assessing Risks to Debt Sustainability

Any forward-looking exercise is subject to uncertainty, a fact that also needs to inform any judgment about debt sustainability.[25] Unexpected developments can affect both policy implementation and economic and financial conditions, possibly pushing debt trajectories way off the "baseline," as illustrated in Figure 1. Like the determinants of fiscal behavior, growth and interest rates, the sources of policy uncertainty and macroeconomic volatility are country-specific. DSA tools typically rely on two distinct methodologies to assess uncertainty: stress tests and probabilistic approaches displayed in the form of "fan charts." We briefly review these two approaches.

3.3.1. Stress Tests

In a purely deterministic world, the determinants of debt dynamics discussed in Section 3.2 would suffice to inform debt sustainability assessments. In an uncertain world, however, that is not the case. Relative to the baseline, growth could disappoint, interest rates could skyrocket, the exchange rate could collapse, fiscal measures may not be implemented as planned (or have unexpected effects), and contingent liabilities could materialize. While good surprises could also happen, the real concern is that the trajectory of the debt-to-GDP ratio could be significantly more worrisome than envisaged in the baseline.

One approach to account for such uncertainties is to design adverse scenarios that capture particularly bad states of the world for debt dynamics. These so-called "stress tests" aim at gauging the sensitivity of the relevant debt (service) indicators to unfavorable conditions. Typical scenarios include shocks to the primary balance, real GDP growth, the rate of interest, and/or the exchange rate. The bare-bone stress tests, such as those embedded in the IMF DSA template, typically consider worse-than-expected realizations for one single determinant of debt dynamics taken in isolation, leaving projections for all other variables unchanged. Such tests are usually calibrated on country-specific circumstances as described by unconditional distributions of relevant variables. Since real-world macroeconomic variables do move together (e.g. lower growth reduces tax revenues, which in turn deteriorates the primary balance), a full-fledged alternative forecast scenario or combined shocks obtained from an estimated model could improve the realism of stress tests.[26]

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[25] Note that the uncertainty surrounding debt sustainability assessment is explicitly recognized in the IMF lending framework. The latter used to require public debt to be "sustainable with high probability" before a member country could be given "exceptional access" to IMF resources. Reforms initiated in 2013 now apply that criterion to guide debt reduction measures in countries where debt is clearly unsustainable to start with.

[26] A middle ground approach is to consider a combination of all shocks happening at the same time. The IMF DSA contains such a combined stress test. However, assuming that all bad news are perfectly positively correlated is barely more convincing than assuming the strict opposite.

Stress tests are also used to analyze the impact of the materialization of contingent liabilities. These are potential liabilities, such as the need to cover losses in state-owned enterprises (SOEs), systemically important private companies, or public-private partnerships (PPPs).[27] Proper calibration requires extensive information about the government's on- and off-balance sheet operations, including exposure to potential SOEs losses, the size of the financial sector (which is of systemic importance in most countries), and the stock of PPPs. While countries following leading international practice in fiscal transparency publish detailed reports on contingent liabilities, this is not the case everywhere, and most stress tests end up being grossly calibrated (e.g. a one-off shock of x percent of GDP).

While well-designed stress tests should in principle inform analysts about sensible boundaries to potentially bad realizations of debt trajectories, they remain a deterministic exploration. In other words, any individual stress scenario, regardless of the care and sophistication underlying its design, has exactly zero chance of ever materializing. Hence, stress tests can even be a distraction as time and resources are wasted to figure out the minutiae of a "plausible" scenario. Shortcuts consisting of standardized stress tests do save time and resources, but they come at the cost of being mostly irrelevant. Indeed, knowing the impact on the projected debt path of a one-standard-deviation reduction in GDP growth for one year (while nothing else happens in the economy compared to the baseline) is not particularly informative.

3.3.2. Probabilistic Methods

A more convincing approach to assess uncertainty is to prepare a very large number of different scenarios to obtain distributions of possible debt outcomes for each year of the forecast. Such information can be summarized in the form of a chart showing these distributions around the baseline debt path. Those so-called "fan charts" not only give a more informative visual of the uncertainty around debt forecasts, they allow for an explicitly probabilistic analysis of debt sustainability. Compared to implausible stress tests, the value of being able to say that public debt has a less than 10 percent probability of reaching its official target by the time of the next election is clear enough.

That said, fan charts are only as informative as the inputs and methods used to generate them. There are indeed many ways to obtain from a computer a large number of randomly generated debt paths, and they are not born equal in terms of the information they capture. Transparency about the "black box" behind random simulations is therefore essential.

[27] A public-private partnership is an arrangement in which a private party provides a public service (being paid for by the government). These arrangements have significant fiscal implications for governments—not only as the PPP contract might require the government to purchase the provider's services for a certain amount of time, but also because the PPP might end up in distress (bringing significant transaction/renegotiation costs).

In a nutshell, there are two polar approaches to build a probabilistic DSA. A simple but rather crude method is to randomly generate alternative debt paths that reproduce past forecast errors. This "reduced-form" approach thus shows analysts how uncertain their current assessment is if they can be assumed to remain as wrong in the future as they were in the past. At the other end of the spectrum, empirical relationships between all relevant variables for debt dynamics (mainly growth, interest rate, and exchange rates) can be estimated and used to generate a series of shocks over the forecasting horizon as well and forecasts for the determinants of debt consistent with those shocks (see Penalver and Thwaites, 2004; Garcia and Rigobon, 2005; and Celasun et al. 2006).[28]

Although it is more demanding in terms of data requirement and maintenance, the second approach is much richer. First, econometric models such as Vector Autoregressions (VAR) are well suited to (i) capture the dynamic linkages between the determinants of debt as well as their underlying steady state values, (ii) to generate plausible sets of random disturbances, and (iii) to produce consistent forecasts for all relevant variables feeding into the debt accumulation equation. The second key advantage of a model-based approach is the possibility to incorporate an estimated fiscal reaction function. Indeed, one thing is to properly account for all the exogenous sources of what can derail the debt path (low growth, high interest rates, etc.), another is to acknowledge that governments do not stay idle in the face of rising public debt. An estimated reaction function also allows to incorporate empirically plausible shocks emanating directly from the budget process and to account for more structural dimensions of a country's capacity to generate primary surpluses, such as good institutions. Table 3 below compares stress testing to the probabilistic DSA.

[28] A host of intermediate options exist to generate random shocks and the corresponding debt paths, including ad-hoc shock distributions.

Table 3. DSA and Risk Assessment

	Deterministic stress-testing	Probabilistic approach (model-based)
Diagnostic based on...	...a few stylized, isolated shocks; exogenous policies.	...many random shocks drawn from an estimated joint distribution; endogenous fiscal policy.
Calibration of shocks	Fraction or multiple of historical standard deviations of underlying variables.	Based on the estimated joint distribution of disturbances.
Output	Large temporary shocks provide a probabilistic upper bound to the debt ratio; small permanent shocks delineate interval of most probable outcomes.	Frequency distributions of the debt ratio over time, "fan charts." Gives a sense of the most likely range within which future values of the relevant debt (service) indicators are likely to lie.
Main advantages	Amenable to standardized stress tests across countries; low data requirement.	Better reflection of country specificity (in terms of shocks and fiscal policy behavior); explicitly probabilistic output.

Sources: Adapted from Celasun et al. (2006).

3.4. Other Approaches

Interest in public debt sustainability and the risks around it goes well beyond official institutions and country authorities. This section focuses on rating agencies and financial market participants.

3.4.1. Rating Agencies

Rating agencies have a direct business interest in providing well-informed assessments of public debt sustainability. Their views not only affect investment decisions (particularly those of institutional investors who are often constrained in terms of the quality of their portfolio), but also put ceilings on the ratings assigned to debt issued by domestic companies, and influence interest rate spreads (see Section 3.4.2.).

Typically, sovereign debt ratings indicate a sovereign's ability and willingness to service debts to nonofficial creditors. While a failure to repay financial obligations to official creditors by itself is not considered a default event, the implications for the servicing of debt to nonofficial creditors are taken into account.

There are about 150 credit rating agencies worldwide, only a handful of which rate sovereign debt on a global scale (Fuchs and Gehring, 2017). Within this limited set, the landscape is dominated by the "Big Three" consisting of Fitch, Moody's, and Standard & Poor's (S&P)—who, together, hold the lion's share of the market.

Comparing the rating methodologies employed by the Big Three (described in Fitch (2016), Moody's (2016), and S&P (2015)) shows a great deal of similarity in terms of focus areas. By and large, they arrive at their final rating by assessing the following five factors:[29]

i) *Institutional/structural factors*: government quality and stability, default history, transparency and accountability of government.
ii) *Macroeconomic factors*: level of economic development, growth, volatility.
iii) *Fiscal factors*: debt stock, fiscal balance, contingent liabilities.
iv) *Monetary factors*: inflation level and volatility, credibility of monetary policy, impact of monetary policy on the real economy.
v) *External factors*: Balance of Payments, Net International Investment Position, external debt stock.

The similarity in relevant considerations and underlying approaches translate into a broad agreement among the Big Three on the rating of debt issued by different countries: Hill, Brooks, and Faff (2010) report that 82 percent of country ratings accord within one grade.

In practice, ratings combine the output of an econometric model and qualitative judgment (see e.g. Fitch (2016)). This judgment is typically forward-looking—thereby incorporating information which the (mostly backward-looking) econometric exercise lacks. Judgment-based adjustments can for example occur if a country bases its projections upon an inconsistent macroeconomic framework or in the presence of large contingent liabilities; S&P also assigns lower ratings to debt issued by members of a monetary union (citing reduced flexibility relative to sovereigns with their own Central Bank; S&P, 2015).

While the above considerations suggest several key drivers of sovereign credit ratings, the empirical literature has sought to identify the main ingredients. In a seminal study of the ratings issued by Moody's and S&P, Cantor and Packer (1996) find the level of economic development (+), GDP growth (+), inflation (-) external debt (-), and default history (-) to be the most important determinants. Interestingly, fiscal and external imbalances do not seem to matter much. One plausible explanation is that markets ultimately force countries with poor ratings to run strong fiscal and current account balances. Their results furthermore suggest that Moody's and S&P largely base their ratings upon the same criteria, although Moody's appears to put more weight on external debt and less weight on default history relative to S&P.

[29] Note that the first three factors are at the core of the fiscal space literature pioneered by Ostry et al. (2010) and Ghosh et al. (2013). Several rating agencies, including Moody's, rank advanced economies in terms of fiscal space availability.

More recently, Afonso et al. (2011) study sovereign credit ratings issued by the Big Three over the period 1970-2005, using a refined methodology and a larger sample than Cantor and Packer (1996), their results suggest that government effectiveness (+), the level of external debt (-), that of foreign reserves (+), as well as a country's default history (-) have a significant and persistent impact on a sovereign's credit rating.[30] At the same time, per capita GDP (+), real GDP growth (+), government debt (-), and the fiscal deficit (-) have a significant short-run effect. Compared to earlier studies, the greater influence of fiscal variables is worth noting.

Aside economic fundamentals, Fuchs and Gehring (2017) suggest that sovereign credit ratings are subject to a home bias. Using data from nine credit rating agencies, headquartered in six different countries, they report that agencies assign higher ratings to their home countries, as well as to countries which are aligned along cultural, geopolitical, and economic dimensions. In particular, sovereigns to which home-country banks have greater risk exposure obtain more favorable ratings than warranted by their fundamentals. They conjecture that the main transmission channel is cultural proximity, which is argued to be associated with a more optimistic perception of risk because of higher mutual trust.

In the end, the real hard question about credit ratings is whether they provide information that would not be (or be less easily) available otherwise, notably in sovereign bond spreads. Reinhart (2002) argue that ratings merely lag market developments, while Ferri et al. (2001) suggest that they amplify ongoing developments, making bad situations worse, and good ones, exuberant. As explained in Cavallo et al. (2011), ratings really add value if they help make sense of other market variables (while controlling for sovereign spreads and other fundamentals). A failure to do so implies that credit ratings can—at best—be seen as lagging market developments. Exploiting a large dataset of 75,000 daily observations from emerging markets, Cavallo et al. (2011) find that sovereign ratings are significant *predictors* of future spreads, stock market indices, and exchange rates, suggesting that they do add value over and above the information contained in sovereign bond spreads.

3.4.2. Sovereign Spreads

In principle, debt sustainability assessments can also be informed by sovereign spreads, which can be thought of as proxying default risk. These spreads can be obtained from bond prices or from Credit Default Swaps (or CDS, financial agreements whereby the seller guarantees to compensate the buyer in case of default on an underlying debt contract). Of these two, the latter offers the purest measure of default risk, particularly since bond spreads may also embed inflation expectations (Aizenman, Hutchison, and Jinjarak, 2013). However, for countries that borrow in foreign currency (e.g. US dollars or euros, as many emerging markets do), the

[30] The negative impact of a country's repayment history on perceived creditworthiness is also documented in Reinhart, Rogoff, and Savastano (2003).

inflation premium can be eliminated. Consequently, many studies analyzing sovereign spreads in emerging markets have used the EMBIG spread index. [31]

The most relevant empirical question in the context of this chapter is whether fiscal fundamentals have the expected influence on sovereign spreads. The literature analyzing the determinants of sovereign spreads goes back to Edwards (1984), who find a positive but statistically weak association with the debt- and debt-service-to-GDP ratios. By contrast, higher investment and international reserves (also scaled to GDP) tend to reduce spread, with the latter playing a particularly big role. While the variables included in Edwards' regression mostly capture a country's *ability* to service its debt, a country's *willingness* to do so is important as well (Bulow and Rogoff, 1989). Proxying willingness by the level of external payment arrears, Boehmer and Megginson (1990) find that countries that signal a reluctance to service debt (by accumulating arrears) face higher spreads. Using bond-based primary yield data,[32] Min (1998) identifies a wider set of variables to play a role in determining the spread—including the terms-of-trade, the real exchange rate, the rate of inflation, and the level of net foreign assets. Like Edwards (1984), Min (1998) does not find a significant role for fiscal variables in explaining spreads. Other studies report mixed results.

However, once one accounts for *the composition* of fiscal policy, a clearer picture emerges: bond markets seem to distinguish between government spending and government investment—with Peppel-Srebny (2017) reporting that a higher deficit solely due to higher public investment lowers borrowing costs. This suggests that markets believe that the returns to such investments improve the sustainability of a given debt level. Akitoby and Stratmann (2008) moreover find that revenue-based adjustment reduces spreads more than spending-based adjustment, while debt-financed spending widens spreads.

With respect to the fiscal positions of countries, evidence suggests that its impact on spreads might be non-linear: after adding the square of the ratio of public debt to GDP, Ardagna, Caselli, and Lane (2007) for example document that an increase in debt only leads to a widening of spreads when the country's debt stands at above-average levels (about 65 percent of GDP in the OECD-sample underlying their study).

[31] JP Morgan's emerging markets-focused EMBIG database includes Brady bonds, Eurobonds, traded loans denominated in US dollars, and local market debt instruments. Only issues with a remaining maturity of 2.5 years or more (and face value greater than USD 500 million) are included (see JP Morgan (1999) for details).

[32] As argued by Eichengreen and Mody (2000), looking at secondary spreads is preferable and care should be taken in using primary yields. The reason is that results from primary issues are likely to suffer from a selection bias: when financing conditions toughen, riskier borrowers might drop out of the market and not place any new bonds. As a result, it is possible that poor market conditions lead to a situation in which primary and secondary spreads move in opposite directions.

As noted before, sovereign risk can also be proxied by CDS spreads (arguably in a cleaner way). In principle, spreads on bonds and CDSs of similar maturity should reflect the same information about creditworthiness, implying that there should be no profitable arbitrage possible between them. Indeed, the empirical literature generally finds similar determinants for CDS spreads as for bond spreads (Aizenman, Hutchison, and Jinjarak, 2013). However, their relative pricing seems more involved than basic theory suggests.

Discrepancies between CDS and bond spreads of similar maturities (known as the CDS-bond "basis") reflect somewhat subtle differences in specific features of these contracts. Ammer and Cai (2011), who document considerable short-run deviations in emerging markets over the period 2001-2005, suggest that the non-zero basis reflects the "cheapest-to-deliver" (CTD) option associated with CDSs. Standard CDS contracts do not insure a specific debt contract, but *any* debt claim on the underlying country. In such a case, a CDS holder would present the least valuable eligible claim for payment. This "freedom of choice" offered by the CDS contract is valuable for the insured and would explain observations of non-zero a basis. Different degrees of liquidity could also get in the way of the zero-basis hypothesis, as documented by Fontana and Scheicher (2016) during the Euro Area sovereign debt crisis—a time when many arbitrage traders left the market.

3.5. Beyond Debt

Like all economic agents, the government has a balance sheet, an account that collects the *stock* of all assets and liabilities of the public sector. However, unlike most private agents, and certainly listed companies, it is often difficult to know what that balance sheet really looks like, either because it is not published or does not even exist. In the context of this chapter, ignoring the government balance sheet is more than a mere issue of fiscal transparency (or the lack of it). It forces us to ask whether the focus on *gross* public debt—only one component of the balance sheet—might not be too partial a way to look at the financial sustainability of the public sector.

Because the government is big, assembling its balance sheet is a daunting task. The asset side includes items like the present value of all future tax revenues over the indefinite future, financial assets, and publicly owned natural resource waiting to be extracted. To that, one should add non-financial assets like public infrastructure, national parks, architectural wonders, and cultural treasures, all of which have no market value and whose price is consequently unknown. Aside gross public debt, liabilities include pension obligations and other public employees' benefits, clearly big-ticket item, particularly in countries with unfunded pension systems. The difference between assets and liabilities—the "bottom line"—is the government net worth.

Like solvency, net worth is a theoretically clear-cut concept, and a conceptually attractive basis to define "sustainability." And like solvency, net worth is fully intertemporal and defined over the very long-term. For instance, pension liabilities reflect the *present value* of all future payments currently committed to specific beneficiaries at a certain date (e.g. public employees), and all this at *unchanged policies*. Arrow et al. (2004) suggest that a *non-decreasing net worth* is the right concept of sustainability.

However, like solvency, achieving a non-decreasing intertemporal net worth (INW) has no operational meaning. It is arguably too far from anyone's immediate concerns, which have more to do with liquidity and the ability to service current obligations. To be clear, concerns about net worth do not cause sovereign crises or defaults, unmanageable gross debt dynamics does. This likely explains why most countries (New Zealand is one notable exception) do not explicitly refer to net worth in the framework guiding the conduct of fiscal policy. For the sake of transparency, more and more countries publish balance sheet and the related analysis. However, the very fact that they do not seem concerned to publish abysmally negative numbers for net worth is telling. Either they do not realize that they are declaring insolvency, or nobody (especially bondholders) cares. After all, exhibiting a negative net worth of 300 percent of GDP is the same as publishing 50-year projections showing gross public debt rising to 500 percent of GDP at the end of the horizon: it has no current impact. Taxpayers may ultimately care, which will prompt gradual policy correction, which will in turn invalidate these numbers.

Aside conceptual considerations, the impracticalities related to INW are comparable to those associated with solvency.[33] The number obtained is just too sensitive to key assumptions to mean much. Data availability and valuation issues are pervasive: what is the value of a free museum, a freeway, or a natural park? The intertemporal nature of INW also makes it highly sensitive to assumptions about discount rates, just in the same way as the intertemporal budget constraint can be fulfilled with small variations in long-term growth or interest rate assumptions.

In practice, interest in the balance sheet approach suggests considering net public debt instead gross debt to assess sustainability. For countries with significant liquid financial assets, DSA tools could arguably be run using a net debt metric, an approach supported by the IMF when countries' own fiscal framework use net debt as a reference (or anchor). That said, question marks remain about whether fire sales of state assets would be feasible to cope with severe liquidity stress, even if the capital losses that such sales could entail probably pale in comparison of the costs of a default or restructuring.

[33] Solvency and the IWN are intimately linked. For instance, the European Commission estimates the IWN as the difference between the current net worth and the present value of all future primary balances *required to fulfill the intertemporal budget constraint* (the so-called S2 indicator). The Commission's S2 sustainability indicator is simply the wedge in the intertemporal budget constraint (i.e. the difference between current gross debt and the present value of all future primary balances).

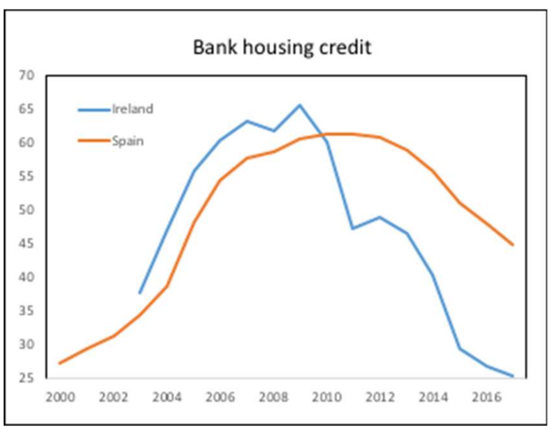
If the value of the INW for sustainability purposes remains in doubt, a complete government balance sheet nevertheless allows a much better grasp on the risks facing the entire public sector. Shocks to the balance sheet are often absorbed by the public debt. Hence designing stress tests reflecting the full exposure of the balance sheet to adverse developments can certainly improve upon the stress testing approach included in DSA tools and discussed above (Clements et al., 2016). Better knowing the overall exposure of public debt to shocks elsewhere in balance sheet certainly has value in light of the potentially devasting effects of such shocks, as illustrated in Box 3.

Box 3—Doom Loops or When Contingent Liabilities Strike: Ireland and Spain

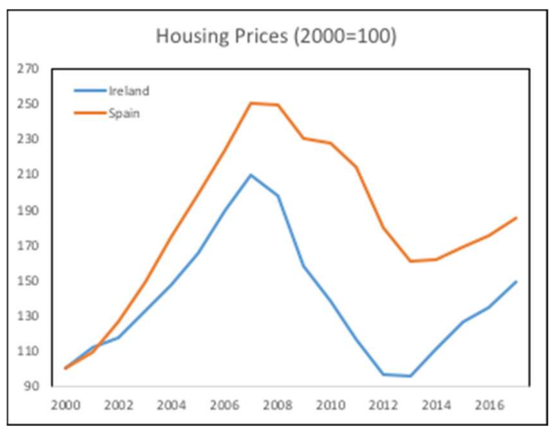
In the years that preceded the Eurozone crisis, Ireland and Spain had successfully reduced their public debts, from 108 percent of GDP in 1988 to 24 percent in 2008 for Ireland, and from 57 percent in 1996 to 36 percent in 2007 for Spain. Not only were these debt ratios among the lowest among the developed countries, there was no doubt that both countries were fiscally disciplined and had sustainable debt levels. Following the crisis, the Irish government debt nearly quintupled, the Spanish one nearly tripled. This was the 'doomed loop' effect (Brunnermeier et al., 2016; Farhi and Tirole, 2018).

The Irish and Spanish crises are classic example of implicit liabilities, first ignored and then unavoidably acknowledged. In this case, they arise from poor bank supervision. The right-hand chart in Figure 4 shows the rapid buildup of house lending by banks that started in the early 2000s to almost double by 2010. This buildup fueled and was fueled by increases in housing prices, which more than doubled in bubble-like fashion. Following the turnaround in US housing prices, which had followed a similar pattern, Irish and Spanish prices also peaked and started to decline.

Figure 4. Housing credit and prices (% of GDP)



Source: WEO

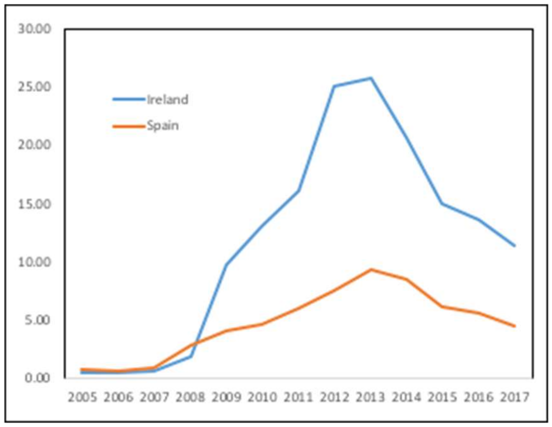


Source: BIS

As prices fell, borrowers started hold on to loan servicing, which led to a sharp increase in non-performing loans, especially acute in Ireland. Facing a banking crisis, the Irish authorities declared that the totality of bank deposits would be protected. This required injecting considerable amounts of money in banks. As member of the Euro Area, Ireland could not use the central bank as lender of last resort. Consequently, the government had to borrow on the markets. Even though its debt was initially small, its ratio to GDP rose to 110% of GDP in 2010, partly because the ensuing recession slashed nominal GDP by 15 percent between 2007 and 2010. Ireland all but lost market access. When Ireland was forced to apply for emergency support from the IMF and the European Stability Mechanism, other weaknesses were revealed, including an excessive rise in labor costs during the Great Moderation (Lane, 2011). The current account saw a sharp correction (from a 6.4 percent of GDP deficit in 2007, to a 1.4 percent deficit when the crisis hit, and a surplus soon thereafter, as labor costs were slashed).

As Figure 5 shows, the impact of falling real-estate prices on Spanish banks was more moderate. This allowed the authorities to borrow less (in proportion to GDP) and to retain market access, although at an increasingly higher cost. As a result, Spain was able to successfully negotiate support from the European Stability Mechanism without a full-blown program. Yet, public debt reached 100 percent of GDP by 2014 and has since stabilized close to that level.

Figure 5. Bank Nonperforming lows (% of total bank gross loans)



Source: WEO.

In both cases, the doom loop means that bank liabilities were transferred to the government. As a result, the government had to borrow at higher interest rates, which affected the whole economy. This worsened the situation of banks, requiring further public support.

4. Debt Limits and Fiscal Space

Because unsustainable debt paths are ultimately not sustained, accidents happen. Or are these not really accidents? This section reviews the main reasons why governments find themselves unable or unwilling to service their debt.

4.1. When Do Countries Lose Control of Debt Dynamics?

The model of fiscal fatigue (Ostry et al., 2010; Ghosh et al., 2013) discussed earlier leads in a straightforward way to the concept of debt limit. In particular, if following some threshold, the primary balance no longer keeps up with the higher effective interest payments needed to service the debt, then there will necessarily be a debt level above which the debt dynamics become explosive. At that point, the government must either undertake extraordinary fiscal adjustment (where extraordinary means a break with its fiscal reaction function based on history) or default on its debt. Default in this setup occurs because of an inability to pay, not for strategic reasons (willingness to pay).

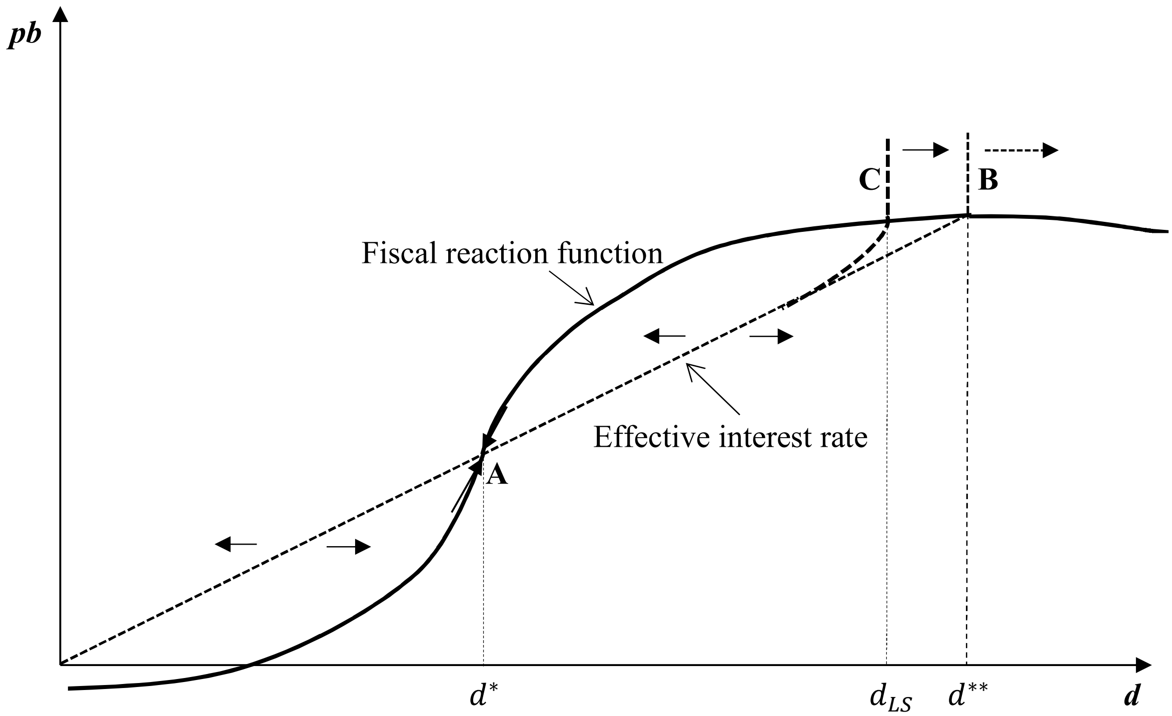
Of course, creditors will not be willing to lend to the sovereign at or near the point where default is imminent and will instead demand an increasing risk premium as debt approaches its limit. The general stochastic case, discussed and solved in the aforementioned papers, is rather complicated owing to the joint endogeneity of the risk premium and the default probability. Figure 6 provides a heuristic treatment. The solid line is a stylized representation of the behavior of the primary balance as a function of debt. At very low levels of debt, there is little response of the primary balance to debt. As debt increases, the balance responds more vigorously, but eventually the adjustment effort peters out as it becomes increasingly difficult to raise taxes or cut primary expenditures. The dotted line in the figure represents the effective interest rate schedule. At low debt levels, the interest rate is the risk-free rate and, assuming that output growth is independent of the level of public debt or the interest rate, this schedule is simply a straight line with slope given by growth-adjusted risk-free real interest rate.

The lower intersection A between the primary balance and interest rate schedules defines the long-run public debt ratio d^* to which the economy normally converges. This equilibrium is conditionally stable: if a shock raises debt above this point (but not beyond the upper intersection), the primary balance in subsequent periods will more than offset the higher interest payments, returning the debt ratio to its long-run average.

There is another (upper) intersection as well, however. Abstracting from stochastic shocks and the endogeneity of the interest rate, this intersection B yields a debt limit d^{**} above which debt is unsustainable: if debt were to exceed this point, it would rise forever because, in the absence of extraordinary adjustment, the primary surplus would never be enough to offset the growing debt service. At such a point, the interest rate becomes infinite as the government loses market access and is unable to rollover its debt. In the presence of stochastic shocks to the primary balance and an endogenous response of the interest rate to rising risk, the interest rate schedule of course is not simply the extrapolation of the risk-free rate, but rather bends upward as debt approaches its limit. In such a case, the debt limit d_{LS} is defined by the point C at which there is no finite interest rate that solves the fixed-point problem between the default probability and the interest rate (as debt rises, default risk rises which requires a higher yield to compensate investors; and the higher yield in turn raises the default probability).[34]

[34] Above the debt limit, the fixed-point problem no longer has an interior solution with a default probability strictly less than unity, and the sovereign must default because it can no longer borrow at a finite interest rate.

Figure 6: Determination of Debt Limit



Source: Ostry et al. (2010)

It is readily seen that the debt limit is increasing in the country's willingness to undertake fiscal adjustment (as captured by the position and shape of the fiscal reaction function) and decreasing in the growth-adjusted real interest rate.[35] A shock to either one of these schedules could thus put the country on the wrong side of the debt limit. Indeed, the mere recognition that negative shocks to the primary balance could be larger than previously considered would imply a higher probability of default for any given level of debt—and a larger risk premium. This would steepen the growth-adjusted interest rate schedule and shift the debt limit leftward—possibly driving a formerly sustainable country into a situation of unsustainability.

A key policy implication, given the potential for surprises that can push a country from a sustainable to an unsustainable position rather quickly, is that if a country is close to its estimated public debt limit, timely policy measures are needed to increase the probability that debt will remain on a sustainable path and convince markets that fiscal policy is not proceeding on a "business as usual" basis. Moreover, since the analysis does not take into account rollover

[35] Bi (2012) studies the interactions among sovereign risk premia and fiscal limits, whose state-dependent distributions depend on the growth of lump-sum transfers, the size of government, and the degree of counter-cyclicality of policy. A nonlinear relationship emerges in equilibrium between sovereign risk premia and the level of government debt, which is in line with the empirical evidence that once risk premia begin to rise, they do so rapidly (a finding also in the Ostry et al. (2010) and Ghosh et al. (2013) papers).

risk, governments will typically want to keep their debt well below the estimated limit, to ensure that fiscal space remains comfortably positive (Debrun et al. 2018).

4.2. Why Do Countries with Sustainable Debts Default?

Implicitly, ability to pay is about solvency. However, the concept of solvency does not strictly constrain governments ex-ante for two crucial reasons.[36] First, governments cannot be liquidated like corporations. Second, government incomes are a proportion of the country's GDP. While the proportion is bounded, the effective upper bound is a fuzzy technical issue (in the spirit of the Laffer curve) but more realistically a political matter. Thus strategic and political considerations enter into the determination of the maximum primary surplus and the period during which it can be kept large enough to service the debt. This is the intuition of the seminal contribution by Eaton and Gersovitz (1981) who propose to focus on the willingness, not the ability, to pay.

Since a government cannot be liquidated, default results from a cost-benefit analysis. Since defaulting amounts to a capital gain, there must be associated costs. Without them, default would always be the dominating option, which would imply that governments could never borrow. In this view, the very existence of public bonds is predicated on the presence of costs of defaulting. A large literature examines what these costs are.

Another key insight of the willingness to pay approach is that, if the driver of defaults is not a mechanical implication of the balance sheet, lenders must assess when and why governments choose to withdraw debt service. This, in turn, implies that lenders will act to avoid being caught inadvertently and, if possible, to affect the borrowers' actions, both in building up reasons to default (moral hazard) and in defaulting. Lenders are not passive agents that can only recognize the inability to pay. They are likely to carefully evaluate the borrower, including its likely options in the future. The implication is that debt service and default should be analyzed in a strategic interaction context (game theory). Each side will influence the other side. Decisions concern all possible instruments, including loan size and design, the interest charge, the formal write-up of the contract, the use of courts by lenders and avoidance measures by borrowers, political pressure and more. In this context, the IMF, as a lender of last resort, becomes a part of the game because its role affects the penalty—the Fund provides an alternative source of borrowing, and affects the costs through program conditionality while improving the eventual outcome.

[36] "In a formal sense, insolvency is not really an issue in lending to foreign governments. The debt of a country in almost all instances is less than the value of the assets owned by nationals and the government of the country. There may be limits on the extent to which governments can appropriate the assets, but these limits themselves are, in general, not hard and fast constraints, but involve trade-offs" (Eaton et al. 1986, p. 484).
"'Ability' to pay is never truly an issue" (Bulow and Rogoff, 1989a, p. 156).

The literature on willingness to pay therefore considers three related issues. First, what are the costs that make sovereign lending possible? Second, what are the legal avenues that lenders can use? Finally, how have these aspects evolved over time as the game has become more sophisticated. We start with the costs.

4.2.1. Exclusion from financial markets

Eaton and Gersovitz (1981) assume that the loss of market access costs is the penalty imposed in the event of a default. A country that defaults is not able to borrow anymore. In their early contribution they reach several conclusions, most of which have been upturned subsequently, including in their own work.

A first result is that the exclusion must be permanent. Otherwise, once the default has occurred, both borrowers and lenders will have an incentive to renegotiate a new loan as explained in Section 5. Permanent exclusion is not a renegotiation-proof equilibrium so it must be imposed no matter what. Second, the cost to the borrower of permanent exclusion must be higher than the gain from defaulting. To that effect, lenders must impose a ceiling on the total amount lent, since this is potentially what the defaulting country can appropriate. The larger the cost of market access loss, the higher is the ceiling.

The cost of exclusion, therefore, must be specified. Governments wish to borrow to smooth adverse shocks or to invest in presumably productive spending. The frequency and amplitude of likely shocks therefore determine the benefits from borrowing: the larger they are, the more the borrower stands to lose from loss of market access and, hence, the more can be lent. Similarly, 'good' governments that can spur long run growth through well-chosen investments are more likely to suffer from exclusion than 'bad' governments that misuse public spending.

The evidence surveyed in Panizza et al. (2009) is that exclusion is never permanent. In fact, it is quite short-lived (a few years, at most, see e.g. Sandleris, 2016; Sandleris et al., 2004; Richmond and Dias, 2009). The period of exclusion has also become shorter over time, especially so once an agreement with stakeholders is reached. Market exclusion occasionally occurs before defaults and ends with the default. In addition, agreements with stakeholders take an average on two years.

Temporary exclusion can be justified theoretically. Kovrijnykh and Szentes (2007) show how it may be in the interest of the lender to resume lending after a default prompted by a series of adverse shocks. Eventually, positive shocks will allow the lender to recover some of the defaulted debt *and* to make profits on new loans. The situation is a little bit trickier when a large number of lenders compete, but the conclusion remains. First, lenders who hold senior debts can block other lenders to step in as long as they are all subject to court decisions. Second, all lenders share the same interest. They should therefore act in the same way, especially if they are bound by common rules, such as bondholders' committees and *pari passu* agreements enforceable in the lenders' jurisdictions.

4.2.2. Reputation and borrowing costs

Another penalty that affects the willingness to pay is the implication of a default for a country reputation. When there is uncertainty about a debtor's willingness to pay, lenders will require a risk premium. A long period of faithful debt service should lead to a gradual reduction of the risk premium. A default will reveal the country's unwillingness to pay and this reputation loss will lead to a high premium. Loss of market access is the limit case when the risk premium become infinite or just unbearably high and the same questions arise as in the case of exclusion: how much and how long?

The empirical evidence suggests that the borrowing costs rise following a default, once the country can re-access markets. The increase may be steep but it is not lasting very long (Borensztein and Panizza, 2009). This matches the evidence that market access loss is temporary. It also suggests that higher borrowing costs are unlikely to explain why countries serve sovereign debts and why lenders provide loans to sovereigns.[37]

4.2.3. Legal sanctions

Another way of imposing costs on defaulting borrowers is to seek legal authorization to impose a variety of sanctions. Long gone is the use of gunboats and other military means. Instead, lenders may pursue legal avenues. In theory, assets may be seized and trade forbidden, either directly of by withholding trade credit or by the use of the banking system to settle payments. This range of options bring to the fore the issue of the powers of courts to constrain sovereigns. Panizza et al. (2009) provide an extensive review of this issue.

At stake is whether a court, say in the United States, has the legal authority to judge a foreign government at the request of domestic private lenders. For a long time in the 20th century, such an authority was severely limited. Over the last half-century, the legal doctrine has evolved. The US adopted in 1976 the Foreign Sovereign Immunities Act (FSIA) and a number of countries followed suite. Private lenders may take to court sovereign entities (governments and their agencies) in the case of commercial disputes. Even so, restrictive conditions apply, which makes the legal firepower far more limited than in the case of disputes among private entities. Importantly, it may be impossible to enforce judgment decisions.

Seizing assets is a natural starting point. Assets to be seized include pledged collateral but also state-owned subsidiaries, exports of state-owned firms, payments for exports, government assets and reserves of central banks. Collateral is shunned by borrowers because it effectively

[37] Bulow and Rogoff (1989a) showed that reputation effects are unlikely to sustain lending.

makes defaulting a non-option (Eaton et al., 1986). Over the years, courts have been more open to order asset seizure, but many defaulting countries – or would-be defaulting countries – have managed to shield much of these assets. A striking example is foreign exchange reserves that are deposited with the Bank for International Settlements, which is protected by an international treaty. Of course, lenders and the courts have also improved their legal tactics in a never-ending game.[38]

Collective lending is a particularly important battleground. Increasingly, lenders to sovereigns, which used to be mostly a few large international banks, are bondholders of various sizes. This makes default negotiations cumbersome. As a result, collective action clauses have become widespread and are the default option in many jurisdictions. They usually require super majorities, e.g. 85% of bondholders, and may include mandatory changes in the initial loan agreements. This has led a number of investors – called distressed debt funds or vulture funds – to buy bonds at deep discounts on the secondary market and then start legal proceedings to achieve better terms than the investors who reached an agreement with the sovereign. The holdouts attempt to buttress their position by asking the right to attach assets that are part of the agreement, which stands to fatally undermine the agreement. The jurisdiction has been in flux, oscillating between conflicted legal logics, as described in Panizza et al. (2009). On the one hand, the courts have recognized the rights of holdouts to litigate, on the other hand they have shown an interest in upholding agreements between sovereign and a strong majority of bondholders. Vulture funds first achieved some successes but the assessment of Panizza et al. (2009) was that "full repayment has remained the exception, and many holdouts have received nothing". This changed in 2014 when a New Court backed a group of vulture funds (led by NML Capital, a subsidiary of Elliott Management) that was blocking the agreement reached after more than a decade between Argentina and a majority of its creditors. NML Capital achieved a return of some 1000%.

This is not the end of the story. Some countries have indicated interest in adopting legislation against holdouts. What is clear, though, is that the traditional view, that sovereigns are largely protected from the courts, no longer applies. In the extreme case when holdout rights are always fully protected, rational sovereign borrowers would probably never default. Backward induction suggests that they would exercise great caution in the first place while lenders will have incentive to increase loan sizes. This issue remains unexplored.

4.2.4. Renegotiation

Another view is proposed by Bulow and Rogoff (1989b), whereby sanctions are only a threat, which is not to be carried out. They view the relationship between borrowers and lenders as an infinitely continuing process of negotiations and renegotiations, not a one-off loan. In this view, each side compares what is to be lost from breaking up negotiations. They argue that,

[38] Lastra and Buccheit (2014) provides an excellent collection of relevant papers.

usually, an agreement to restructure the debt is beneficial to both. Under some conditions, the borrower obtains some debt write-down and serves the rest, which is less costly than the sanctions, while the lender receives higher payments than if the relationship breaks downs. Thus, sanctions are never applied but they serve as a threat point in the renegotiation process.

4.2.5. Domestic costs

So far, the costs of defaulting have been those imposed by the lenders. Another branch of the literature points out that defaulting implies large domestic costs as well. While exclusion from financial markets may not last very long, limiting the financial socks, it lasts long enough to impose large macroeconomic costs. Indeed, we know from Reinhart and Rogoff (2009) that sovereign defaults are systematically accompanied by deep recessions. Many channels may be at work. Some may be related to sanctions, for example on trade or on loss of market access for the private sector that weakens domestic financial institutions and therefore depresses domestic borrowing. Mendoza and Yue (2012) shows that this accords well with the evidence. Panizza et al. (2009) consider that the domestic costs may be much larger than those of international sanctions considered so far. Other domestic costs may be related to distributional issues. Thus, D'Erasmo and Mendoza (2016) note that a large fraction of public debts is typically owned domestically. If the government cannot discriminate between domestic and foreign bondholders, for instance because sufficient information is not available, a default affects domestic bondholders while benefitting the remaining citizens. This redistribution effect – which is well documented – carries serious domestic implications.

Finally, a debt default may have domestic signaling effects (pooling equilibria in the case of asymmetric information). Sandleris (2016) gives an example where the quality of the government or the state of the economy are unknown. Defaulting provides information on either of these characteristics, which may have wide repercussions. It can affect investment decisions by firms, saving decisions by households, lending strategies of banks or trade union militancy. In all cases, a default can precipitate domestic shocks and capital outflows that result in a deep recession and lead to economic and political disturbances, as is found in the data.

4.2.6. The role of the IMF [preliminary]

The IMF affects the implications of the willingness to pay theory. The availability of IMF loans reduces the costs of exclusion from the markets, tilting the game in favor of borrowers. At the same time, the conditions associated to loans represent another type of cost to be borne by a defaulting government, through both economic and political implications. The balance of these effects on the willingness to pay can go either way depending on each case. This may be a reason why the IMF cannot, and does not, walk away from the default decision. For a while, the IMF seemed to have been sympathetic to defaults until it became more prudent not to appear as siding with the borrower, with possible legal implications as lenders blame the Fund for their losses. Debt Sustainability Analysis (DSA) represents an attempt to frame the question within a solid technical framework. However, as argued in Section xxx, DSA inevitably rests on assumptions that extend far into the future so that the results are inherently uncertain. Eichengreen et al. (2018) show that the results are highly sensitive to the assumptions so that the degree of uncertainty is large.

4.2.7. Conclusion

The willingness to pay approach yields results that seem to match the evidence. First, the decision to default is not driven by solvency conditions but by the strategic considerations of borrowers and lenders.[39] Second, if legal venues cannot be binding, loans to sovereigns can only exist if the borrower suffers from costs in the event of a default. Third, there are many potential costs. Empirically, domestic implications of defaulting and the benefits from an on-going relationship seem to be the most important factors sustaining sovereign borrowing. Fourth, the evidence is that sanctions and interest rate penalties are rather short lived. Fifth, the possibility to use legal means available to lenders to impose sanctions and recover losses is continuously evolving in favor of lenders.

4.3. Empirical approaches to assessing debt sustainability

In parallel to the theoretical contributions discussed above, purely empirical approaches have been developed to identify debt limits. This literature is rooted in studies that aim to find the determinants of fiscal stress episodes, typically defined as instances of a default, a restructuring, or an IMF program of significant size. Manasse, Roubini, and Schimmelpfennig (2003) made an early contribution to this so-called "early warning" literature. Combining data from 47 emerging markets over the period 1970-2002 with logistic regression,[40] they find that fiscal crises are more likely in the presence of high external debt, high short-term debt, high debt-service payments, a negative current account balance, tight US monetary policy, low real GDP growth, high inflation (volatility), as well as political uncertainty.

Manasse, Roubini, and Schimmelpfennig (2003) solely focused on emerging markets. Building upon their contribution, Kraay and Nehru (2006) extended their work to low-income countries—finding that fiscal crises are also more likely to arise in countries with weak institutions: when worsening a country's institutions from the 75th percentile to the 25th (whilst holding all other variables at their median), the probability of distress more than doubles to 27 percent (from 12 percent). In low-income countries, the effect of institutional quality is reported to be even larger (while it is not significantly different from zero in middle-income countries).

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[39] Illiquidity may be another reason to default, but this should be a symptom of market failure.

[40] Logit estimation is a statistical approach to do regression analysis on a binary dependent variable ("in fiscal stress" versus "not in fiscal stress"). The specification ensures that the resulting predicted probability of fiscal stress always lies between 0 and 1 (as probabilities should be).

Armed with an empirical specification with which one can estimate the probability of fiscal stress (given a country's fundamentals included in the regression), it is also possible to obtain thresholds above which debt (or debt service) is deemed unsustainable. This is the approach underlying the IMF's debt sustainability framework for low-income countries (see IMF (2017)). The idea is to set a cutoff probability π^* beyond which the risk of a fiscal crisis occurring (as implied by the regression equation) is deemed too high.[41] One can then back-out the associated threshold values of the debt (service) indicators which would imply a π^* probability of a fiscal crisis developing, given average values for other regressors in the equation. Table 4 shows the resulting debt thresholds in IMF's current debt sustainability framework for low-income countries.

Table 4. Thresholds in the IMF's debt sustainability framework for low-income countries

Debt carrying capacity	PV of PPG external debt		PPG external debt service		PV of total public debt
Weak	30% of GDP	140% of exports	10% of exports	14% of revenue	35% of GDP
Medium	40% of GDP	180% of exports	15% of exports	18% of revenue	55% of GDP
Strong	55% of GDP	240% of exports	21% of exports	23% of revenue	70% of GDP

Note: Debt carrying capacity is country-specific and determined by a country's score on a composite indicator, combining the quality of institutions, its growth rate, remittances, reserve levels, and world growth. PV = present value. PPG = public and publicly guaranteed.
Source: IMF (2018).

Thresholds may however be highly country-specific, as suggested by Reinhart, Rogoff, and Savastano (2003). They note that certain countries seem much more willing to default than others, calling such countries "debt intolerant". The country-specific level of debt intolerance typically remains unobserved, which might explain why some countries default when debt stands at 40% of GDP, while others continues to service their debt at such a ratio (and might even be happy to borrow more). The authors conjecture that a sequence of past defaults can weaken a country's institutions (including financial intermediation)—lowering default costs and thereby making future defaults more likely.

As an alternative to the regression-driven methods set out above, it is also possible to follow a more data-driven approach—employing machine learning techniques. Such an approach offers greater freedom, particularly in handling non-linearities. A prominent example is the study by Manasse and Roubini (2009). They estimate a classification tree, which can be seen as a simple set of data-based yes/no-questions to determine whether a country is at risk of debt distress. The goal of a classification tree algorithm is to identify characteristics that are most closely associated with class membership (in this case: countries which are about to enter a debt crisis, versus those that are not). It does so by determining which questions to ask, and in what order. The questions are always of the type "Is variable *X* greater/small than a certain threshold *T*?",

[41] These probabilities can for example be chosen using data on past fiscal crises, with the objective of minimizing a loss function that penalizes missed crises and false alarms.

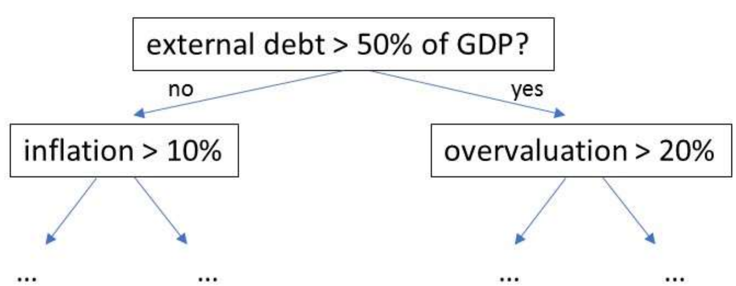
where T is set optimally for each variable X such that it is best able to separate crisis cases from non-crisis ones. The order of questions is determined with the same objective.

This approach is particularly well-equipped to handle state dependence and interactions among variables—thereby moving beyond the linear structure typically underlying the aforementioned "early warning" literature. For example, if the level of exchange rate overvaluation solely matters in countries with high levels of external debt, the algorithm will only ask about exchange rate overvaluation in a node that applies to countries where external debt is indeed large (see Figure 7).

Using data on 47 emerging market economies from 1970-2002, Manasse and Roubini (2009) find that 10 (out of the 50 candidate) variables matter for classification/prediction purposes (all in the expected direction): total external debt/GDP, short-term debt/reserves, real GDP growth, public external debt/revenues, CPI inflation, number of years to the next Presidential election, US T-bill rate, external financing requirements, exchange rate overvaluation, and exchange rate volatility. In the end, their tree identifies four types of countries:

(i) Countries that are relatively safe, with strong fundamentals. These are characterized by low external debt ratios, low inflation, and strong growth.
(ii) Countries that are prone to a liquidity crisis. They have a high ratio of short-term debt to reserves, coupled with political uncertainty, a relatively fixed exchange rate, and tight monetary conditions in international markets.
(iii) Countries that are prone to a solvency crisis. They feature high levels of external indebtedness, together with high inflation.
(iv) Countries that are prone to an exchange rate/macro crisis. For them, the exchange rate is overvalued, while growth is low.

Figure 7: example of non-linearity in regression tree



5. Conclusions

To be completed

References

To be completed ê é