Knowledge Diffusion, Trade and Innovation across Countries and Sectors

Jie Cai, Nan Li and Ana Maria Santacreu*

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Abstract

We develop and quantify a multi-country and multi-sector endogenous growth model in which productivity evolves through innovation and knowledge diffusion. We quantify the effect of a trade liberalization on innovation, productivity and welfare in a framework that features intersectoral production and knowledge linkages that are consistent with the data. A reduction in trade frictions induces a reallocation of innovation and comparative advantage across sectors. Knowledge spillovers imply convergence in relative productivity. In contrast to standard one sector models of trade and innovation, we obtain significant dynamic gains from trade. We find that, while intersectoral production linkages and innovation are the main drivers of dynamic welfare gains from trade, intersectoral knowledge flows are key to explain convergence in relative productivity.

Keywords: Technology Diffusion; R&D; Patent Citations; International Trade

JEL Classification: F12, O33, O41, O47

*Cai, Shanghai University of Finance and Economics, cai.jie@mail.shufe.edu.cn; Li, International Monetary Fund, address: 700 19th Street NW, Washington DC 20431, email: nanli1@gmail.com; Santacreu, Federal Reserve Bank of St. Louis, email: am.santacreu@gmail.com. We are grateful to Jonathan Eaton, Dan Trefler, Lorenzo Caliendo, Fernando Parro and participants of the seminars at the Saint Louis Fed, Penn State University, RIDGE, Fudan University for helpful comments. The views in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Banks of St. Louis, or the Federal Reserve System. The views expressed herein are those of the authors and should not be attributed to the IMF, its Executive Board, or its management.

1 Introduction

The world has increasingly become a highly interconnected network of countries and sectors which not only trade goods and services between each other, but at the same time, exchange ideas with one another. Recently, a growing strand of the trade literature has examined how the benefits of trade liberalization may spread across sectors through heterogeneous production input-output linkages (Caliendo and Parro 2015). However, sectors are also linked along a different dimension—innovation and knowledge diffusion. Indeed, technological advances never happen in isolation (David 1990; Rosenberg 1982). Knowledge in one sector can be used to enhance innovation in another, and much alike the cross-sector production input-output linkages, knowledge diffusion across sectors is far from uniform. Therefore, in a world with multiple sectors, when changes in trade costs alter the knowledge composition of the economy, the latter also conditions trade patterns and aggregate growth (as shown in the empirical research by Hausmann, Hwang, and Rodrik 2007; Hidalgo, Klinger, Barabási, and Hausmann 2007). Furthermore, although trade flows often serve as a vehicle for knowledge diffusion (Alvarez, Buera and Lucas, 2014), there may be other channels of diffusion of ideas across countries and sectors. The literature so far has either treated these two as separate issues or has modeled them together as one channel (e.g. more trade necessarily implies more knowledge spillovers).

We develop a multi-sector and multi-country endogenous growth model in which technology evolves endogenously through innovation and knowledge diffusion to study the effect of changes in trade costs on innovation, productivity and welfare. Knowledge diffusion occurs when a firm in an industry learns about ideas that have been developed in other industries. We assume this learning to be exogenous. Our structural model introduces realistic features of intersectoral linkages in production and knowledge diffusion. The production side of the model is a multi-sector version of on Eaton and Kortum 2002, as the one developed in Caliendo and Parro 2015. Different from those papers, which assume that technology is exogenous, we introduce dynamics through an endogenous process of innovation and and exogenous process of knowledge diffusion. In contrast to models that do not feature knowledge spillovers, our model delivers convergence in relative productivity at the sector level. Several empirical studies have documented convergence in relative productivity at the sector level (see Levchenko and Zhang 2016, Hausmann and Klinger 2007, Cameron, Proudman, and Redding 2005, Proudman and Redding 2000, Bernard and Jones 1996a, and Bernard and Jones 1996b.) In a Ricardian framework, the evolution of sectoral productivity causes changes comparative advantage.

Our model has implications for welfare gains from trade that differ from both static models of trade and dynamic one-sector models of trade and innovation. Relative to static models, the endogenous evolution of productivity provides an additional source of welfare gains. Changes in trade costs cause changes in innovation which, through knowledge flows, spread across countries and generates changes in revealed comparative advantage, hence welfare. The effect of trade on innovation is driven by the heterogeneous multi-sector dimension of our model. Standard models of trade and innovation that do not account for heterogeneity in production, innovation, and knowledge flows find a negligible impact of trade on innovation and welfare (Atkeson and Burstein 2010, Eaton and Kortum 1996 and Eaton and Kortum 1999). Accounting for these sources of heterogeneity across sectors is important to study the effect of openness on the reallocation of R&D and production across sectors, and ultimately on the cross-country distribution of welfare gains from trade. Our model generates dynamic gains from trade. Recent papers that also emphasize the importance of this heterogeneity are Somale 2014 in a multi-sector model of trade and innovation without knowledge spillovers, and Sampson 2016 in a theoretical Armington framework of innovation and learning. Different from Somale 2014 , who focuses only on innovation as a source of endogenous comparative advantage, we introduce knowledge spillovers in our model. Furthermore, we use data on R&D at the sector level to discipline the innovation process of the model, so that we can analyze explicitly the effect of trade on innovation. Different from Sampson 2016, our emphasis is on the quantification of the model, which allows us to do counterfactuals.

Despite of its complexity, the model comes with the benefit of tractability, as we build upon the Ricardian trade model of Eaton and Kortum 2002 with Bertrand Competition (Bernard, Eaton, Jensen, and Kortum 2003). The innovation and international technology diffusion processes are modeled in a similar fashion as in Eaton and Kortum 1996 and Eaton and Kortum 1999. The diffusion lags—backed by empirical observations—have an exponential distribution. All these features allow us to estimate the set of parameters based on observables in trade and citation data from steady-state relationships. This production structure delivers a gravity equation at the sector level that can be estimated to obtain both trade barriers and the level of technology of a sector-country pair. Both shape the comparative advantage of that sector-country (Levchenko and Zhang 2016). The level of technology reflects the stock of knowledge of the sector-country. Technology evolves over time through two channels: (i) innovators in each sector invest final output to introduce with a new idea which, if successful, can be used to to produce an intermediate good. The innovation process is also affected by an externality: the larger the stock of knowledge of a sector-country pair, the larger the efficiency of innovation in that sector-country pair; (ii) Ideas diffuse both across sectors and countries according to an exogenous process of diffusion. The novel feature of this model is that sectors and countries are connected not only through trade in intermediate goods but also through knowledge diffusion.

The model is solved in two stages. Given the probability distribution of firm's productivity together with trade barriers at the country-pair and sector level, we solve for a static competitive equilibrium for the world economy. The equilibrium is static in that we take as given the technology level that determines the patterns of trade. We then allow for the technology profile to evolve endogenously due to a process of innovation and diffusion. The second stage allows us to determine the characteristics of the innovation process that drives the endogenous evolution of comparative advantage and dynamic welfare gains from trade. A similar approach has been used in Alvarez, Buera, and Lucas Jr 2008. Different form their paper, our diffusion channel produces a Frechet distribution of productivity, as in Eaton and Kortum 1999. Furthermore, Alvarez, Buera, and Lucas Jr 2008 abstract from innovation, which is a key channel in our model. Buera and Oberfield

2016 study a model with technology diffusion and innovation that delivers the same Frechet distribution. However, different from the predictions of our model, trade has no impact on innovation in their framework.

We calibrate the model to data on intersectoral patent citations, R&D intensity and international trade. Following Levchenko and Zhang 2016, we estimate gravity equations at the sector level to uncover the trade costs and technology parameters. Cross-country and cross-sector patent citations allow us to discipline the direction and intensity in which knowledge in a particular sector is utilized in the innovation of other sectors. Finally, data on R&D intensity at the sector-country level allow us to calibrate the parameters that govern the evolution of technology.

We conduct a counterfactual exercise to study the effect of a trade liberalization on innovation, productivity and welfare. In contrast to Eaton and Kortum 1996 and Eaton and Kortum 1999 , Buera and Oberfield 2016, and Atkeson and Burstein 2010, changes in trade frictions have non-negligible effect on innovation, as there is a reallocation toward sectors in which the country has comparative advantage. Somale 2014 obtains similar predictions on the reallocation of innovation in a semi-endogenous model of growth with multiple sectors and no knowledge diffusion. We consider international technology diffusion as an additional source of technological progress in our paper, which is important to explain convergence in relative productivity found in the data or to explain growth miracles as in Buera and Oberfield 2016. We find that this reallocation is more important in sectors with stronger knowledge spillovers. Furthermore, different from Somale 2014, we use data on R&D and calibrate the model in levels which allows us to study explicitly the role of trade on R&D.

Our quantitative framework has implications for welfare gains from trade. A trade liberalization strengthens a country's comparative advantage, and hence the static gains from trade. In addition, there are dynamic gains from trade due to higher R&D investment in those sectors in which the country has comparative advantage. Moreover, knowledge diffusion has two opposite effects on welfare. On the one hand, it enables faster productivity convergence and makes countries more similar to each other, which dampens the static gains from trade. On the other hand, it provides strong dynamic gains, because countries can innovate with access to a larger foreign knowledge pool.

Finally, we study the role of different sources of sector heterogeneity on innovation, productivity and welfare. We find that, after a trade liberalization: (i) Larger countries experience lower gains from trade; (ii) Innovation reallocates towards sectors that experience larger increases in revealed comparative advantage; this is especially the case in those countries that have larger gains from trade; (iii) an increase in innovation translates into an increase in the growth rate and income per capita on the BGP. As a result, we find significant dynamic gains from trade, which are heterogeneous across countries. Dynamic gains are larger in those countries that experience larger increases in innovation (iv) Knowledge spillovers imply convergence of relative productivity, hence comparative advantage after a trade liberalization. If we do not allow for knowledge spillovers, welfare gains from trade are larger, as countries are more disperse in their comparative advantage;

(v) Symmetric production linkages deliver welfare gains from trade that are substantially lower than in the baseline multi-sector model with heterogeneity in production linkages; and (vi) One-sector models generate negligible dynamic gains. Furthermore, in a model without royalties, there is no effect of changes in trade costs on innovation, hence dynamic welfare gains from trade are zero.

Related Literature Our paper merges and extends several strands of existing literature. First, the literature on innovation, diffusion and international trade. Eaton and Kortum 1996 and Eaton and Kortum 1999 posit technological innovations and their international diffusion through trade as potential channels of embodied technological progress. Santacreu 2015 develops a model in which trade allows countries to adopt innovation developed abroad, and thus diffusion does not take place without trade. Our main departure from these previous papers is that we allow knowledge diffusion and trade to operate separately, even though common economic forces may contribute to the development of both and diffusion and trade may benefit and reinforce each other. In addition, we extend these studies into multi-sector environment in which sectors interact both in the product space and in the technology space.

The second is the multi-sector trade literature which extends Eaton and Kortum 2002 trade model to multiple sectors (Chor 2010; Costinot, Donaldson, and Komunjer 2012). A recent growing body of research in this area also explores the trade and growth implications of interdependence across different sectors through intermediate input-output relationships (Eaton, Kortum, Neiman, and Romalis 2016, Caliendo and Parro 2015). Our paper differs in several dimensions. First, our focus is on innovation and knowledge diffusion. Second, besides the factor demand linkages, this paper also simultaneously consider the intrinsic interconnections of technologies embodied in different sectors, which turns out to be significant and relevant when studying innovation and diffusion. Related to the current work, Cai and Li 2016 study knowledge spillovers across sectors within a country and how trade costs affect the distribution of endogenous knowledge accumulation across sectors. Different from our paper, however, cross-sector knowledge diffusion is not considered across countries and intermediate input demand linkages across sectors are absent. Perla, Tonetti, and Waugh 2015 study the effect of trade on growth in a symmetric country model in which firms learn from existing knoledge by other firms. Levchenko and Zhang 2016 provide evidence of relative productivity convergence across 72 countries over 5 decades: productivity grew systematically faster in initially relatively less productive sectors. These changes have had a significant impact on trade volumes and patterns, and a modest negative welfare impact.

Led by Hidalgo, Klinger, Barabási, and Hausmann 2007, several papers have shown that producing goods with strong synergy with each other can improve growth, as it is easier to adapt existing ideas and enter new sectors (e.g. Hausmann and Klinger 2007, Kali, Reyes, McGee, and Shirrell 2013, Hausmann and Klinger 2007). However, these studies mostly adopt the regression based approach which is hard to establish causality and to examine the general equilibrium implications of changing trade structure. Moreover, none of these studies consider at the same time the product complementarity along the intermediate input-output dimension.

2 The Model

We develop a general equilibrium model of trade in intermediate goods, with sector heterogeneity and input-output linkages, in which technology evolves endogenously through innovation and knowledge diffusion. The model builds upon the Ricardian trade model of Eaton and Kortum 2002 with Bertrand Competition (Bernard, Eaton, Jensen, and Kortum 2003). The innovation and diffusion processes are modeled as Eaton and Kortum 1996 and Eaton and Kortum 1999.

There are M countries and J sectors. Countries are denoted by i and n and sectors are denoted by j and k. Labor is the only factor of production and we assume it to be mobile across sectors within a country but immobile across countries. In each country, there is a representative consumer who consumes a non-traded final good and saves. A perfectly competitive final producer combines a composite output of all J sectors in the domestic economy with a Cobb-Douglas production function. In each sector there is a producer of a composite good that operates under perfect competition and sells the good to the final producer and to intermediate producers from all sectors in that country. Intermediate producers use labor and composite goods to produce varieties that are traded and are used by the composite producer of that sector, either domestic or foreign. These firms operate under Bertrand competition and are heterogeneous in their productivity. Trade is Ricardian. Finally, the technology of each sector evolves endogenously through innovation and technology diffusion. The innovation process follows the quality-ladders literature in that new innovations increase the quality of the product in a given sector. Diffusion is assumed to be exogenous. Foreign firms that decide to use a domestic innovation pay royalties to the innovator.

2.1 Consumers

In each country there is a representative households who choose consumption optimally to maximize their life-time utility

$$U_{nt} = \int_{t=0}^{\infty} \rho^t u\left(C_{nt}\right) dt, \tag{1}$$

where $\rho \in (0,1)$ is the discount factor, and C_{nt} represents consumption of country n at time t. The households finance R&D activities of the entrepreneurs and own all the firms.

We assume that households' preferences are represented by a CRRA utility function

$$u\left(C_{nt}\right) = \frac{C_{nt}^{1-\gamma}}{1-\gamma}$$

with an intertemporal elasticity of substitution, $\gamma > 0$.

2.2 Final production

Domestic final producers use a composite output from every domestic sector j in country n at time t, Y_{nt}^{j}, to produce a non-traded final output Y_{nt} according to the following Cobb-Douglas production function

$$Y_{nt} = \prod_{j=1}^{J} \left(Y_{nt}^{j}\right)^{\alpha^{j}}, \tag{2}$$

with $\alpha^j \in (0,1)$ the share of sector production on total final output, and $\sum_{j=1}^{J} \alpha^j = 1$.

Final producers operate under perfect competition. Their profits are given by:

$$\Pi_{nt} = P_{nt} Y_{nt} - \sum_{j} P_{nt}^{j} Y_{nt}^{j},$$

where P_{nt} is the price of the final produce, and P_{nt}^j is the price of the composite good produced in sector j from country n.

Under perfect competition, the price charged by the final producer to the consumers is equal to their marginal cost, that is

$$P_{nt} = \prod_{j=1}^{J} \left(\frac{P_{nt}^{j}}{\alpha^{j}}\right)^{\alpha^{j}}.$$

The demand by final producers for the sector composite good is given by:

$$\alpha^{j} P_{nt} \frac{Y_{nt}}{Y_{nt}^{j}} = P_{nt}^{j}.$$

2.3 Intermediate producers

In each sector j there is a continuum of intermediate producers indexed by $\omega \in [0,1]$ that use labor, $l_{nt}^{j}(\omega)$, and a composite intermediate good from every other sector k in the country, $m_{nt}^{jk}(\omega)$ to produce a variety ω according to the following constant returns to scale technology[1]

$$q_{nt}^{j}(\omega) = z_{n}^{j}(\omega)[l_{nt}^{j}(\omega)]^{\gamma^{j}} \prod_{k=1}^{J} [m_{nt}^{jk}(\omega)]^{\gamma^{jk}}, \tag{3}$$

with $\gamma^j + \sum_{k=1}^{J} \gamma^{jk} = 1$. Here γ^{jk} is the share of materials from sector k used in the production of intermediate ω is sector j, and γ^j is the share of value added. Firms are heterogeneous in their productivity $z_n^j(\omega)$.

The cost of producing each intermediate good ω is

$$c_{nt}^{j}(\omega) = \frac{c_{nt}^{j}}{z_{nt}^{j}(\omega)},$$

where c_n^j denotes the cost of input bundle. In particular, given constant returns to scale:

[1] The notation in the paper is such that every time there are two subscripts or two superscripts, the right one corresponds to the source country and the left one corresponds to the destination country.

$$c_n^j = \Upsilon^j W_{nt}^{\gamma^j} \prod_{k=1}^{J} (P_n^k)^{\gamma^{jk}},$$

with $\Upsilon^j = \prod_{k=1}^{J} (\gamma^{jk})^{-\gamma^{jk}} (\gamma^j)^{-\gamma^j}$ and W_{nt} the nominal wage rate. Intermediate producers operate under Bertrand competition.

2.4 Composite intermediate goods (Materials)

Each sector j produces a composite good combining domestic and foreign varieties from that sector. Composite producers operate under perfect competition and buy intermediate products ω from the minimum cost supplier.

The production for a composite good in sector j and country n is given by the Ethier 1982 CES function,

$$Q_{nt}^j = \left(\int r_{nt}^j(\omega)^{1-1/\sigma} d\omega \right)^{\sigma/(\sigma-1)}, \tag{4}$$

where $\sigma > 0$ is the elasticity of substitution across intermediate goods, and $r_{nt}^j(\omega)$ is the demand of intermediate goods from the lowest cost supplier in sector j.

The demand for each intermediate good ω is given by

$$r_{nt}^j(\omega) = \left(\frac{p_{nt}^j(\omega)}{P_{nt}^j} \right)^{-\sigma} Q_{nt}^j,$$

where

$$P_{nt}^j = \left(\int p_{nt}^j(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}, \tag{5}$$

The sector composite producer uses varieties from its own sector, but only from the lower cost producer, since there is perfect competition.

Composite intermediate goods are used final goods in the final production and as materials for the production of the intermediate goods.

$$Q_{nt}^j = Y_{nt}^j + \sum_{k=1}^{J} \int m_{nt}^{kj}(\omega) d\omega.$$

2.5 International trade

We follow Bernard, Eaton, Jensen, and Kortum 2003 and assume Bertrand competition. Trade in goods is costly. In particular, there are iceberg transport costs from shipping a good in sector j from country i to country n, d_{ni}^j.The p'th most efficient producer of variety ω from sector j in country i can deliver a unit of good to country n at the cost:

$$c_{pni}^j(\omega) = d_{ni}^j \frac{c_i^j}{z_{pi}(\omega)},$$

With Bertrand competition, as with perfect competition, composite producers in each sector buy from the lowest cost supplier. The cost of a good ω in country n is given by

$$c^j_{1n}(\omega) = min_i \left\{ c^j_{1ni}(\omega) \right\},$$

In addition, Bertrand competition implies that the price charged by the producer will be the production cost of the second lowest producer

$$c^j_{2n}(\omega) = min \left\{ c^j_{2ni*}(\omega), min_{i \neq i^*} \{ c^j_{1ni}(\omega) \} \right\},$$

where i^* satisfies $c^j_{1ni*}(\omega) = c^j_{1n}(\omega)$. The low cost supplier will not want to charge a mark-up above $\bar{m} = \sigma/(\sigma - 1)$. Hence,

$$p^j_n(\omega) = min \left\{ c^j_{2n}(\omega), \bar{m} c^j_{1n}(\omega) \right\}.$$

Ricardian motives for trade are introduced as in Eaton and Kortum 2002, since productivity is allowed to vary by sector and country. The productivity of producing intermediate good ω in country i and sector j is drawn from a Frechet distribution with parameter T^j_i and shape parameter θ. A higher T^j_i implies a higher average productivity of that sector-country pair, while a lower θ implies more dispersion of productivity across varieties.

$$F(z^j_i) = Pr \left[Z \leq z^j_i \right] = e^{-T^j_i z^{-\theta}},$$

and,

$$Pr \left[p^j_{ni,t} < p \right] = 1 - e^{-T^j_{it} \left(d^j_{ni} c^j_{it}/p \right)^{-\theta}}.$$

We make the following assumption:

$$T^j_{it} = A^j_{it} T^j_{p,i}. \tag{6}$$

where A^j_{it} represents a measure of the quality of ideas in country i and sector j, or "knowledge-related productivity", and $T^j_{p,i}$ is a component of productivity unrelated to research and is assumed to be constant over time. In the next sections, we determine how A^j_{it} evolves endogenously over time through innovation and diffusion.[2]

Because each sector j in country n buys goods from the second cheapest supplier, the cost of producing good ω in sector j and country n is $p^j_{nt}(\omega) = min \left\{ p^j_{nit}(\omega) \right\}$. Then, $c^j_{nt}(\omega)$ are realizations from

$$G^j_n(p) = 1 - \prod_{i=1}^{M} \left(Pr \left[p^j_{nit} > p \right] \right) = 1 - \prod_{i=1}^{M} e^{-T^j_{it} \left(d^j_{ni} c^j_{it}/p \right)^{-\theta}} = 1 - e^{-\Phi^j_{nt} p}$$

with $\Phi^j_{nt} = \sum_{i=1}^{M} T^j_{it} (d^j_{ni} c^j_{it})^{-\theta}$ each country n and sector j accumulated technology. From here, we

[2]This formulation is similar to the one introduced in Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple 2013.

can obtain the distribution of prices of goods in sector j in country n as

$$P_{nt}^{j} = B\left(\Phi_{nt}^{j}\right)^{-1/\theta}, \tag{7}$$

with $B = \left[\frac{1+\theta-\sigma+(\sigma-1)(\bar{m})^{-\theta}}{1+\theta-\sigma}\Gamma\left(\frac{2\theta+1-\sigma}{\theta}\right)\right]^{1/(1-\sigma)}$. For prices to be well defined, we assume $\sigma < (1+\theta)$.

2.6 Expenditure shares

The probability that country i is the low cost supplier of a good in sector j that is to be exported to country n is

$$\pi_{nit}^{j} = \frac{T_{it}^{j}\left(c_{it}^{j}d_{ni}^{j}\right)^{-\theta}}{\Phi_{nt}^{j}}, \tag{8}$$

π_{nit}^{j} is also the fraction of goods that sector j in country i sells to any sector in country n. In particular, the share country n spends on sector j products from country i is

$$\pi_{nit}^{j} = \frac{X_{nit}^{j}}{X_{nt}^{j}}. \tag{9}$$

2.7 Endogenous growth: Innovation and international technology diffusion

We model the innovation process within each industry as in Kortum 1997. Innovation follows the quality-ladders literature, in that a blueprint (i.e., an idea) is needed to produce an intermediate good. Ideas are developed with effort and they increase the efficiency of production of an intermediate good. In each sector j and country n, there are entrepreneurs that invest final output to come up with an idea. Within each sector, research efforts are targeted at any of the continuum of intermediate goods. In each country n and sector j, ideas are drawn at the Poissson rate λ_{nt}^{j}. If a fraction of final output s_{nt}^{j} is invested into R&D by the entrepreneur, then ideas are created at the rate

$$\lambda_{nt}^{j}\left(s_{nt}^{j}\right)^{\beta_r} \tag{10}$$

with $\lambda_{nt}^{j} = \lambda_{n}^{j}A_{nt}^{j}$ and λ_{n}^{j} a scaling parameter that captures the efficiency of innovation in sector j of country n, and $\beta_r \in (0,1)$ a parameter of diminishing returns to investing into R&D. This process has been microfounded in Eaton and Kortum 1996 and Eaton and Kortum 1999 and it ensures that there is a balanced growth path without scale effects.

Ideas from sector j and country n may become an intermediate product in that sector and country. An idea is the realization of two random variables. One is the good ω to which the idea applies. An idea applies to only one good in the continuum. The good ω to which it is associated is drawn from the uniform distribution $[0,1]$. The other is the quality of the idea, $q^{j}(\omega)$ which is drawn from the Pareto distribution $H(q) = 1 - q^{-\theta}$. In equilibrium, only the best idea for each input in each sector and country is actually used to produce an intermediate good in any sector and country. In that case, the idea can be used to produce an intermediate product in sector j and country n with efficiency $z_n^j(\omega)$. Therefore, the efficient technology $z_n^j(\omega)$ for producing good ω in country n is the best idea for producing it yet discovered. This modeling choice follows Eaton and Kortum 2006, with a few modifications.

The stock of ideas at each pint in time in sector j and country n is A_{nt}^j. Because there is a unit interval of intermediate goods, the number of ideas for producing a specific good is Poisson with parameter A_{nt}^j. This Poisson arrival implies that the probability of k ideas for producing a good by date t in sector j and country n is $\left(A_{nt}^j\right)^k e^{-A_{nt}^j}/k!$. If there are k ideas, th eprobability that the best one is below the best quality z is $[H(z)]^k$. Summing over all possible k, $F(z) = e^{-A_{nt}z^{-\theta}}$.

Once an idea has arrived in sector j and country n there is no forgetting. New ideas created in each sector j and country n increase its average productivity, A_{nt}^j. Ideas may also diffuse exogenously to other sectors and countries. An idea discovered at time t in country i and sector k diffuses to country n and sector j at time $t + \tau_{ni}^{jk}$. We assume that the diffusion lag τ_{ni}^{jk} has an exponential distribution with parameter ε_{ni}^{jk} as the speed of diffusion, so that $Pr[\tau_{ni}^{jk} \leq x] = 1 - e^{-\varepsilon_{ni}^{jk}x}$.

Through diffusion, the stock of knowledge in a country-sector pair is composed of knowledge that has been developed in all sectors and countries, $A_{ni,t}^{jk}$. That is, $A_{nt}^j = \sum_i \sum_k A_{ni,t}^{jk}$. Therefore, the flow of ideas diffusing to country n and sector j is given by the accumulation of the past research effort of each sector k in country i that has already been diffused, according to

$$\dot{A}_{nt}^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \varepsilon_{ni}^{jk} \int_{-\infty}^{t} e^{-\varepsilon_{ni}^{jk}(t-s)} \lambda_{is}^k \left(s_{is}^k\right)^{\beta_r} ds, \tag{11}$$

with $\lambda_{is}^k = \lambda_i^k A_{is}^k$. If $\varepsilon_{ni}^{jk} \to \infty$, then there is instantaneous diffusion. If $\varepsilon_{ni}^{jk} \to 0$, then there is no diffusion. The growth of the stock of knowledge in a particular sector j and country n at time t depends on the past research effort that has been done by each other sector k in country i up to time t, and that has diffused at the rate ε_{ni}^{jk}.

2.7.1 The incentives to innovate

Entrepreneurs finance R&D issuing equity claims to the households. These claims pay nothing if the entrepreneur is not successful in introducing a new technology in the market, and it pays the stream of future profits from selling the good in a particular sector either domestically or abroad if the innovation succeeds. The value of a successful innovation in a particular sector is the expected flow of profits that will last until a new producer is able to produce the good at a lower cost. Because of the probabilistic distribution of productivity, entrepreneurs will be indifferent on what product ω to devote its efforts, since in expectation, all products within a sector deliver the same expected profit. As in the quality ladders literature, we focus on a situation in which all products within an industry are targeted with the same intensity. Following the quality-ladders literature, a new idea will interact with the set of existing technologies in a particular sector and country if

$Q > Z_n^j$, which occurs with probability

$$Pr[Q > Z_n^j] = \int_0^\infty Pr[Q > z] dF_n^j(z) = 1/A_{nt}^j$$

This introduces a competitive effect, by which the larger the stock of knowledge in a sector-country pair, the lower the probability that the new idea lowers the cost there.

Then, the distribution of Q conditional on $Q > Z_n^j$ is

$$Pr[Q \leq q | Q > Z_n^j] = e^{-A_{nt}^j q^{-\theta}}$$

Therefore, conditional on joining the set of best technologies, the quality of a new idea has the same distribution of the quality of existing technologies.

The profits of an innovator in sector j in country n have two components. First, the expected profit from selling the product in that sector an country:

$$\frac{1}{(1+\theta)} \frac{\sum_{i=1}^M \pi_{int}^j X_{it}^j}{A_{nt}^j}, \tag{12}$$

Second, in addition to the profits from selling the product, the innovator gets royalties from the technologies that have diffused to other countries and sectors. We assume that royalty payments are proportional to the profits that successful intermediate good producers in other sectors and countries obtain from using that technology. The expected royalty payment to an entrepreneur in country n and sector j from a technology that has been diffused and adopted by a producer in sector k of country i is

$$\chi_{in,t}^{kj} \frac{\Pi_{it}^k}{A_{it}^k} = \chi_{in,t}^{kj} \frac{1}{(1+\theta)} \frac{\sum_{m=1}^M X_{mt}^k \pi_{mit}^k}{A_{it}^k}, \tag{13}$$

where $\chi_{in,t}^{kj}$ is the fraction of technologies developed by entrepreneurs from from sector j in country n that are used by sector k in country i. Note that $\chi_{nn,t}^{jj} = 1$.

The value of an idea that has been developed in country n and sector j is the expected present discounted value of the stream of future profits

$$V_{nt}^j = \sum_{i=1}^M \sum_{k=1}^J \int_t^\infty \left(\frac{P_{it}^k}{P_{is}^k} \right) e^{-\rho(s-t)} \chi_{in,s}^{kj} \frac{\Pi_{is}^k}{A_{is}^k} ds, \tag{14}$$

The first order condition for optimal R&D is:

$$\beta_r \lambda_{nt}^j V_{nt}^j \left(s_{nt}^j \right)^{\beta_r - 1} = P_{nt} Y_{nt}, \tag{15}$$

Therefore, the optimal R&D investment is a positive function of the value of an innovation, V_n^j and the efficiency of innovation λ_n^j.[3]

2.8 Balance of payments

The current account balance equals the trade balance plus the net foreign income derived from net royalty payments. Total imports in country n are given by:

$$IM_{nt} = \sum_{i=1}^{M}\sum_{k=1}^{J} X_{nit}^k = \sum_{k=1}^{J} X_{nt}^k \sum_{i=1}^{M} \pi_{nit}^k, \tag{16}$$

Total exports in country n are given by:

$$EX_{nt} = \sum_{i=1}^{M}\sum_{k=1}^{J} X_{int}^k = \sum_{i=1}^{M}\sum_{k=1}^{J} \pi_{int}^k X_{it}^k$$

Net royalty payments are given by

$$RP_{nt} = \sum_{j=1}^{J} RP_{nt}^j$$

and

$$RP_{nt}^j = \sum_{i=1}^{M}\sum_{k=1}^{J} \left(\chi_{in,t}^{kj}\Pi_i^{kt} - \chi_{ni,t}^{jk}\Pi_{nt}^j\right)$$

The balance of payments implies

$$EX_{nt} = IM_{nt} - RP_{nt}$$

3 Endogenous growth along the balanced growth path (BGP)

In our model, all countries and sectors grow at the same rate along a unique BGP. International and sector diffusion guarantees that the "knowledge-related productivity" A_{nt}^j, and by assumption (6), also average productivity, T_{nt}^j, grow at a common rate across countries and sectors, which we denote by g_A. We normalize all the endogenous variables so that they are constant on the BGP. We denote the normalized variables with a hat.

From the resource constraint in equation (27), the fraction of final output that is invested into R&D, s_n^j, is constant on the BGP. This result, together with the expression for the value of an

[3] The optimization problem of the innovator is as follows. Innovators choose the amount of final output to be allocated into R&D. In our model, s_n^j is the fraction of final output that is spent into R&D activity. Therefore, innovators choose $S_n^j = s_n^j Y_n$ to maximize

$$\dot{A}_n^j V_n^j - P_n S_n^j$$

subject to equation (11).

innovation, imply that

$$\hat{V}_n^j = \sum_{i=1}^{M}\sum_{k=1}^{J}\chi_{in}^{kj}\frac{\hat{\Pi}_i^k}{\rho - g_y + g_A}\frac{1}{\hat{A}_n^j}$$

with $\hat{V}_n^j = \frac{V_n^j A_n^j}{P_n Y_n}$, $\hat{A}_n^j = \frac{A_n^j}{A_M^J}$, and χ_{in}^{kj} is the fraction of profits that a firm in sector k, country i pays to the innovator in sector j, country n as royalties. We impose $\rho - g_y + g_A > 0$ and we derive and expression for g_y in Appendix D.

Expected profits are given by,

$$\frac{\hat{\Pi}_n^j}{\hat{A}_n^j} = \frac{\sum_{i=1}^{M}\pi_{in}^j \hat{X}_i^j}{(1+\theta)\hat{A}_n^j}$$

with $\hat{\Pi}_n^j = \frac{\Pi_n^j}{W_M}$.

We can show that, on the BGP, the fraction of profits paid by producers from sector k in country i, that use technologies from sector j in country n is

$$\chi_{in}^{kj} = \frac{\varepsilon_{in}^{kj}}{g_A + \varepsilon_{in}^{kj}}\frac{\hat{A}_i^k}{\hat{A}_n^j}$$

We can now use the expression for the value of an innovation together with the optimal investment into R&D to obtain

$$\hat{V}_n^j = \sum_{i=1}^{M}\sum_{k=1}^{J}\frac{\varepsilon_{in}^{kj}}{g_A + \varepsilon_{in}^{kj}}\frac{1}{(1+\theta)}\sum_{m=1}^{M}\hat{X}_m^k \hat{\pi}_{mi}^k \frac{1}{\rho - g_y + g_A}$$

To gain some intuition on why trade has an effect on R&D, let's assume that there are no royalties, that is $\chi_{in}^{kj} = 0$. Then,

$$s_n^j = \left(\beta_r \lambda_n^j \frac{1}{(1+\theta)}\frac{1}{\rho - g_y + g_A}\frac{\sum_{i=1}^{M}\pi_{in}^j \hat{X}_i^j}{\hat{Y}_n}\right)^{\frac{1}{1-\beta_r}}$$

with $\hat{X}_i^j = \frac{X_i^j}{W_M}$ and $\hat{Y}_n = \frac{P_n Y_n}{W_M}$. Trade affects optimal investment into R&D at the sector level to the extent that it affects the reallocation of production into particular sectors. This result differs from previous papers in the literature that find that trade has no impact on R&D intensity. In our paper, R&D reallocates towards sectors in which the country has comparative advantage, through $\frac{\sum_{i=1}^{M}\pi_{in}^j \hat{X}_i^j}{\hat{Y}_n}$. In section E, we show how in the one sector version of our model without royalties, changes in trade costs have no effect on innovation, hence on the growth rate, even when we allow for knowledge spillovers.

Substituting into the growth rate of stock of knowledge in

$$g_A = \sum_{i=1}^{M}\sum_{k=1}^{J}\frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}}\lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j}\left(s_i^k\right)^{\beta_r}$$

$$1 = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}/g_A}{g_A + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{A}_i^k}{\hat{A}_n^j} \left(\frac{1}{\rho - g_y + g_A} \beta_r \lambda_i^k \frac{1}{(1+\theta)} \frac{\sum_{n=1}^{M} \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}}$$

Rearranging, we obtain an expression for the growth rate of the stock of knowledge in steady state,

$$g_A \hat{A}_n^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_A + \varepsilon_{ni}^{jk}} (\lambda_i^k)^{\frac{1}{1-\beta_r}} \hat{A}_i^k \left(\frac{1}{\rho - g_y + g_A} \beta_r \frac{1}{(1+\theta)} \frac{\sum_{n=1}^{M} \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}} \quad (17)$$

The growth rate of the stock of knowledge on the BGP depends positively on the speed of diffusion, the expected profits and negatively on the dispersion parameter. Following Eaton and Kortum 1999, the Frobenius theorem guarantees that there is a unique balanced growth path in which all countries and sectors grow at the same rate g_A. The expression for the growth rate can be expressed in matrix form as:

$$g_A A = \Delta(g_A) A$$

If the matrix $\Delta(g_A)$ is definite positive, then there exists a unique positive balanced growth rate of technology $g_A > 0$ given research intensities and diffusion parameters. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country-sector pair relative level of knowledge along that balanced growth path.

In what follows, we report the equations of the model after normalizing the endogenous variables.

(1) Probability of imports

$$\pi_{ni}^j = \hat{T}_i^j \frac{\left(\hat{c}_i^j d_{ni}^j\right)^{-\theta}}{\hat{\Phi}_n^j}, \quad (18)$$

where $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$ and $\hat{\Phi}_n^j = \frac{1}{T_M^J} \frac{\Phi_n^j}{(W_M)^{-\theta} (T_M^J)^{\Lambda_j}}$ with Γ^j defines in Appendix D.

(2) Import shares

$$\hat{X}_{ni}^j = \pi_{ni}^j \hat{X}_n^j, \quad (19)$$

(3) Cost of production

$$\hat{c}_n^j = \Upsilon^j \hat{W}_n^{\gamma^j} \prod_{k=1}^{J} (\hat{P}_n^k)^{\gamma^{jk}}, \quad (20)$$

(4) Intermediate good prices in each sector

$$\hat{P}_n^j = B \left(\hat{\Phi}_n^j\right)^{-1/\theta}, \quad (21)$$

(5) Cost distribution

$$\hat{\Phi}_n^j = \sum_{i=1}^{M} \hat{T}_i^j \left(d_{ni}^j \hat{c}_i^j \right)^{-\theta}, \tag{22}$$

(6) Price index

$$\hat{P}_n = \prod_{j=1}^{J} \left(\frac{\hat{P}_n^j}{\alpha^j} \right)^{\alpha^j}, \tag{23}$$

(7) Labor market clearing condition

$$\hat{W}_n L_n = \sum_{j=1}^{J} \gamma^j \sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j, \tag{24}$$

(8) Sector production

$$\hat{X}_n^j = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} \pi_{in}^k \hat{X}_i^k + \alpha^j \hat{Y}_n, \tag{25}$$

where $\hat{Y}_n = \frac{P_n Y_n}{W_M}$.

(9) Final production

$$\hat{Y}_n = \hat{W}_n L_n + \frac{\sum_{j=1}^{J} \sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{1+\theta}, \tag{26}$$

(10) Resource constraint

$$\hat{Y}_n = \hat{C}_n + \sum_{k=1}^{J} s_n^k \hat{Y}_n, \tag{27}$$

(11) R&D expenditures

$$\beta_r \lambda_n^j \hat{V}_n^j \left(s_n^j \right)^{\beta_r - 1} = \hat{Y}_n, \tag{28}$$

(12) Value of an innovation

$$\hat{V}_n^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{in}^{kj}}{g_A + \varepsilon_{in}^{kj}} \frac{1}{(1+\theta)} \sum_{m=1}^{M} \hat{X}_m^k \hat{\pi}_{mi}^k \frac{1}{\rho - g_y + g_A}, \tag{29}$$

4 Welfare Gains from Trade

We compute welfare gains from trade after a trade liberalization between the baseline and the counterfactual BGP. Welfare in our model is defined in equivalent units of consumption. We can use equation (1) to obtain the lifetime utility in the initial BGP as

$$\bar{U}_i^* = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{1-\gamma} e^{g^{*(1-\gamma)t}} dt = \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{\rho - g^*(1-\gamma)}$$

and in the counterfactual BGP as

$$\bar{U}_i^{**} = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{1-\gamma} e^{g^{**}(1-\gamma)t} dt = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}$$

with $*$ denoting the baseline BGP, and $**$ denoting the counterfactual BGP.

Welfare gains are defined as the amount of consumption that the consumer is willing to give up in the counterfactual BGP to remain the same as in the initial BGP. We call this, λ_i, which is obtained as:

$$\bar{U}_i^{*}(\lambda_i) = \bar{U}_i^{**}$$

$$\frac{\left(\hat{C}_i^{*}\lambda_i\right)^{1-\gamma}}{\rho - g^{*}(1-\gamma)} = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}$$

From here,

$$\lambda_i = \frac{\hat{C}_i^{**}}{\hat{C}_i^{*}} \left(\frac{\rho - g^{*}(1-\gamma)}{\rho - g^{**}(1-\gamma)}\right)^{\frac{1}{1-\gamma}}$$

Welfare gains depend on changes in normalized consumption between the BGPs and the change in growth rates. From equation (27), normalized consumption in the BGP is equal to income per capita net of R&D expenditures. That is,

$$\hat{C}_i = \hat{Y}_i - \sum_{k=1}^{J} s_i^k \hat{Y}_i = \left(1 - \sum_{k=1}^{J} s_i^k\right) \hat{Y}_i$$

In static models or one-sector models of trade and innovation in which changes in trade costs do not have an effect on innovation, $g^* = g^{**}$ and $s_i^k = 0$. In that case, welfare gains from trade are computed as changes in the real wage. As in Caliendo and Parro 2015, we can obtain an expression for the real wage in country i as

$$\frac{W_i}{P_i} \propto \prod_{j=1}^{M} \left(\frac{W_i}{P_i^j}\right)^{\alpha^j}$$

Using the first order conditions for prices and import shares, it can be shown that

$$\frac{W_i}{P_i^j} = \left(\frac{T_i^j}{\pi_{ii}^j}\right)^{1/\theta} \frac{W_i}{c_i^j} \propto \left(\frac{T_i^j}{\pi_{ii}^j}\right)^{1/\theta} \prod_{k=1}^{J} \left(\frac{W_i}{P_i^k}\right)^{\gamma^{jk}}$$

Therefore,

$$\frac{W_i}{P_i} \propto \prod_{j=1}^{J} \left(\left(\frac{T_i^j}{\pi_{ii}^j} \right)^{\alpha^j/\theta} \prod_{k=1}^{J} \left(\frac{W_i}{P_i^k} \right)^{\alpha_i^j \gamma^{jk}} \right) \tag{30}$$

Note that this formula resembles the standard welfare formula in Arkolakis, Costinot, and Rodríguez-Clare 2012. In a one sector version of our model, in which $j = 1$ and, γ^{jk}=0, $\alpha^j = 1$, equation 30 becomes

$$\frac{W_i}{P_i} \propto \left(\frac{T_i}{\pi_{ii}} \right)^{1/\theta} \tag{31}$$

This is the standard formula for welfare gains from trade that has been used in the literature and it depends on aggregate productivity, the home trade shares and the trade elasticity.

Our formula for welfare in equation (30) is dynamic. Dynamics are driven by the evolution of the stock of ideas captures in T_i^j. In this sense, our formula is the multi-sector version of the one derived in Buera and Oberfield 2016.

5 Quantitative Analysis

We quantify our model to evaluate the role that sector heterogeneity and interlinkages in production and knowledge flows have on innovation, productivity and welfare. We study the effect of a trade liberalization that consists of a uniform reduction of trade barriers of 40%. We compare the economy in the baseline and counterfactual BGPs. We consider four versions of our model: (i) our baseline model with heterogeneity in innovation, production and knowledge linkages; (ii) a model with sector heterogeneity but where diffusion is almost negligible, (iii) a model in which the production structure is symmetric across sectors; and (iv) a one sector model, in which there are no production and knowledge linkages across sectors. In all cases, we recalibrate the parameters of the model to match the same moments of the data.

5.1 Calibration

We use data on bilateral trade flows, R&D intensity, production, and patent citations to calibrate the main parameters of the model. We assume that the world is on a BGP in 2005. Here we explain in more detail the calibration of the average productivity parameters T_i^j, the diffusion parameters ε_{in}^{jk}, and the parameters governing the innovation process—the elasticity of innovation, β_r, and the efficiency of innovation, λ_i^j. Details on the data used in the calibration are relegated to Appendix B, and the description of the calibration procedure to recover other parameters of interest is provided in Appendix C.

5.1.1 Estimation of T_i^j: Gravity equation at the sector level

To estimate the technology parameters for tradable sectors, $j \leq J-1$, we follow the procedure in Levchenko and Zhang 2016 by estimating standard gravity equations for each sector in 2005. We start from the trade shares in equation (9):

$$\pi_{ni}^j = \frac{X_{ni}^j}{X_n^j} = \frac{T_i^j \left(c_i^j d_{ni}^j\right)^{-\theta}}{\Phi_n^j}. \quad (32)$$

Dividing the trade shares by their domestic counterpart as in Eaton and Kortum 2002 and, assuming $d_{nn}^j = 1$, we have

$$\frac{\pi_{ni}^j}{\pi_{nn}^j} = \frac{X_{ni}^j}{X_{nn}^j} = \frac{T_i^j \left(c_i^j d_{ni}^j\right)^{-\theta}}{T_n^j \left(c_n^j\right)^{-\theta}}. \quad (33)$$

Taking logs of both hand sides, we have

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j \left(c_i^j\right)^{-\theta}\right) - \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right) - \theta \log(d_{ni}^j). \quad (34)$$

The log of the trade frictions can be expressed as

$$\log(d_{ni}^j) = D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + ex_i^j + \nu_{ni}^j \quad (35)$$

Following Eaton and Kortum 2002, $D_{ni,k}^j$ is the contribution to trade costs of the distance between country n and i falling into the k^{th} interval (in miles), defined as [0,350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, maximum). The other control variables include common border effect, B_{ni}, common currency effect CU_{ni}, and regional trade agreement RTA_{ni}, between country n and country i. We include an exporter fixed effect, ex_i^j, to fit the patterns in both country incomes and observed price levels as shown in Waugh (2010). ν_{ni}^j is the error term.

Substituting (35) back into (34) results in the following gravity equation at the sector level:

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \log\left(T_i^j \left(c_i^j\right)^{-\theta}\right) - \theta ex_i^j - \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right) - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \quad (36)$$

Define $\hat{F}_i^j = \log\left(T_i^j \left(c_i^j\right)^{-\theta^j}\right) - \theta ex_i^j$ and $F_n^j = \log\left(T_n^j \left(c_n^j\right)^{-\theta}\right)$. We then estimate the following equation using fixed effects and observables related to trade barriers, taking θ as known.

$$\log\left(\frac{X_{ni}^j}{X_{nn}^j}\right) = \hat{F}_i^j - F_n^j - \theta(D_{ni,k}^j + B_{ni}^j + CU_{ni}^j + RTA_{ni}^j + \nu_{ni}^j). \quad (37)$$

The productivity of the tradable sector in country n relative to that in U.S., T_n^j / T_{US}^j, is then recovered from the estimated importer fixed effects as in

$$S_n^j = \frac{\exp(F_n^j)}{\exp(F_{US}^j)} = \frac{T_n^j}{T_{US}^j} \left(\frac{c_n^j}{c_{US}^j} \right)^{-\theta} \tag{38}$$

in which the relative cost component can be computed by expressing (20) as

$$\frac{c_n^j}{c_{us}^j} = \left(\frac{W_n}{W_{US}} \right)^{\gamma^j} \prod_{k=1}^{J-1} \left(\frac{P_n^k}{P_{US}^k} \right)^{\gamma^{jk}} \left(\frac{P_n^J}{P_{US}^J} \right)^{\gamma^{jJ}}, \tag{39}$$

where J indicates the nontradable sector. Using data on wages (in USD), estimates of price levels in the tradable sector and the nontradable sector relative to the United States, we can back up the relative cost. The nontradable relative price is obtained using the detailed consumer price data collected by the International Comparison Program (ICP). To compute the relative price of the tradable sector, we follow the approach of Shikher (2012) by combining (18), (19) and (21) and get the following expression for relative prices of tradable goods

$$\frac{P_n^j}{P_{US}^j} = \left(\frac{X_{nn}^j / X_n^j}{X_{US,US}^j / X_{US}^j} \frac{1}{S_n^j} \right)^{\frac{1}{\theta}}. \tag{40}$$

The right hand side of this expression can be estimated using the observed expenditure shares of domestic product in country n and in U.S. and the estimated importer fixed effects.

To compute the relative productivity in nontradable sectors, we combine (21), (22) and set the trade cost in nontradable sector d_{ni}^J to infinity for all i and n. This implies $\Phi_n^J = T_n^J \left(c_n^J\right)^{-\theta}$ based on equation (22). Substituting this expression into (21), we express the nontradable good price as

$$p_n^J = \frac{c_n^J}{(T_n^J)^{1/\theta}}. \tag{41}$$

The relative technology in nontradable sector can then be constructed based on

$$\frac{T_n^J}{T_{US}^J} = \left(\frac{c_n^J}{c_{US}^J} \frac{P_{US}^J}{P_n^J} \right)^{\theta} \tag{42}$$

Again, the cost ratios are calculated following (39) and the price ratios for the non-tradable sectors are from the ICP database.

We now have estimated the relative productivity for all countries relative to U.S. in every sector. To estimate the level of productivity, we need the productivity level in the U.S. First, using OECD industry account data, we estimate the empirical sectoral productivity for each sector in the U.S.

by the Solow residual (without capital in the production function)

$$\ln Z_{US}^{j} = \ln Y_{US}^{j} - \gamma^{j} \ln L_{US}^{j} - \sum_{k=1}^{J} \gamma^{jk} \ln M_{US}^{jk}, j = 1, 2, ..., J, \tag{43}$$

where Z_{US}^{j} denotes the measured productivity in U.S. in sector j, Y_{US}^{j} is the output, L_{US}^{j} is the labor input and M_{US}^{jk} is the intermediate input from sector k. Finicelli et al. (2013) show that trade and competition introduce selection in the productivity level, and the relationship between empirical productivity and the level of technology T_{US}^{j} in an open economy is given by

$$T_{US}^{j} = \left(Z_{US}^{j}\right)^{\theta} \left[1 + \sum_{i \neq US} S_{i}^{j} \left(d_{US,i}^{j}\right)^{-\theta}\right]^{-1}, \tag{44}$$

in which S_{i}^{j} and $d_{US,i}^{j}$ are estimated using (38) and (35) respectively. To obtain the exporter fixed effect in trade cost, ex_{i}^{j}, we use the importer and exporter fixed effects from the Gravity equation (37). That is, $ex_{i}^{j} = (F_{i}^{j} - \hat{F}_{i}^{j})/\theta$. Lastly, we normalize the nontradable technology in the U.S. to one, and express all T_{US}^{j} relative to T_{US}^{J} as

$$\hat{T}_{US}^{j} = \left(\frac{Z_{US}^{j}}{Z_{US}^{J}}\right)^{\theta} \left[1 + \sum_{i \neq US} S_{i}^{j} \left(d_{US,i}^{j}\right)^{-\theta}\right]^{-1}. \tag{45}$$

Throughout our analysis we assume that θ is common across countries and set it equal to 8.28.[4]

Figure 1 plots the distance parameters that we obtain from the sectoral gravity equations, d_{in}^{j}, against the trade share from the data that we use to estimate the gravity equations at the sector level, using $\theta = 8.28$.

[4]We have also run our gravity equation at the sector level using $\theta = 4$ and a sector specific θ from Caliendo and Parro 2015. We find that the technology parameters estimated under different θ are highly correlated, as it has been documented in Levchenko and Zhang 2016. In particular, the calibration of technology parameters for $\theta = 4$ and $\theta = 8.28$ is 0.98, whereas the correlation of the technology parameter when θ is common and when we use the θ from Caliendo and Parro 2015 is 0.8.

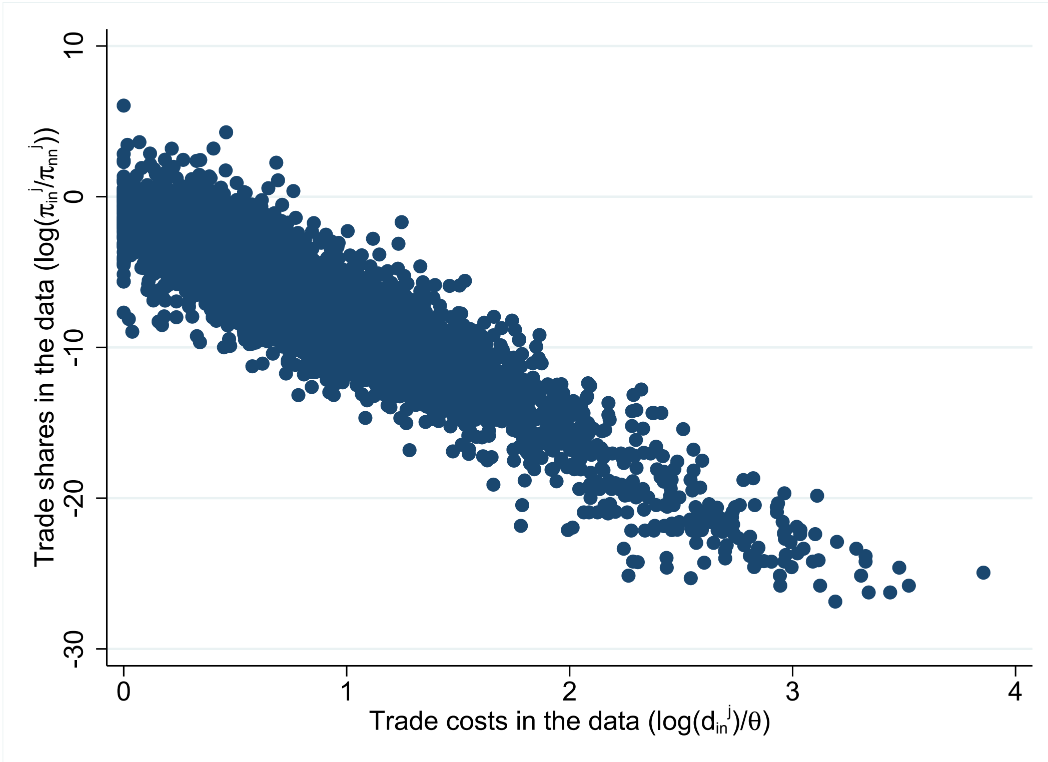


Figure 1: Trade shares and distance

5.1.2 The speed of knowledge diffusion

We discipline the speed of knowledge diffusion, ε_{ni}^{jk}, using citation data across countries and sectors obtained from the U.S. Patent and Trade Office (USPTO) for the period of 2000-2010. In the innovation literature, citation data have been used to trace the direction and intensity of knowledge flows between economic units (such as firms or countries) and across technological classes.[5] In the dataset, each patent is assigned to one of the 428 three-digit United States Patent Classification System (USPCS) technological fields (NClass) and belongs to one to seven out of the 42 two-to-four-digit Standard Industrial Classification (SIC) categories. We use the probability mapping provided by USPTO to assign patents into different SIC categories, which are then mapped into one of our 19 sectors.[6]

Based on our model assumption of the exponential distribution of citation lags, we can express the share of total citations from country n sector j made in year t to patents applied in year s by

[5] Although patent statistics have been widely used in studies of firm innovations, not all innovations are patented, especially process innovations, which are often protected in other ways such as copyright, trademarks and secrecy (see Levin et al.,1987). Our measure implicitly assumes that for any sector, the unpatented and patented knowledge utilizes knowledge (patented or unpatented) from other sectors in the same manner, with the same likelihood and intensity.

[6] Details of the concordance are available at http://www.uspto.gov/web/offices/ac/ido/oeip/taf/data/sic_conc.

country i sector k as

$$\widehat{citeshare}^{jk,t}_{ni,s} \equiv \frac{citation^{jk,t}_{ni,s}}{\sum_{s=0}^{t}\sum_{ik} citation^{jk,t}_{ni,s}} = \frac{\varepsilon^{jk}_{ni} e^{-\varepsilon^{jk}_{ni}(t-s)} P^k_{i,s}}{S^{j,t}_n}, \tag{46}$$

where $P^k_{i,s}$ is the total number of patent applications in sector k of country i in year s, $S^{j,t}_n = \sum_{s=1}^{t}\sum_{i,k}\varepsilon^{jk}_{ni} e^{-\varepsilon^{jk}_{ni}(t-s)} P^k_{i,s}$ is the knowledge stock available to country n sector j at time t that was ever invented at $s \leq t$, and $citation^{jk,t}_{ni,s}$ is the number of citations from patents in nj, t to patents ik, s. $P^k_{i,s}$, similar to the term $\lambda^k_{is}\left(s^k_{is}\right)^{\beta_r}$ in Eqn.(11), measures the new knowledge generated in ik at time s, and $\varepsilon^{jk}_{ni} e^{-\varepsilon^{jk}_{ni}(t-s)}$ is the share of $P^k_{i,s}$ that arrives nj at time t.

In addition, we assume that $S^{j,t}_n$ is a stock variable that grows at a constant rate g^j_n, i.e. $S^{j,t}_n = S^{j,0}_n e^{tg^j_n}$. Suppose (t_0, s_0) is the base year-pair. Dividing both sides of (46) by the base year-pair observation leads to

$$\frac{\widehat{citeshare}^{jk,t}_{ni,s}}{\widehat{citeshare}^{jk,t_0}_{ni,s_0}} = \frac{\varepsilon^{jk}_{ni} e^{-\varepsilon^{jk}_{ni}(t-s)} P^k_{i,s}}{S^{j,0}_n e^{tg^j_n}} \Big/ \frac{\varepsilon^{jk}_{ni} e^{-\varepsilon^{jk}_{ni}(t_0-s_0)} P^k_{i,s_0}}{S^{j,0}_n e^{t_0 g^j_n}} = \frac{P^k_{i,s}}{P^k_{i,s_0}} e^{-\varepsilon^{jk}_{ni}[(t-s)-(t_0-s_0)]} e^{(t_0-t)g^j_n} \tag{47}$$

Parameters $\{\varepsilon^{jk}_{ni}, g^j_n\}$ are then estimated using General Moment of Method (GMM) by the quadratic distance between the empirical counterpart of the citation share and (47). For each (nj, ik) country-sector pair over T peroids, we have $T(T-1)/2$ observations of (t, s) year-pairs and 2 unknowns.

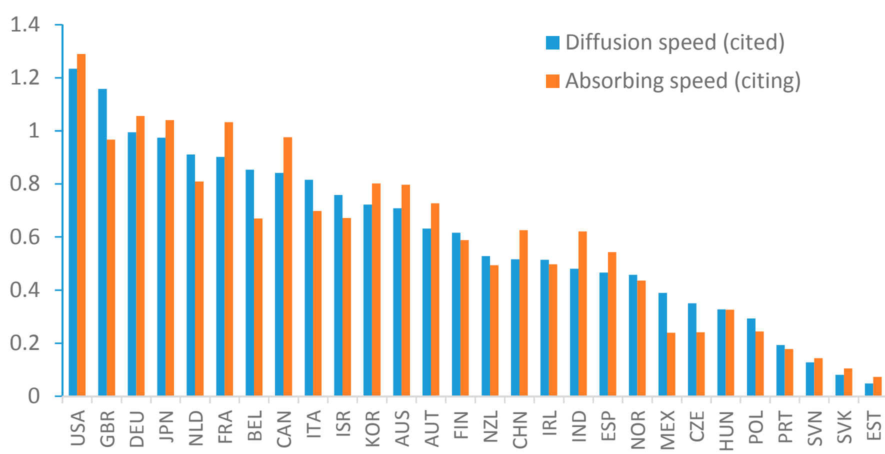
Figure 3 shows that the distribution of diffusion speed across countries and sectors is highly heterogeneous and skewed. In addition, Table 1 reports the average speed of diffusion by cited sector and citing sector. It shows that Chemicals, Computer, Electronic and Medical Instruments are the sectors that have the largest diffusion speed, while patents in Wood Products has the lowest diffusion speed. The citing speed (or speed of absorption) is highly correlated with the cited speed. Figure 2 shows the average speed of diffusion and absorption by country. Unsurprisingly, new knowledge in US, UK, Germany and Japan diffuse the fastest. The speed of diffusion of knowledge in US and UK on average diffuses in less than a year (captured by $1/\epsilon$). Countries which diffuse knowledge (get cited) rapidly also tend to acquire new knowledge from other countries (citing others) fast. Canada, France, and emerging innovation powerhouse like China and India are faster at acquiring new knowledge than diffusing its own knowledge.

The determinants of diffusion speed Table 2 examines the determinants of cross-country-sector knowledge diffusion speed by estimating a gravity equation extended to include measures of linguistic and religious distance as well as common history variables that potentially affect effectiveness of interaction and communication, all obtained from CEPII. We also investigate whether trade plays any role in driving the diffusion speed once distances and historical variables are controlled for. Citing and cited country fixed effects are included to control for country-specific characteristics such as size, level of development, and geography. Since we are interested not only on

Table 1: Average diffusion speed by sectors

ISIC	Industry	Cited	Citing
C24	Chemicals and chemical products	0.948	0.853
C30T33X	Computer, electronic and medical instruments	0.939	0.931
C01T05	Agriculture, hunting, forestry and fishing	0.932	0.912
C17T19	Textiles, textile products, leather and footwear	0.771	0.888
C29	Machinery and equipment, n.e.c.	0.752	0.850
C10T14	Mining and Quarrying	0.793	0.747
C28	Fabricated metal products, except machinery and equipment	0.667	0.736
C40T95	Nontradables	0.630	0.633
C21T22	Pulp, paper, paper products, printing and publishing	0.619	0.595
C15T16	Food products, beverages and tobacco	0.628	0.611
C25	Rubber and plastics products	0.570	0.537
C27	Basic metals	0.594	0.581
C23	Coke, refined petroleum products and nuclear fuel	0.478	0.506
C34	Motor vehicles, trailers and semi-trailers	0.406	0.377
C26	Other non-metallic mineral products	0.445	0.471
C31	Electrical machinery and apparatus, n.e.c.	0.473	0.467
C36T37	Manufacturing n.e.c. and recycling	0.336	0.319
C35	Other transport equipment	0.310	0.310
C20	Wood and products of wood and cork	0.175	0.144

Figure 2: Average speed of diffusion by country



Note: This figure presents the average diffusion speed and absorbing speed by country. Average diffusion (absorbing) speed is calculated as the average ϵ by cited (citing) country.

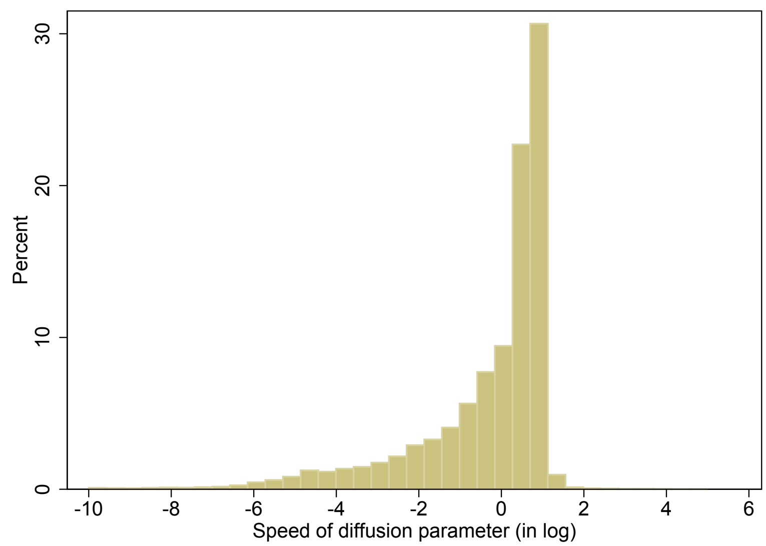


Figure 3: Distribution graph of ϵ_{ni}^{jk}

cross-country but also on cross-sector knowledge diffusion, we also include patent stock in citing and cited country-sector, and directional sector-pair fixed effects to capture the innate knowledge spillover relationship between different technologies (Cai and Li, 2016) that are independent of the source and destination countries.

Column (1) to (3) show that knowledge in sectors of country i diffuses faster to sectors of country n when the two countries are linguistically closer to each other or share a common language, share a border or in the same continent, both belong to the same regional free trade agreement (FTA) or currency union, were ever in a colonial relationship before 1945, have shared a common colonizer or were once the same country, have a different latitude. One country being landlocked or one country once being the colony of the other reduces the knowledge diffusion speed. Interestingly, georgraphic distance does not play a significant negative role and even an insignificant positive role on knowledge diffusion once trade linkages—that is exports between any country-sector pair combinations— are controlled for. Trade linkages are significantly and positively associated with knowledge linkages. The size of knowledge stock, as reflected in the patent stock, also matters. Higher the stock of knowledge the faster the diffusion speed, while countries with similar knowledge structure tend to diffuse slower.

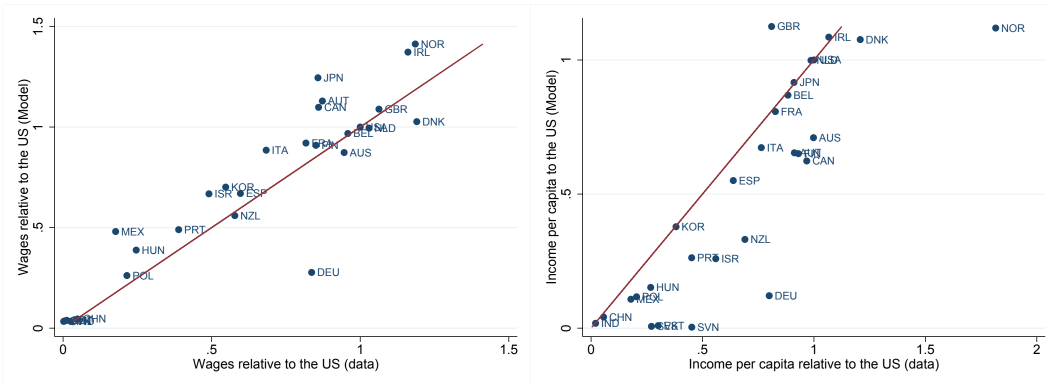
5.1.3 Parameters of innovation

We calibrate the parameters of innovation $\{\beta_r, \lambda_n^j, \hat{A}_n^j\}$ in two steps. First, we solve for the the static trade equilibrium taking as given the estimated sectoral productivity T_i^j, the estimated trade barriers d_{in}^j, and production input-output linkages parameters that we have obtained from the OECD input-output database $\{\alpha^j, \gamma^j, \gamma^{jk}\}$. The static equilibrium delivers relative wages, costs, prices and trade shares on the BGP.

Table 2: Determinants of Knowledge Diffusion Speed across Countries and Sectors

Dependent variable: $\log \varepsilon_{ni}^{jk}$			
	(1)	(2)	(3)
Georgraphic Distance$_{ni}$)	-0.071	0.038	0.014
	(-1.66)	(0.86)	(0.31)
Border$_{ni}$	0.349***	0.355***	0.348***
	(5.64)	(5.73)	(5.61)
FTA$_{ni}$	0.259***	0.214***	0.225***
	(5.66)	(4.67)	(4.88)
Currency Union$_{ni}$	0.582***	0.596***	0.585***
	(9.83)	(10.06)	(9.87)
Common language$_{ni}$	1.035***	0.997***	0.988***
	(18.77)	(18.01)	(17.82)
landlock$_{ni}$	-1.091***	-1.154***	-1.162***
	(-9.25)	(-9.80)	(-9.84)
absolute distance in lattitude$_{ni}$	0.015***	0.016***	0.016***
	(9.07)	(9.18)	(9.31)
absolute distance in longtitude$_{ni}$	-0.000	-0.000	0.000
	(-0.21)	(-0.40)	(0.01)
Common continent$_{ni}$	0.309***	0.289***	0.277***
	(5.00)	(4.68)	(4.47)
Linguistic Distance$_{ni}$	-1.732***	-1.663***	-1.663***
	(-9.80)	(-9.40)	(-9.40)
Religious Distance$_{ni}$	0.301*	0.263	0.203
	(2.12)	(1.86)	(1.41)
Colony$_{ni}$	-0.551***	-0.555***	-0.552***
	(-7.13)	(-7.17)	(-7.13)
Common Colonizer$_{ni}$	1.754***	1.730***	1.722***
	(6.23)	(6.16)	(6.14)
Colony after 1945$_{ni}$	0.998***	0.991***	1.021***
	(5.89)	(5.85)	(6.01)
Same country$_{ni}$	2.185***	2.145***	2.141***
	(20.72)	(20.36)	(20.32)
$\log X_{in}^{jk}$		0.025***	0.027***
		(5.18)	(5.48)
$\log X_{ni}^{jk}$		0.015**	0.013**
		(3.07)	(2.59)
$\log X_{in}^{kj}$		0.012*	0.010*
		(2.52)	(2.02)
$\log X_{ni}^{kj}$		0.029***	0.030***
		(5.86)	(6.17)
log Patent Stock$_n^j$			0.025***
			(3.66)
log Patent Stock$_i^k$			0.026***
			(3.87)
$Similarity_{ni}$			-0.256**
			(-2.72)
Citing country FEs	Yes	Yes	Yes
Cited Country FEs	Yes	Yes	Yes
Sector-pair FEs	Yes	Yes	Yes
R^2	0.52	0.52	0.52
Num of obs	272,916	272,916	272,916

Figure 4 shows relative wages and income per capita in the data and in the model. Our calibration strategy delivers relative wages and relative income per capita that are consistent with those observed in the data.

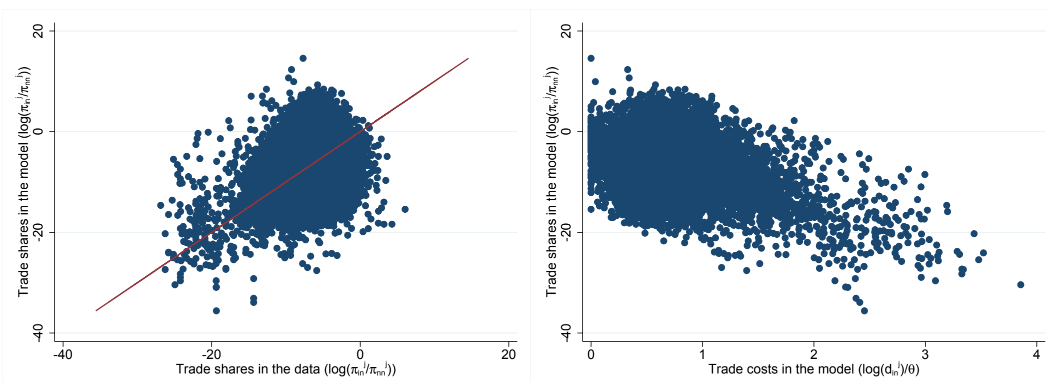


(a) Wages relative to the US

(b) Income per capita

Figure 4: Validation: Relative wages and relative income per capita

Figure 5 shows that our calibration strategy delivers trade shares that are consistent with the data.



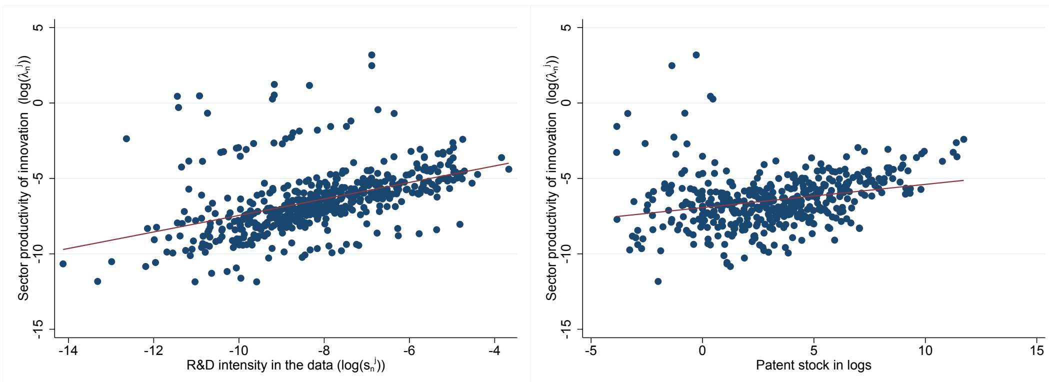
(a) Trade shares

(b) Trade shares and distance

Figure 5: Validation: Trade shares and distance

Having computed wages and trade shares, in the second step we use the estimated parameters for knowledge diffusion, ε_{in}^{kj}, data on R&D intensity at the country-sector level, s_n^j, and the growth rate of the economy on the BGP in equation (17) to calibrate the innovation parameters $\lambda_n^j, \beta_r, A_n^j$. We proceed as follows: First, we assume that all countries' productivity grow at $g_y = 3\%$ along the BGP, which corresponds to a growth rate for the stock of knowledge on the BGP of $g_A = \theta\left(1 + \sum_{j=1}^{J} \alpha_j \Lambda_j\right)^{-1} g_y = 0.25$ (see Appendix D for details on the derivation). Second, we use the Frobenius theorem and equation (17) to obtain a value for the efficiency of innovation, λ_i^k, and the elasticity of innovation, β_r. Given data for s_n^j, the estimated values for ε_{ni}^{jk}, and g_A, we can use the

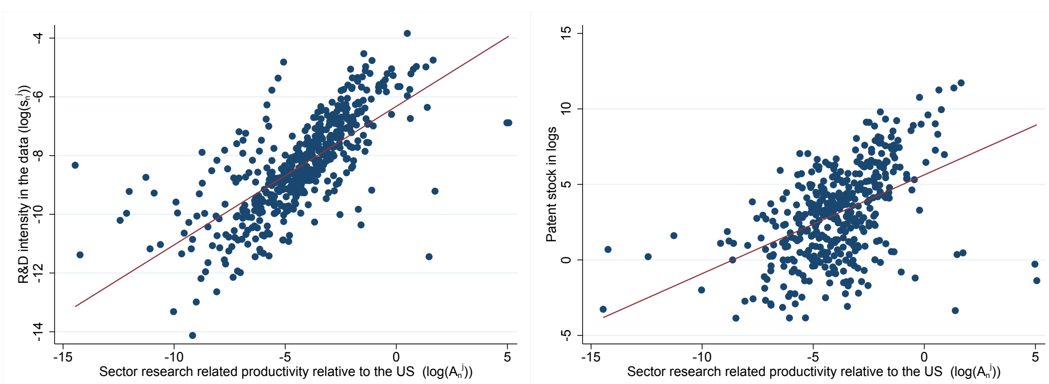
Frobenius theorem and iterate on equation (17) to obtain β_r and λ_n^j. We obtain that $\beta_r = 0.24$ and λ_n^j ranges from $7 * 10^{-6}$ to 24, with mean 0.10 and standard deviation 1.2. Figure 6 plots our estimated λ_i^k against both R&D intensity and the stock of patents. As the figure shows, there is a positive relationship between the productivity of innovation, λ_i^k, and both R&D intensity (as an input of innovation and a flow variable) and the stock of patents (as an output of innovation and a stock variable).



(a) R&D intensity (b) Stock of patents

Figure 6: The productivity of innovation, R&D intensity and the stock of patents

Given these parameter values, and using again the properties of the Frobenius theorem, the associated eigenvector to the growth rate of $g_A = 0.25$ corresponds to the normalized"knowledge-based productivity" $\hat{A}_n^j$. Figure 7 shows that there is a strong positive relation between the "research-related productivity", relative to the United States in sector J, $\hat{A}_i^j$, and both R&D intensity and the stock of patents.



(a) R&D intensity (b) Stock of patents

Figure 7: Research-related productivity and innovation

Figure 8 shows, by industry, the relationship between R&D intensity and the "productivity of research". We observe a strong positive correlation between the two measures. The correlation is larger for machinery and equipment, computer, electronic and optical equipment, and electrical machinery sectors.

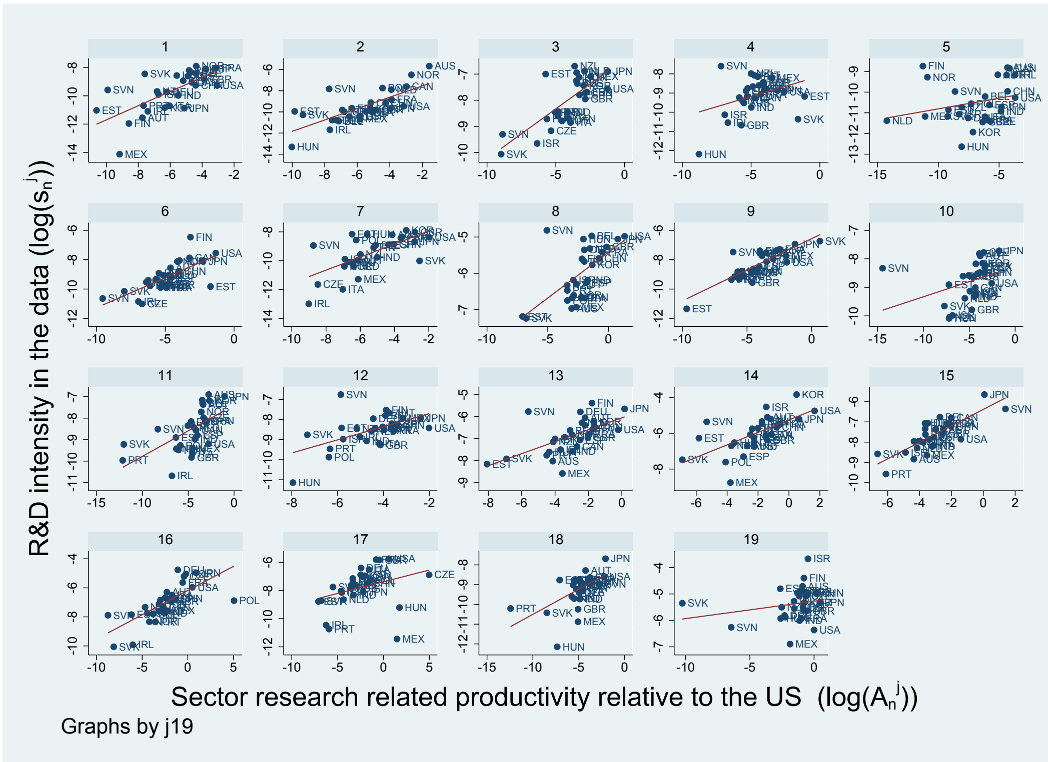
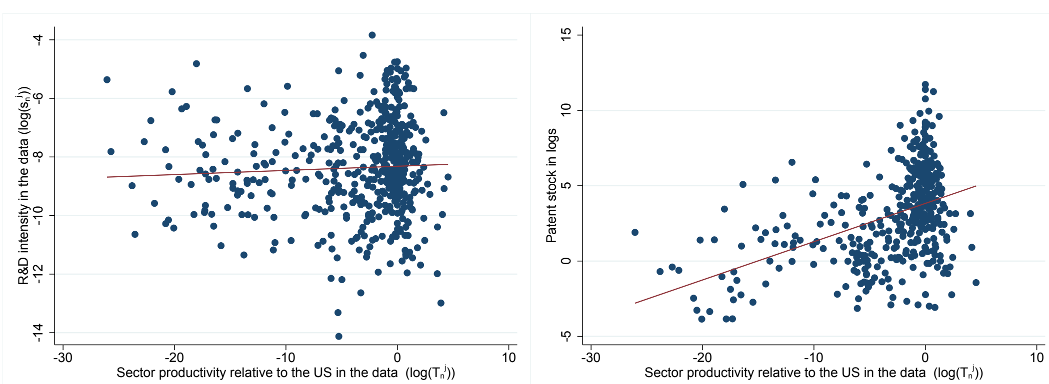


Figure 8: Research-related productivity and innovation by sector

When we compare R&D intensity with total average productivity relative to the United States, T_n^j, the relationship is not as strong (see figure 9). Average productivity in our model can be explained by two components: (i) A_n^j, which is highly correlated with R&D intensity, and (ii) $T_{p,n}^j$, which is not correlated with R&D intensity.



(a) R&D intensity (b) Stock of patents

Figure 9: Relative productivity and innovation

The algorithm

The calibration of the parameters of innovation, $\{\lambda_n^j, \beta_r, A_n^j\}$ follows a recursive algorithm. First, knowing $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, T_n^j, d_{in}^j\}$, we use the trade structure of the model to obtain wages, prices, expenditures, trade shares, and output, from equations (18), (19), (20), (21), (22),(24), (25), (26), and (27).

Then, knowing $\{\varepsilon in^{jk}, g_A, s_n^j\}$ we iterate over equation (17) to obtain $\{\lambda_n^j, \beta_r\}$. We do this in an iterative process in which, we guess over λ_n^j, and β_r we use R&D data, s_n^j, and we keep iterating until $g_A = 0.25$. We use (28) and (29) and the Frobenious theorem. The Frobenius theorem guarantees that there is a unique balanced growth path in which all countries and sectors grow at the same rate g_A. The expression for the growth rate can be expressed in matrix form as:

$$g_A A = \Delta(g_A) A$$

If the matrix $\Delta(g_A)$ is definite positive, then there exists a unique positive balanced growth rate of technology $g_A > 0$ given research intensities. Associated with that growth rate is a vector A (defined up to a scalar multiple), with every element positive, which reflects each country and sector relative level of knowledge along that balanced growth path. We update β_r so that $g_A = 0.25$ and we update λ_n^j so that R&D intensity matches the data. Then, we obtain A_n^j from the eigenvector associated to $\Delta(g_A = 0.25)$. Knowing T_n^j from the gravity regressions, and A_n^j from the Frobenius theorem, we can obtain $T_{p,n}^j$ from equation (6).

5.2 Counterfactual Analysis

We perform a uniform reduction of trade barriers for each country-sector pair of 40%. We analyze the effect of this reform on welfare gains from trade, innovation and productivity across the baseline and counterfactual BGP. First, we describe briefly the algorithm that we develop to compute the counterfactual BGP. Then, we report our main results for our multi-country and multi-sector endogenous growth model featuring heterogeneous interlinkages in production and knowledge flows.

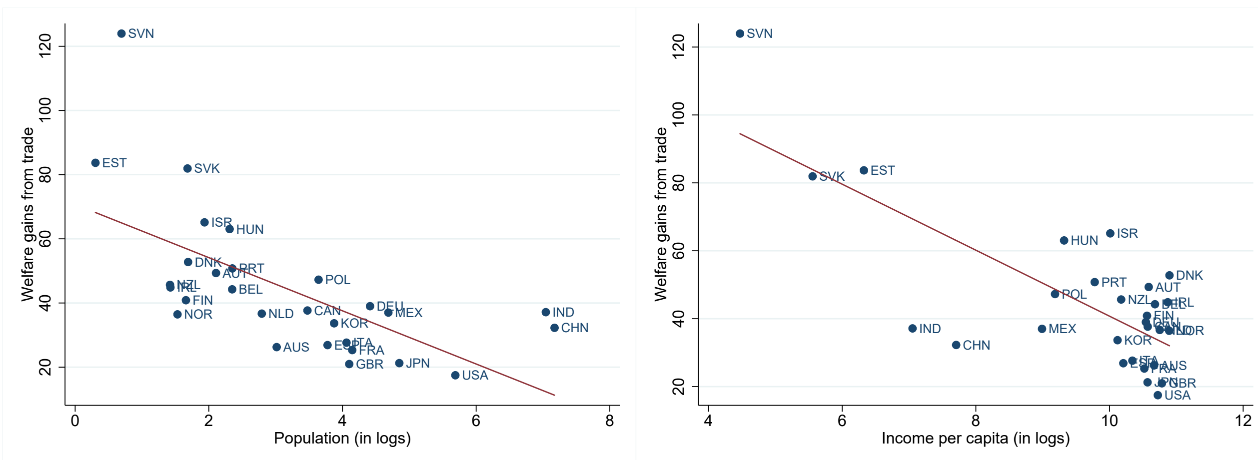
The algorithm

In our calibration exercise, we took the level of technology, $\hat{T}_n^j$, as given by the estimated values from the gravity regressions. However, $\hat{T}_n^j$ changes across counterfactuals due to changes in $\hat{A}_n^j$ that are induced by changes in innovation intensity, s_n^j. Our algorithm to solve for the counterfactual equilibrium uses the properties of the Frobenius theorem and allows T_n^j to evolve over time through changes in A_n^j. We proceed as follows. First, we take $\{\gamma^j, \gamma^{jk}, \alpha^j, \sigma, T_{p,n}^j, \hat{T}_n^j, \beta_r, \lambda_n^j\}$ as given, and compute the static equilibrium that corresponds to the new trade barriers, d_{in}^j. With that equilibrium, we compute the new optimal R&D intensity s_n^j and use the Frobenius theorem to obtain the new g_A and associated eigenvector $\hat{A}_n^j$. We do this by iterating over equation (17) until $g_A(t-1) = g_A(t)$. The new A_n^j delivers a new $\hat{T}_n^j$ (we keep $T_{n,p}^j$ constant across counterfactuals). We then repeat the procedure until $\hat{T}_n^j$ converges.

Welfare Gains from Trade, Innovation and RCA

We compute welfare gains from trade using equation (4). Welfare gains across two BGPs depend on two components: (1) changes in normalized consumption, $\hat{C}_i$, and (2) changes in the growth rate, g . From equation (4), the change in normalized consumption depends on two additional factors: (i) the change in R&D intensity s_i^k, and (ii) the change in income per capita, $\hat{Y}_i$. The change in R&D has an effect both on the growth rate g and on income per capita $\hat{Y}_i$.

We find that welfare gains from trade are heterogeneous across countries, raging from 17.5% in the United States to 124% in Slovenia, with an average gain of 44.6%. The gains depend negatively on population and the level of income per capita (see Figure 10).



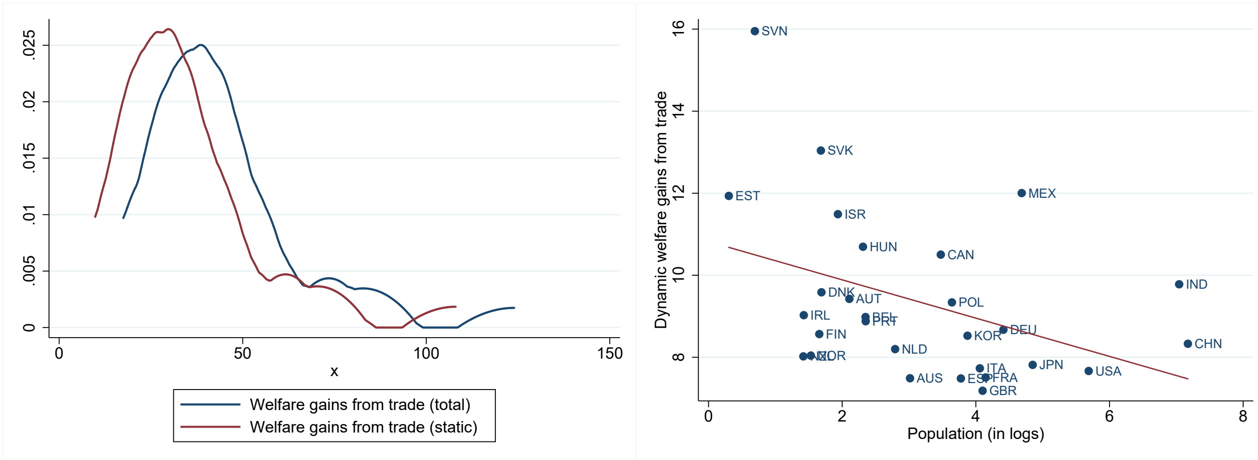
(a) Population

(b) Income per capita

Figure 10: Welfare gains from trade against population and income per capita

Welfare gains in equation 4 can be divided into static an dynamic gains. Static gains correspond to those obtained in a model where the stock of knowledge is not allowed to change over time. These are the gains that are obtained in standard static models of trade and are driven by increased specialization and comparative advantage. Dynamic gains operate through innovation by increasing income per capita, and the growth rate g. Knowledge diffusion has an additional effect on dynamic gains from trade. On the one hand, it increases the productivity of innovation of a sector though a spillover effect, as we can see from equation (10). On the other hand, knowledge spillovers cause convergence in relative productivity, dampening the total welfare gains from trade that are driven by differences in comparative advantage. We explore this point further in Section 5.2.

Figure 11a compares welfare gains from trade in our baseline model to those static gains in which the stock of technology is kept constant across counterfactuals. The difference accounts for the dynamic gains from trade (see Figure 11b). The cross-country distribution of static gains is shifted to the left, which implies that dynamic gains are positive. We find that, after the trade liberalization, the average country experiences dynamic gains from trade that are around 9%.

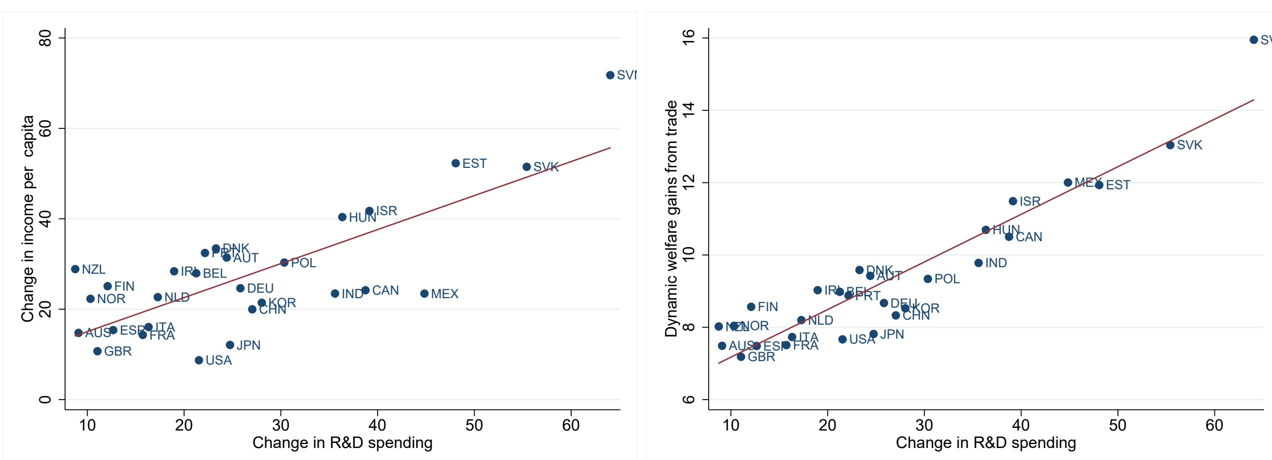


(a) Welfare gains from trade: Baseline against static

(b) Dynamic welfare gains from trade

Figure 11: Welfare gains from trade against population and income per capita

Dynamic gains from trade are driven by innovation, which generates increases in the growth rate and in income per capita. In our counterfactual exercise, the growth rate increases from 3% in the initial BGP to 3.1% in the counterfactual BGP. Income per capita, hence consumption and welfare also increase, from equation (4). We find that countries with larger increases in R&D spending also experience larger increases in income per capita (see Figure 12a). Those countries with larger increases in R&D experience larger dynamic welfare gains from trade (see Figure 12b).



(a) Income per capita

(b) Dynamic welfare gains from trade

Figure 12: The effect of changes in R&D

After a trade liberalization, all countries in our sample experience an increase in R&D spending. However, there is heterogeneity across sectors within a country. In general, R&D increases are correlated with increases in revealed comparative advantage (RCA) at the sector level. We find that this correlation is stronger for those countries that experience larger gains from trade (see

Figure 13).

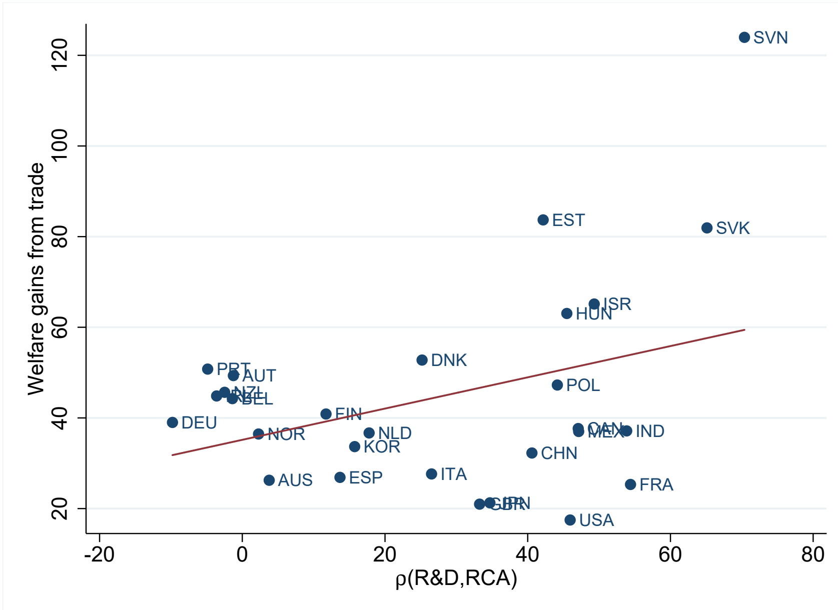


Figure 13: Welfare gains from trade and correlation of R&D and RCA

Welfare Gains from Trade: The role of knowldege spillovers

We study the role of knowledge diffusion on welfare gains from trade by recalibrating a model in which the diffusion parameters ε_{ni}^{jk} are set to a very small value of 0.0001 (we set $\varepsilon_{nn}^{jj} \to \infty$, that is, we assume instantaneous diffusion within the same country-sector pair).[7] We recalibrate the model parameters, β_r and λ_n^j, by using the same input-output linkage parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$, estimated technology, T_n^j, R&D intensity s_n^j and growth rate g_A values as those in the baseline model. We obtain $\beta_r = 0.28$ and a mean λ_n^j of 0.18 with standard deviation 2.2 .We then perform the same trade liberalization exercise as in the baseline model and evaluate the effect that a reduction of 40% in trade barriers has on innovation and welfare.

Figure 14 shows that the welfare gains from trade in a model with very low diffusion are larger than in a model where we allow for diffusion. As trade barriers go down, diffusion implies convergence in the stock of knowledge across countries. Because of evolving comparative advantage forces, a convergence in technology reduces the gains from trade as the forces of comparative advantage become more similar. This is consistent with the findings in Levchenko and Zhang 2016. When countries are more dissimilar in relative productivity, changes in trade costs have a larger impact on welfare gains from trade, through comparative advantage forces. This effect dominates the spillover effect on the productivity of innovation that allows countries to benefit from foreign innovations through diffusion in equation (10).

[7] The Frobenius theorem is only valid if there is at least some diffusion across all country-sector pairs. Setting ε_{ni}^{jk} to a very small number allows us to make use of the properties of the Frobenius theorem, while allowing for very slow diffusion, or virtually no diffusion.

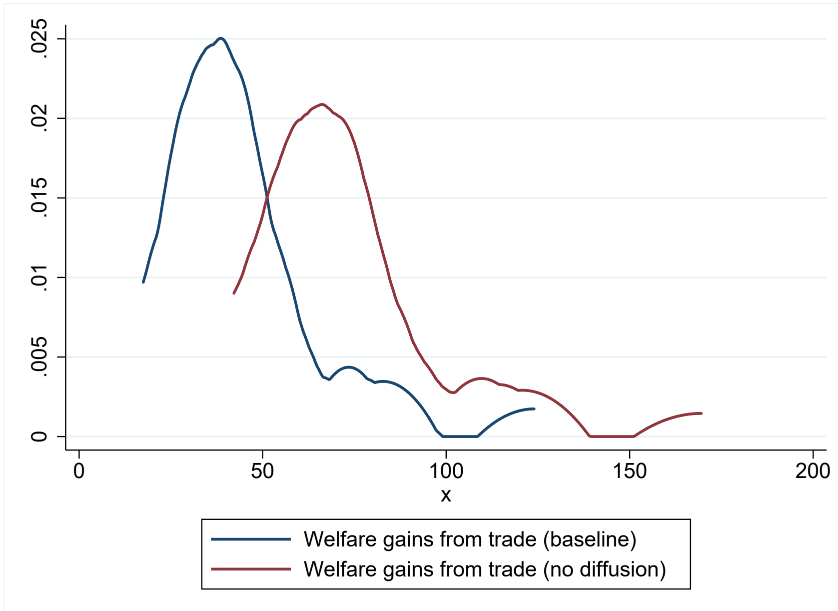
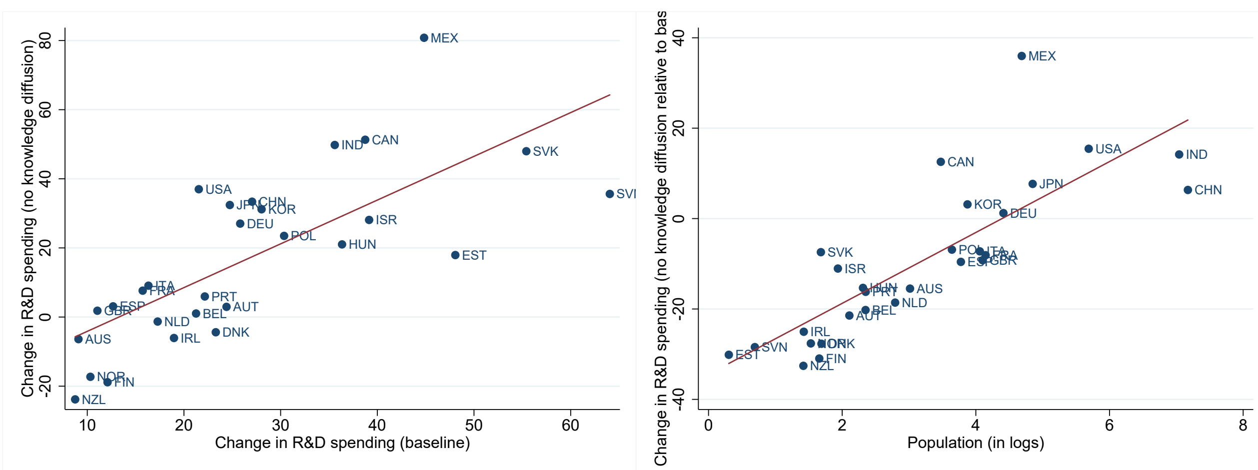


Figure 14: Welfare gains from trade with negligible knowledge spillovers

In a model in which there are no knowledge spillovers, R&D increases more in large countries. Figure 15a compares, for a cross-section, the change in R&D in the baseline model with the corresponding change in R&D in a model with negligible knowledge spillovers. The figure shows that there are substantial differences in how R&D changes in the two models. Figure 15b that large countries experience larger gains in R&D in the no knowldge spillover model. These are the countries that were gaining the least in a model with knowledge spillovers.



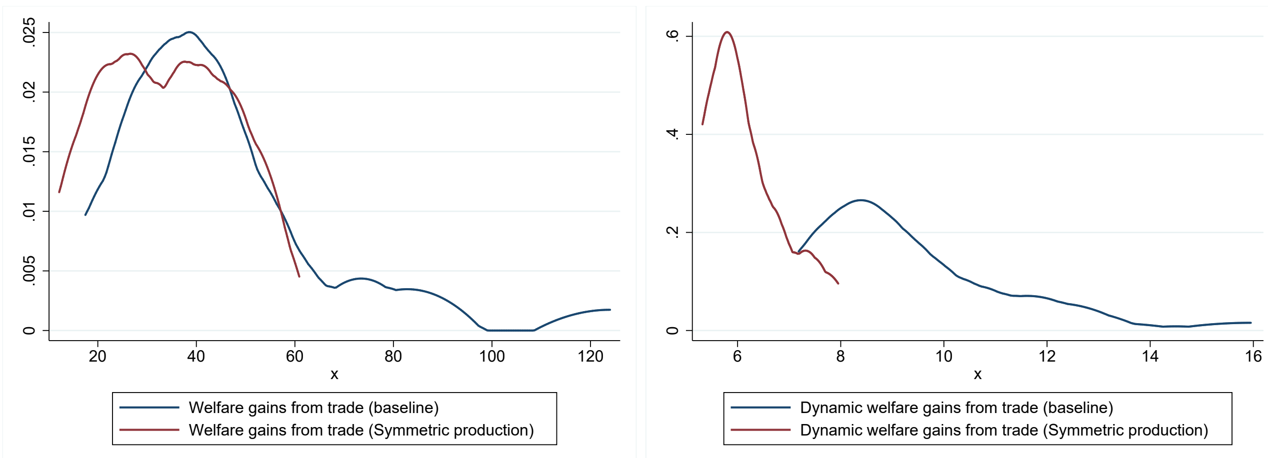
(a) Changes in R&D spending

(b) Population

Figure 15: Welfare gains from trade and R&D

Welfare Gains from Trade: The role of heterogeneity in production linkages

We now recalibrate a version of the model in which we shut down heterogeneity in production linkages, that is, we set γ^{jk}, γ^{j} and α_n^j to their average across all j and k. We also need to recalibrate T_n^j. We keep $\{\varepsilon_{in}^{kj}, s_n^j, g_A\}$ as in the calibrated baseline model. We obtain $\beta_r = 0.39$ and λ_n^j with mean 0.057 and standard deviation 0.18. We then perform the same trade liberalization exercise as in the baseline model and evaluate the effect that a reduction of 40% in trade barriers has on welfare. Figure 16 shows that welfare gains from trade are smaller in a model with homogeneous input-output linkages.

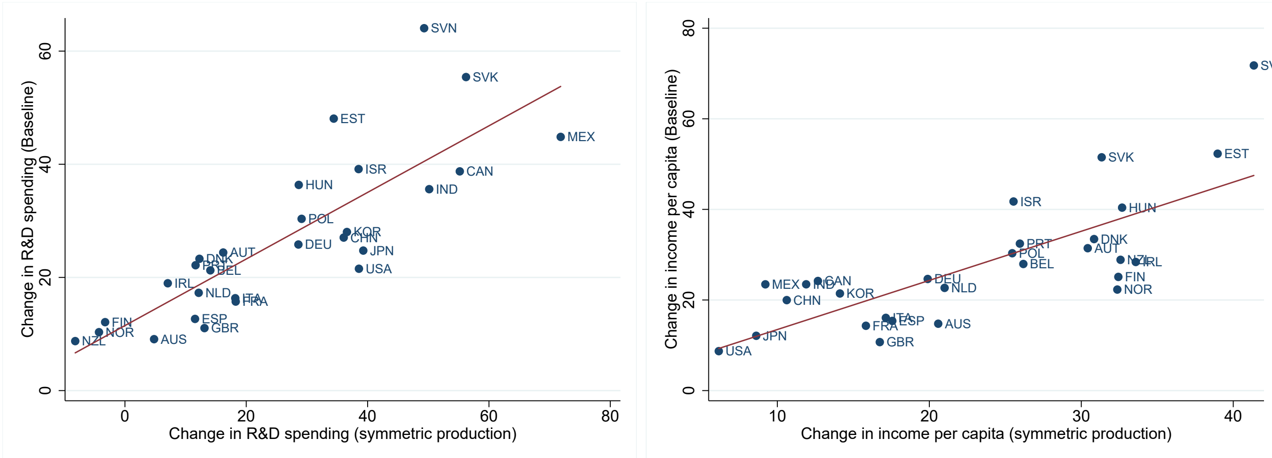


(a) Welfare gains form trade (symmetric production)

(b) Dynamic welfare gains form trade (symmetric production)

Figure 16: Welfare gains from trade in a model with symmetric production linkages

Dynamic welfare gains are even smaller than total gains when production linkages are symmetric. As Figure 17 shows, the increase in R&D and income per capita is lower with symmetric input-output linkages in those countries that were getting larger gains from trade when production linkages were asymmetric, such as Slovenia and Slovakia. Large countries, however, experience larger increases in R&D and income per capita when production linkages are symmetric.

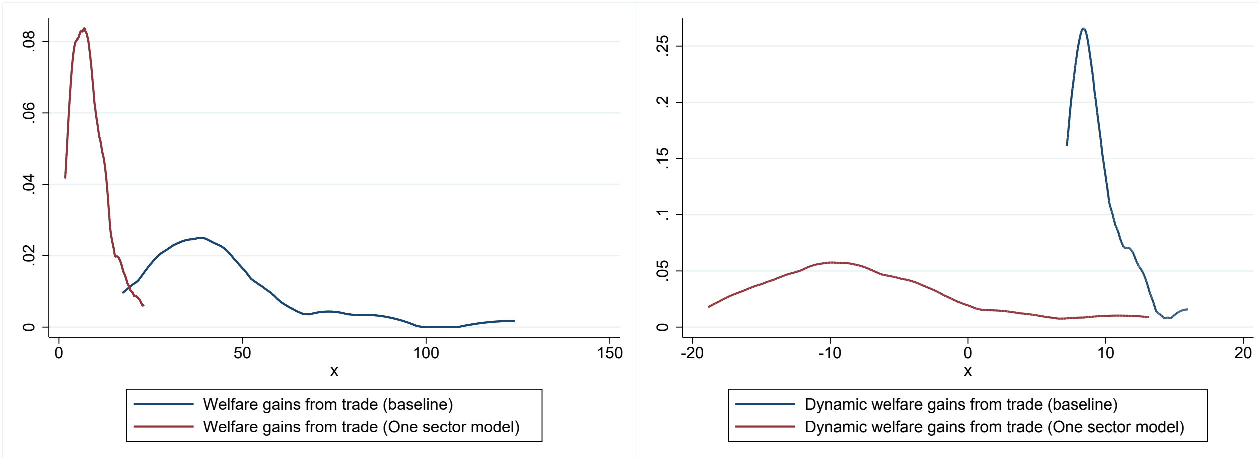


(a) Change in R&D (b) Change in income per capita

Figure 17: R&D and income per capita changes with symmetric production linkages

Welfare Gains from Trade: The role of multiple sectors

In Appendix E we show that in a one sector model without royalties changes in trade barriers have no effect on R&D and the growth rate of the economy. We recalibrate our baseline model to a one-sector model. We need to re-estimate the technology parameters, T_n by running gravity equations at the country level. We also recalibrate the production and knowledge linkages parameters. We obtain country-level data for R&D intensity, s_n. Then, assuming the same g_A as in the baseline model, we obtain a $\beta_r = 0.28$ and λ_n with mean 0.22 and standard deviation 0.13. We find that this model delivers substantially lower gains from trade than our multi-sector growth model with production and knowledge interlinkages (see Figure 18). In the multi-sector version of the model R&D was reallocating towards sectors that experienced larger increases in comparative advantage. That channel is no longer present in a one-sector model and R&D increases are much smaller.

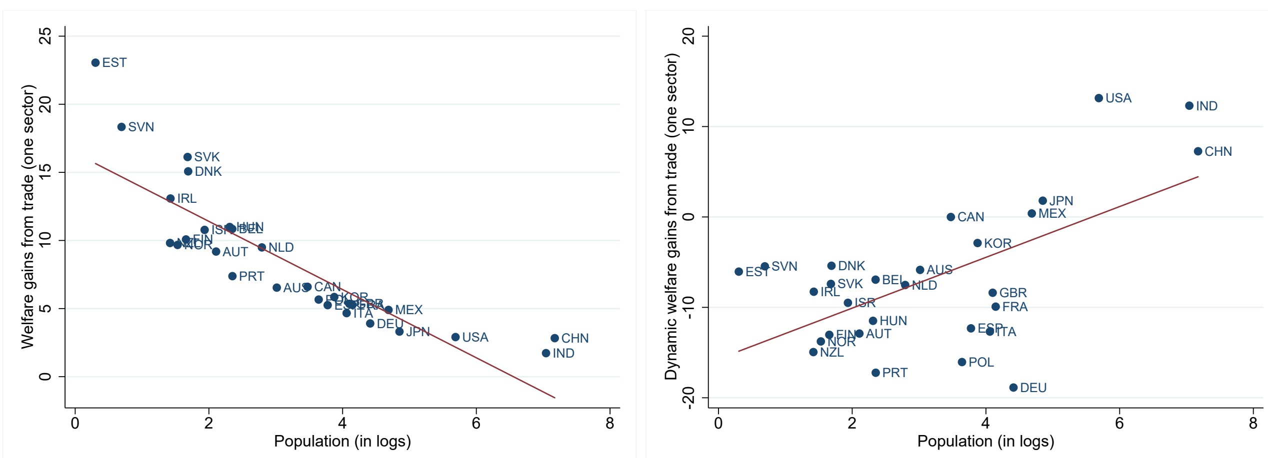


(a) Total gains

(b) Dynamic gains

Figure 18: Welfare gains from trade in a one-sector model

In a one sector model, large countries experience lower gains from trade. However, large countries experience larger dynamic gains, and some small countries experience negative dynamic gains from trade.



(a) Welfare gains from trade

(b) Dynamic welfare gains from trade

Figure 19: Welfare gains from trade and population in a one sector model

The dynamic gains from trade in this model are zero (see Apendix E). In a model with royalties, changes in trade costs do have an effect on R&D, but the effect is very small.

6 Conclusion

We develop a quantitative framework to study the interconnections between trade, knowledge flows and input-output linkages. In our model, changes in trade barriers have an effect on innovation and productivity. Changes in trade frictions induce a reallocation of R&D toward sectors in which the country has a comparative advantage and larger knowledge flows. Knowledge spillovers result into convergence in relative productivity after a trade liberalization. As a consequence, there are larger gains from trade when knowledge flows are slow, as relative productivities are more different across countries. One sector models deliver almost negligible dynamic welfare gains, and in some cases even negative.

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Appendix

A Model Equations

There are 14 endogenous variables and we need 14 equations. The endogenous variables are

$$\{\pi^j_{in}, T^j_i, c^j_i, W_i, P^j_n, X^j_{ni}, X^j_n, P_n, Y_n, \Phi^j_n, C_n, s^j_n, V^j_n, A^j_n\}$$

The corresponding equations are:

(1) Probability of imports

$$\pi^j_{ni} = T^j_i \frac{\left(c^j_i d^j_{ni}\right)^{-\theta}}{\Phi^j_n}, \tag{48}$$

with

$$T^j_i = A^j_i T^j_{p,i}, \tag{49}$$

(2) Import shares

$$X^j_{ni} = \pi^j_{ni} X^j_n, \tag{50}$$

(3) Cost of production

$$c^j_n = \Upsilon^j W^{\gamma^j}_{nt} \prod_{k=1}^{J} (P^k_n)^{\gamma^{jk}}, \tag{51}$$

(4) Intermediate good prices in each sector

$$P^j_n = A^j \left(\Phi^j_n\right)^{-1/\theta}, \tag{52}$$

(5) Cost distribution

$$\Phi^j_n = \sum_{i=1}^{M} T^j_i \left(d^j_{ni} c^j_i\right)^{-\theta}, \tag{53}$$

(6) Price index

$$P_n = \prod_{j=1}^{J} \left(\frac{P^j_n}{\alpha^j}\right)^{\alpha^j}, \tag{54}$$

(7) Labor market clearing condition

$$W_n L_n = \sum_{j=1}^{J} \gamma^j \sum_{i=1}^{M} \pi_{in}^j X_i^j, \tag{55}$$

(8) Sector production

$$X_n^j = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} X_i^k \pi_{in}^k + \alpha^j P_n Y_n, \tag{56}$$

(9) Final production

$$P_n Y_n = W_n L_n + \frac{\sum_{j=1}^{J} \sum_{i=1}^{M} \pi_{in}^j X_i^j}{1+\theta}, \tag{57}$$

(10) Resource constraint

$$Y_n = C_n + \sum_{k=1}^{J} s_n^k Y_n, \tag{58}$$

(11) Innovation

$$\dot{A}_{nt}^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \varepsilon_{ni}^{jk} \int_{-\infty}^{t} e^{-\varepsilon_{ni}^{jk}(t-s)} \alpha_{is}^k \left(s_{is}^k\right)^{\beta^k} ds, \tag{59}$$

(12) R&D expenditures

$$\beta^j \lambda_{nt}^j V_{nt}^j \left(s_{nt}^j\right)^{\beta^j - 1} = P_{nt} Y_{nt}, \tag{60}$$

(13) Value of an innovation

$$V_{nt}^j = \int_t^{\infty} \left(\frac{P_{nt}^j}{P_{ns}^j}\right) e^{-\rho(s-t)} \Pi_{ns}^j ds, \tag{61}$$

with

$$\Pi_{nt}^j = \frac{1}{(1+\theta) A_{nt}^j} \sum_{i=1}^{M} X_{it}^j \pi_{int}^j. \tag{62}$$

B Data Description and Calculation

This appendix describes the data sources and construction for the paper. 28 countries are included in our analysis based on data availability (mostly constrained by R&D data): Australia, Austria, Belgium, Canada, China, Czech Republic, Estonia, Finland, France, Germany, Hungary, India, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovenia, Spain, Slovakia, Slovenia, United Kingdom, and United States. The model is calibrated for the year 2005. There are 18 tradable sectors and one aggregate nontradable sector under consideration, which correspond to those in Caliendo and Parro 2015 and are reported in Table 3.

Bilateral trade flows at the sectoral level Bilateral trade data at sectoral level Data for expenditure by country n of sector j goods imported from country i (X_{ni}^j) are obtained from the OECD STAN Bilateral Trade Dataset. Values are reported in thousand U.S. dollars at current prices. Sectors are recorded at the ISIC (rev. 3) 2-3 digit level and were mapped into 2-digit tradable 19 sectors as listed in Table 3. We use the importer reported exports in each sector as the bilateral trade flows as it is generally considered to be more accurate than the exporter reported exports.

Value added and gross production Domestic sales in sector j, X_{nn}^j is estimated based on the *domestic* input-output table provided by OECD STAN database, which contains data at ISIC 2-digit level that can be easily mapped into our 19 sectors. OECD provides separate IO table for domestic output and imports. We sum up the values for a given row up to "Direct pruchases abraod by residents (imports)" to obtain X_{nn}^j. We compared this way of estimating the domestic expenditure on domestic product with an alternative calculation based on $X_{nn}^j = Y_n^j - \sum_{i \neq n}^M X_{in}^j$, where both gross production of country n in sector j, Y_n^j and the total exports from n to i in sector j, $\sum_{i \neq n}^M X_{in}^j$, are from OECD STAN Database for Structural Analysis. The first method proves to be superior as the second generates a number of negative observatoins for some country-sectors. However, data are missing for India, for which we use the INDSTAT (2016 version) provided by United Nations Industrial Development Organization (UNIDO).

Trade barriers and gravity equation variables Data for variables related to trade costs used in gravity equations (e.g. distance and common border dummies) at the country-pair level are obtained from the comprehensive geography database compiled by CEPII. WTO's RTA database provides infomraiton on regional trade aggrements.

Wages Average annual wages is reported by OECD Labour statistics at current price in local currency. They are translated into U.S. dolars at the 2005 exchange rates to obtain the variable w_n in the model. However, wage data for China, India, and New Zealand are missing in this database, and are obtained from International Labor Organization (ILO).

Factor shares and final consumption shares Data on the share of materials from sector k used in the production in sector j, γ_n^{jk}, as well as the labor share of production in sector j, γ_n^j, come from the Input-Output Database maintained by OECD STAN. The I-O table gives the value of the intermediate input in row k required to produce one dollar of final otuput in column j. We then divide this value by the value of gross output of sector j to obtain γ^{jk}. Similarly, the labor share is calculated as the ratio of value added to gross output, as capital input does not exist in the model. In our analysis we used the U.S. factor shares in 2005 for all countries. In addition, the final consumption expenditure shares of each sector, α_n^j also come from the I-O matrix.

R&D data R&D expenditures at the country-sector level are obtained from the OECD database of Business enterprise R&D expenditure by industry (ISIC Rev 3). Since sectoral R&D data for China, India and Sweden and several sectors in other countries are missing, we obtain estimates of these missing observations using the following approach. First, we run a regression using existing country-sector specific R&D and patent data from USPTO for 2005:

$$\log(R_n^j) = \beta_0 + \beta_1 \log\left(PS_n^j\right) + \mu_n + \gamma_j + \varepsilon_n^j, \tag{63}$$

where R_n^j is the R&D dollar expenditure of country i in sector j and PS_n^j is the patent stock of country i in sector j. μ_i and γ_j are country and sector fixed effects. This relation is built on the observations that (a) at steady state, R&D expenditure should be a constant ratio of R&D stock, and (b) innovation input (R&D stock) is significantly positively related to innovation output (patent stock). In fact, the coefficient β_1 is large and significant at 99% and R^2 is close to 0.90. Assuming that the relationship captured by (63) holds for China, India and Sweden, we can obtain the fitted value of their sectoral level R&D expenditure:

$$\log(\widehat{R}_n^j) = \widehat{\beta}_0 + \widehat{\beta}_1 \log\left(PS_n^j\right) + \widehat{\mu}_n + \widehat{\gamma}_j$$

For these three countries, we have information on all the right-hand-side variables except for the country fixed effects, $\widehat{\mu}_n$. This allows us to compute the *share* of R&D in a given sector for each country,

$$\widehat{r}_n^j = \frac{\widehat{R}_n^j}{\sum_j \widehat{R}_n^j} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}.$$

Second, we then obtain the aggregate R&D expenditure as percentage of GDP, $R\&D/GDP_n^{WB}$, for each country from the World Bank World Development Indicator Database. The country-sector specific R&D can then be estimated as $s_n^j = \widehat{r}_n^j \times R\&D/GDP_n^{WB}$. For the countries with missing sectors, we estimate the fitted value using the same procedure. To maintain consistency across countries, we correct the OECD data generated total R&D with the World Bank total R&D.

$$s_n^j = R\&D/GDP_n^{WB} \times \frac{R_n^{j,OECD}}{\sum_j R_n^{j,OECD}}$$

This estimated s_n^j is the R&D intensity parameter in Equations (15) and (11) used in our quantitative analysis.

Table 3: List of Industries

Sector	ISIC	Industry Description
1	C01T05	Agriculture, Hunting, Forestry and Fishing
2	C10T14	Mining and Quarrying
3	C15T16	Food products, beverages and tobacco
4	C17T19	Textiles, textile products, leather and footwear
5	C20	Wood and products of wood and cork
6	C21T22	Pulp, paper, paper products, printing and publishing
7	C23	Coke, refined petroleum products and nuclear fuel
8	C24	Chemicals and chemical products
9	C25	Rubber and plastics products
10	C26	Other non-metallic mineral products
11	C27	Basic metals
12	C28	Fabricated metal products, except machinery and equipment
13	C29	Machinery and equipment, nec
14	C30T33X	Computer, Electronic and optical equipment
15	C31	Electrical machinery and apparatus, nec
16	C34	Motor vehicles, trailers and semi-trailers
17	C35	Other transport equipment
18	C36T37	Manufacturing n.e.c. and recycling
19	C40T95	Nontradables

C Calibration

In this section, we describe the procedure that we follow to calibrate all the relevant parameters of our model.

- θ: For the dispersion parameter, we try three different values: Following Levchenko and Zhang 2016, we use $\theta = 4$, $\theta = 8.28$ and θ taken from Table A.1 in Caliendo and Parro 2015. The technology parameters estimated under different θ are highly correlated, as in Levchenko and Zhang 2016.

- σ^j: The elasticity of substitution parameter is taken from Broda and Weinstein (2006) for the United States (this parameter is sector specific but not country-specific. We matched SITC rev 3 into ISIC rev 3 and take the mean σ^i of SITC sectors that belong to the same ISIC sector. Data is based on their estimates for period 1990-2001. We do not need this parameter for any of our results.

- γ_n^j and γ_n^{jk} from the I/O tables. Given our production function, the labor share =value added share (as we don't have capital). So γ_n^j is calculated as value added/gross output V_n^j/Y_n^j for each country-sector, γ_n^{jk} is input value of sector k (row sectors) to the gross output of sector j (column sectors) for country n or the share of intermediate consumption of sector j in sector k over the total intermediate consumption of sector k times $1 - \gamma_n^j$.

- β^j is the elasticity of innovation and we can assume that is the same across countries and sectors.

The remaining parameters that we need to calibrate are d^j_{in}, and T^j_n, and the growth rate of the economy.

1. We use bilateral trade gravity equation to estimate the country-sector specific competitiveness and productivity. We follow as close as possible to Caliendo and Parro 2015 with the same set of countries and sectors. In the production side, sectors are connected by Input-output linkages and trade flows, but service is non-tradable. For robustness, we try two methods to estimate country-sector specific productivity level and distance parameters.

- – Method 1

First, we run sector specific gravity equations with constraints on the importer n and export i fixed effects ($\sum_i S^j_i = 1$ and $\sum_n S^j_n = 1$), to obtain importer-exporter-sector speficit distance $D^j_{ni} = \sum_k \rho^j_k \log(D_k)$ and country-sector fixed effects $\left\{S^j_i\right\}$ and $\left\{S^j_n\right\}$.

$$log\left(\frac{X^j_{nit}}{X^j_{nnt}}\right) = S^j_i - S^j_n - D^j_{ni} \quad (64)$$

$$= S^j_i - S^j_n - \sum_{k=1}^{10} \rho^j_k D_k \quad (65)$$

where D_1 to D_6 are distance dummy variables equal to one if the population weighted distance countries n and i is between 0 and 375 kilometers, 375 and 750 kilometers, 750 and 1500 kilometers, 1500 and 3000 kilometers, 3000 and 6000 kilometers, and above 6000 kilometers; D_7 to D_{10} are dummy variables indicating if countries n and i share common language, common border, belong to the same free trade agreement and costumes union. When $X^j_{nit} = 0$, we enter $log\left(\frac{X^j_{nit}}{X^j_{nnt}}\right)$ as $log\left(\frac{X^j_{nit}*1000+1}{X^j_{nnt}*1000}\right)$.

ρ^j_k is the sensitivity of sector j's trade flow to the k^{th} trade barrier. By allowing sector specific sensitivities, trade libralization in the counterfactual simulation will cause production sturctual change effect, pushing low distance sensitive sectors to remote countries and nontradable service sectors to central countries.

Second, armed with the S^j_i, S^j_n and D^j_{ni} from gravity equations, we then combine Equation (32) to (34) to obtain the country-sector specifc cost c^j_i and productivity T^j_i for three different sets of $\{\theta\}$: (I) $\theta = 4$ for all non-service sectors, (ii) $\theta = 8.28$ for all non-service sectors, and (iii) $\{\theta\}$ from Caliendo and Parro 2015.

- – Method 2

We compute the D_{ni}^j using the sector specific version of Equation (12) in Eaton and Kortum (2002) and P_i^j on the right hand side of the equation from World Bank International Consumer Price dataset for 24 countries, using different sets of $\{\theta\}$ as in Method 1. Then we calculate c_i^j using (32), and substiute c_i^j into (11) to derive T_i^j, also under different sets of $\{\theta\}$.

1. Once we have a value for the fixed effects at the exporter level F_n^k we can plug them into equation (5) to obtain Φ_n^j which is a measure of technology progress in a county.

2. Then, we can use (1) to obtain π_{in}^j

3. Then we can use equation (4) and obtain P_n^j.

4. We then plug this into equation (6) to obtain P_n.

5. We now follow Caliendo and Parro 2015 and guess a vector of wages and use (7), (8) and (9) to obtain wages, expenditure X_n^j and Y_n^j. We guess vector of wages and update using the labor market clearing condition.

6. Then we can obtain the profits and the value of an innovation using (13)

7. Then use (12) to obtain s_n^j

8. Then, use equation (11) to obtain g and T_n^j using the Frobenius theorem

D The Balanced Growth Path

Here, we derive an expression for the growth rate of the economy along the BGP. First, note that through technology diffusion, the level of "knowledge-related productivity", A_n^j, grows at the same rate for every country n and sector j. Therefore, we can pick country M and sector J's technology level to normalize every A_n^j and T_n^j. Normalized variables are denoted with a hat. In particular, $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$.

From equation (60), we normalize the value of an innovation as $\hat{V}_n^j = \frac{V_n^j T_M^J}{W_M}$. Then, from equation (62), profits are normalized as $\hat{\Pi}_n^j = \frac{\Pi_n^j}{W_M}$, and from equation (55) X_i^j are normalized as $\hat{X}_i^j = \frac{X_i^j}{W_M}$ for all j. Hence, expenditures grow at a constant rate for all sectors, since $\pi_i n^j$ is constant in the BGP (see equations (48) and (53)). From equations (55) and (57), $P_n Y_n$ grow at the rate of W_M. Note that $g_{w_n} = g_w$ for all n.

To derive an expression for the BGP growth rate of the real output per capita, Y_n, we start from the fact that $\frac{W_n}{P_n Y_n}$ is constant in steady-state. Hence,

$$g_{Y_n} = g_w - g_{P_n}$$

From equation (54),

$$g_{P_n} = \sum_{j=1}^{J} \alpha^j g_{p_n^j}$$

We then derive the expression for $g_{p_n^j}$ from equations (51), (52) and (53). First, we rewrite equation (51) as

$$\frac{c_n^j}{W_n} = \prod_{k=1}^{J} \left(\frac{p_n^k}{W_n}\right)^{\gamma_n^{jk}}$$

In growth rates,

$$g_{\tilde{c}_n^j} = \sum_{k=1}^{J} \gamma_n^{jk} g_{\tilde{p}_n^k} \tag{66}$$

where $\tilde{c}_n^j = \frac{c_n^j}{W_n}$ and $\tilde{p}_n^k = \frac{p_n^k}{W_n}$.

From equation (53),

$$g_{\Phi_n^j} = g_T - \theta g_{c_n^j} = g_T - \theta g_{c_i^j}$$

with $g_T = g_A$.

Hence, $g_{c_n^j} = g_{c^j}$ for all n. Normalizing by wages,

$$g_{\tilde{\Phi}_n^j} = g_T - \theta g_{\tilde{c}_n^j} \tag{67}$$

where $\tilde{\Phi}_n^j = \frac{\Phi_n^j}{W_n^{-\theta}}$

Then, combining equation (52) and (67),

$$g_{\tilde{p}_n^k} = -\frac{1}{\theta} g_T + g_{\tilde{c}^k} \tag{68}$$

Substitution into (66) and using $\sum_{k=1}^{J} \gamma^{jk} = 1\gamma^j$,

$$g_{\tilde{c}^j} = -\frac{(1-\gamma^j)}{\theta} g_T + \sum_{k=1}^{J} \gamma^{jk} g_{\tilde{c}^k} \tag{69}$$

We can express the previous expression in matrix form so that:

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{1}{\theta} g_T \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} + \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} & \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} & \end{bmatrix} \begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} \tag{70}$$

From here

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{g_T}{\theta}(I-A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} \tag{71}$$

where

$$A = \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} \\ \vdots & \vdots & \vdots & \ddots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} \end{bmatrix}$$

Therefore, the cost of production c_n^j can be normalized as

$$\hat{c}_n^j = \frac{c_n^j}{W_M (T_M^J)^{-\frac{1}{\theta}\Lambda_j}} \tag{72}$$

with Λ_j is the j'th entry of the vector $\Lambda = (I-A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix}$.

With this, we can obtain an expression for the growth rate of real output as

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j g_{p_n^j}$$

From equation (68),

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_{c^j} \right)$$

From equation (72)

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_w - \Lambda_j g_T \right)$$

From here

$$g_{Y_n} = \frac{1}{\theta} \left(1 + \sum_{j=1}^{J} \alpha_j \Lambda_j \right) g_T = g_y, \forall n \tag{73}$$

Note that in a one-sector economy in which $\gamma^{jk} = 0, \forall n, k$ and $\gamma^j = 1, \forall j$, the growth rate is

$$g_y = -\frac{1}{\theta} g_T$$

as in Eaton and Kortum 1996 and Eaton and Kortum 1999. With multiple sectors, however, the growth rate of the economy is amplified by the input-output linkages.

E One Sector Model

We show that in a one-sector version of our model, change sin trade barriers have no effect on the optimal R&D intensity, hence on growth rates along the BGP. In the one-sector model, $\gamma^j = 1$ and $\gamma^{jk} = 0$. The one-sector version of equations (24), (25) and (26) is

$$\hat{W}_n L_n = \sum_{i=1}^{M} \pi_{in} \hat{X}_i, \tag{74}$$

$$\hat{X}_n = \hat{Y}_n, \tag{75}$$

$$\hat{Y}_n = \hat{W}_n L_n + \frac{\sum_{i=1}^{M} \pi_{in} \hat{X}_i}{1+\theta}, \tag{76}$$

Using equations (74) and (76),

$$\hat{Y}_n = \frac{1+\theta}{\theta} \hat{W}_n L_n$$

and

$$\frac{\sum_{i=1}^{M} \pi_{in} \hat{X}_i}{1+\theta} = \frac{\hat{Y}_n}{2+\theta}$$

From equations (28) and (29) in a one sector model with royalties:

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r \lambda_n \frac{\hat{V}_n}{\hat{Y}_n} = \beta_r \lambda_n \frac{1}{\rho - g_y + g_A} \frac{\sum_{i=1}^{M} \frac{\varepsilon_{in}}{g_A + \varepsilon_{in}} \frac{\sum_{m=1}^{M} \hat{X}_m \pi_{mi}}{1+\theta}}{\hat{Y}_n} \tag{77}$$

Using the previous expression

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r \lambda_n \frac{\hat{V}_n^j}{\hat{Y}_n} = \beta_r \lambda_n^j \frac{1}{\rho - g_y + g_A} \frac{1}{2+\theta} \sum_{i=1}^{M} \frac{\varepsilon_{in}}{g_A + \varepsilon_{in}} \frac{\hat{Y}_i}{\hat{Y}_n} \tag{78}$$

In this case, change sin trade costs have an effect on optimal R&D intensity to the extent that they have an effect on $\frac{\hat{Y}_i}{\hat{Y}_n}$.

If there are no royalties, the above expression becomes

$$\left(s_n^j\right)^{(1-\beta_r)} = \beta_r \lambda_n^j \frac{1}{\rho - g_y + g_A} \frac{1}{2+\theta} \tag{79}$$

In this case, changes in trade costs do not have an effect on optimal R&D intensity, hence on the growth rate along the BGP.