Sovereign Debt, Default Risk, and the Liquidity of Government Bonds

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Abstract

Secondary markets for sovereign bonds are illiquid because of trading frictions. I build a framework with endogenous illiquidity to study its implications on credit spreads and default risk. The model integrates directed search in secondary markets into a macro model of sovereign default. In equilibrium, investors face a state dependent trade-off between transaction costs and trading probabilities that generates a time-varying liquidity premium. With trading frictions demand and supply flows in the secondary market are important drivers of bond prices, while they are irrelevant and indeterminate in standard sovereign default models. I also use the model to study the effects of bond purchasing policies in secondary markets. I find that trading frictions significantly tighten the financial constraint of the government and that policy interventions that reduce the sell flows in the secondary markets can partially revert the effect of trading frictions.

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1 Introduction

Governments often issue long-term bonds to sell in the international credit market. Before sovereign bonds mature, they are traded in over-the-counter (OTC) secondary markets where transactions are decentralized, costly and time consuming.[1] Because investors value liquidity, trading frictions in the secondary market affect not only the price of outstanding bonds, but also the price of newly issued bonds and, hence, the government's decision on whether to default on its debt. In turn, the liquidity of sovereign bonds *endogenously* depends on the state of the economy in addition to trading frictions in the secondary market. In this paper, I study sovereign defaults incorporating the role of trading frictions in the secondary market, which has been largely ignored in the sovereign default literature originated in the seminal work of Eaton and Gersovitz (1981).[2]

Recent empirical work has emphasized the role of liquidity on sovereign bond markets. For example, Nguyen (2014) reports that during the 2010-2012 European debt crisis even countries with very liquid bonds faced illiquidity periods. She documents that the relative bid-ask spread - a standard measure of liquidity - of Italian bonds reached 667 basis points,[3] an unprecedented level for Italian bonds which bid-ask spreads are usually below 50 basis points.[4] Large bid-ask spreads were also observed in Ireland and Portugal. However, the most extreme example is Greece. Bloomberg data shows that bid-ask spreads for 10 year Greek bonds were about $2,000$ basis points, on average, in the fourth quarter of 2011, a couple of months before the debt restructuring of March, 2012. However, the standard Eaton and Gersovitz (1981) framework used in the sovereign debt literature cannot be used to understand these issues.

To endogenize the liquidity of bonds, I integrate search frictions in the secondary market into a general equilibrium model of sovereign debt with default risk. Because of search frictions, the flows of demand and supply for bonds are important determinants of bond prices in the secondary market, while quantities traded in the secondary market are usually indeterminate and irrelevant for prices in the sovereign debt literature. Using this model I can qualitatively and quantitatively address the first question of the paper. Namely, how do demand and supply flows in the secondary market for sovereign bonds affect the price

[1] See, for example, Duffie (2012) for details on OTC markets for bonds and World-Bank and IMF (2001) for details structure of sovereign debt markets.

[2] The only exception in the literature is Passadore and Xu (2022), who impose exogenous trading frictions on individuals selling sovereign bonds in the secondary market.

[3] See equation (12) for a formal defintion of the relative bid-ask spread.

[4] For more details on the importance of liquidity shock in the Eurozone debt crisis, see, for example, Nguyen (2014), and Pelizzon et al. (2016). For emerging market bonds, see Hund and Lesmond (2008).

schedule for newly issued bonds and how they interact with default risk?

In addition, because demand and supply flows in the secondary market have an effect on sovereign credit spreads, I can study the effects of policy interventions in the secondary market for sovereign bonds. An example of such intervention in the context of default risk is the Securities Market Programme (SMP) of the European Central Bank (ECB) implemented in 2010-2011 in the context of the European debt crisis. Under the SMP, the ECB directly purchased sovereign bonds in secondary markets to improve liquidity conditions and help stabilize distressed sovereign bond yields.[5] Between May and June, 2010, the ECB purchased around 10% of the outsdaing stock of Greek debt. Trebesch and Zettelmeyer (2018) find that, after the intervention, the yields of Greek bonds purchased by the ECB decreased significantly and persistently. The effects of such are interventions are puzzling from a standard sovereign default model's perspective. This is because bond purchases in secondary markets do not affect the usual determinants of credit spreads in sovereign default models, i.e. the debt-to-GDP ratio of the issuing country, bonds maturity, fiscal deficit, or aggregate production. How do then secondary market interventions reduce bond yields? When are such interventions effective? By endogenizing liquidity in sovereign bonds' market, I highlight the role of such policies in affecting demand and supply flows and contribute to understanding how secondary market interventions affect bonds' price, debt issuance and default.

To model endogenous liquidity, I incorporate frictions in bond markets in the same spirit as Shi (1995) and Trejos and Wright (1995), for fiat money, and Duffie et al. (2005), for corporate bonds. More specifically, in the model the sovereign government sells its debt to dealers in a centralized primary market.[6] Dealers trade bonds with foreign investors in a decentralized secondary market, acting as intermediaries between the government and foreign investors. In the model, dealers do not have reasons to hold bonds other than re-selling them to investors. For their intermediation service, dealers charge investors a transaction fee. On the other side of the market, investors demand sovereign bonds to maximize expected returns and optimize their portfolio composition. In order to be able to trade a bond, investors need to meet dealers in OTC markets, which are subject to search frictions. An investor's valuation for a bond incorporates the cost of intermediation fees, the expected time to trade,

[5] As stated by the ECB Press Release of May 10th, 2010, one of the goals of the SMP interventions was "to ensure depth and liquidity in those market segments which are dysfunctional." See the press release at https://www.ecb.europa.eu/press/pr/date/2010/html/pr100510.en.html.

[6] In reality, only a few large banks can trade in primary bond markets. All other investors, such as individual investors, institutional investors, and investment funds, need to buy bonds in OTC markets. In the model, dealers represent agents of those banks. For the case of Greece, the list of primary dealers can be found at https://www.bankofgreece.gr/Pages/en/Markets/HDAT/members.aspx.

and default risk.

In the secondary market, search is competitive (or directed). Every period, dealers and investors choose to visit one specific submarket to search for a trading counterpart. Each submarket is characterized by a transaction fee that the investor needs to pay to the dealer if they trade. A matching technology determines the number of trades in each submarket given the numbers of dealers and investors. In equilibrium, investors and dealers face a trade-off between the intermediation fee and the trading probability. For an investor, the higher the intermediation fee the investor pays, the higher the investor's probability of trading. For a dealer, the higher the intermediation fee the dealer charges, the lower the probability of trading. Investors' and dealers' entry decision into submarkets endogenously determine transaction fees, trading volumes, and trading frequencies.[7]

Qualitatively, the model generates two main new insights. First, trade flows between investors and dealers in the secondary market affect the price of newly issued government bonds in the primary market. For example, if the flow of demand orders from investors in the secondary market increases, the demand for bonds by dealers in the centralized primary market increases too. In this case, the bond price must increase to clear the centralized primary market and restore equilibrium. Even if the government does not change debt issuances. Second, default risk and illiquidity can in theory be positively or negatively correlated. In a simplified version of the model I show that the sign of this correlation crucially depends on the relative size of potential demand and supply for bonds in the secondary market. All else equal, the higher the default probability, the lower the incentives for investors to purchase bonds and the higher the incentives for bond holders to resale them. This results in less buyers and more sellers matching with dealers in the secondary market, which reduces the net demand for bonds in the primary market. To restore equilibrium prices must fall, and whether the fall is proportionally larger or smaller than the required compensation for the increase in default risk depends on how sensitive demand and supply flows are to changes in prices. When the potential world demand is larger than potential supply of bonds, as is the case for small open economies often studied in the literature, there is a positive correlation between default risk and illiquidity that amplifies the effect of shocks on bonds' interest rates.

To quantify the effects of liquidity frictions and secondary market policy interventions in

[7] Consistent with directed search, Li and Schurhoff (2019) document that, for the case of US municipal bonds, there is a systematic price dispersion in fees charged by different dealers and that those dealers charging higher fees provide more liquidity immediacy to investors.

an equilibrium model of sovereign debt, I calibrate the model to Greece.[8] I find that trading frictions are quantitatively important both for prices and quantities of sovereign bonds. The model produces an average spread 75 basis points larger than a standard model with frictionless secondary market under the same parameters. In addition, the debt to output ratio is 124% in the baseline model while it would be 139% in the model with frictionless secondary markets. That is, trading frictions significantly tighten the borrowing constraint of the government. In addition, I decompose Greek credit spreads during the debt crises and find that without trading frictions in the secondary market the interest rate paid by Greek bonds would have been about 140 basis points lower.

I also use the model to study the role of sale flows in the secondary market and the effects of policy interventions like the ECB's Securities Market Programme of 2010-2011. I find that the size of sale flows are also quantitatively important for bond prices and debt to GDP ratios. Reducing the volume of sell orders to a half reduces the average spreads by 40 basis points and increases the average debt to GDP ratio by 10 percentage points. If instead all investors buy bonds and hold it until maturity, the average spread falls from 3.42% to 2% and the average debt to output ratio increases from 124% to 137%. This result highlights the importance of the type of lenders that hold sovereign bonds. If sovereign bonds are held by investors that rarely adjust their portfolio then bond prices are larger than if bonds are held by investors who adjust their portfolio frequently. In this way, policy interventions like the SMP can increase sovereign bond prices by affecting the frequency at which bonds are traded in the secondary market. If the SMP commits to hold purchased bonds until maturity, as it did, the present and future sell flows are smaller and bond prices increase. Using the model, I find that the SMP reduced sovereign spreads by around 15-35 basis points. If the ECB had purchased twice as many bonds the spreads would have fallen by 32-72 basis points while if the ECB had bought the whole stock of Greek debt spreads would have been 139-313 basis points lower.

Relation to the literature. This paper contributes to the large body of research on sovereign debt with strategic default originated in Eaton and Gersovitz (1981), with a strong quantitative focus after the work by Neumeyer and Perri (2005), Aguiar and Gopinath (2007), and Arellano (2008). Because debt is long term, my work is closer to Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). I contribute to this literature by endogenizing liquidity in the secondary market. The framework provides a tool to understand

[8]Greece is a good case study because of data availability as it is one of the few countries with a default episode since sovereign bonds trade in OTC markets.

how liquidity and default risk interact in equilibrium. In addition, I use information on bid-ask spreads and volumes traded in the secondary market to quantify the importance of liquidity frictions for sovereign bonds' credit spreads, government revenues from new debt issuance, and incentives to default. Most papers in the literature on sovereign default have abstracted from the secondary market completely. Exceptions are Broner et al. (2010) and Bai and Zhang (2012), who consider frictionless secondary markets. Chaumont et al. (2024) considers a short-term debt model where OTC markets for sovereign bonds and credit default swaps interact in equilibrium but there is no role for sale flows as bonds mature in one period. Moretti (2020) uses search frictions in a sovereign debt model to quantify the role of frictions upon introducing GDP-linked bonds as a new asset.

The most closely related paper is Passadore and Xu (2022) who incorporates an exogenous trading frictions in the secondary market into a standard model of sovereign default. They assume that investors that aim to re-sell bonds in the secondary market face an exogenously fixed probability of trading lower than one. Although both papers feature some source of trading friction their work has a different focus. They study how this infrequent selling opportunity interacts with default risk how default risk changes expected maturity of bonds and outside options of sellers. Because trading frictions are exogenously imposed, their model is unable to capture how demand and supply flows in the secondary market endogenously responds to changes in the state of the economy. To capture endogenous liquidity, I assume that investors buy and sell bonds in the secondary market and use directed search to endogenize the trading probabilities in this market. Endogenous liquidity is important to assess the effects of policy interventions in the sovereign bond market, such as the ECB's securities market programme (SMP) of 2010-2011. In my model, interventions of this kind directly affect the price, the bid-ask spreads, and the liquidity premium of newly issued bonds, by changing the net demand for bonds in the secondary market. In contrast, if trading probabilities are exogenous, such interventions may not affect the price of newly issued bonds nor the terms of trade in bilateral meetings, because the interventions do not affect the outstanding level of debt or default probabilities.

This paper is also related to the literature following Duffie et al. (2005) where investors with heterogeneous valuations trade assets in OTC markets and liquidity is modeled using search frictions. To execute mutually beneficial transactions, investors who have relatively high and low valuations for an asset search for each other in frictional markets. In my model, the trading structure in the secondary market is closer to Lagos and Rocheteau (2009) and Lester et al. (2015), where (i) trades occur only between a dealer and an investor but not directly between investors; and, (ii) dealers do not need to hold inventories as they have permanent access to a centralized primary market that serves as a clearing system. The main contribution of my paper to this literature is to build a tractable equilibrium model with endogenous liquidity over the business cycle and strategic default decisions, while this literature usually focuses on steady states and no defaults.[9] Another contribution is to endogenize the supply of assets. Supply's response to market conditions is critical for understanding endogenous liquidity, but it is absent in this literature.

Previous studies also consider endogenous default decisions on corporate bonds. One example is He and Milbradt (2014). However, the model assumes a stationary environment where both the characteristics and the supply of assets are fixed over time. Chen et al. (2018) extend the analysis to allow the aggregate state to take two values and show how exogenous reductions in trading probabilities increase credit spreads. As a result, their model misses two critical features of my model: an endogenous supply of bonds and endogenous trading probabilities in the secondary market. As explained above, both features are necessary to understand how the liquidity of sovereign bonds responds to changes in the state of the economy and affects the price of newly issued bonds.

Finally, this paper complements a growing body of empirical work that investigates how liquidity explains sovereign credit spreads during the European debt crisis and the effects of ECB's policy interventions in the secondary market. See, for example, Beber et al. (2009), Calice et al. (2013), Nguyen (2014), Pelizzon et al. (2016), Eser and Schwaab (2016), Trebesch and Zettelmeyer (2018), and De-Pooter et al. (2018).

Layout. The remainder of the paper is organized as follows. Section 2 describes the model economy and defines an equilibrium. Section 3 illustrates the new channels in the model. Section 4 calibrates the model and provides quantitative results. Section 5 studies the Greek debt crisis. Section 6 concludes.

2 Model

Time is discrete and infinite. There are three types of agents: (i) a sovereign government; (ii) dealers; and (iii) investors. The country of interest faces a random endowment process y_t, distributed according to $F(y_{t+1}|y_t)$. Its sovereign government maximizes the lifetime

[9] One exception is De-Pooter et al. (2018) who incorporate an exogenous default probability into a Duffie et al. (2005) model and studies the differences across steady states with different default probabilities and an exogenously determined supply of assets.

utility of the domestic representative household, given by

$$\sum_{t=0}^{\infty} \beta^t U\left(c_t\right),$$

where $U(\cdot)$ is strictly increasing and concave, c_t is household's consumption, and $\beta \in (0,1)$ is the discount factor. The government can save or borrow from international credit markets, described later in this section, and cannot commit to repay its debt obligations.

At the beginning of each period the stock of debt is B_t. The government chooses whether to default or not on its debt obligations. There are two costs of defaults. First, the government is temporarily excluded from financial markets and cannot borrow or save under financial autarky. While the government is in default, every period it can re-gain access to financial markets with exogenous probability $\phi \in (0,1)$.[10] Upon re-gaining access to credit markets, the government starts with no outstanding debt. Second, there is an output cost. Under default the endowment is given by $h(y_t) \leq y_t$, for all y_t.

If the country repays its debt and is in good credit standings, the government chooses next period's debt, B_{t+1}. Newly issued bonds are sold to dealers in a Walrasian primary market.[11] The sovereign government takes as given the price schedule of bonds $q(y_t, B_t, B_{t+1})$, which is determined in equilibrium. The arguments of this pricing schedule are the current level of endowment, which allows investors to forecast next period's endowment, the current level of debt, B_t, and next period's debt, B_{t+1}. The price of bonds depend on the outstanding stock of debt because it provides information on the *flows* of bonds traded in the secondary market and the liquidity premium, as I it describe in section 2.4.

The maturity of bonds is determined by a parameter λ. Every period, each unit of debt matures with probability $\lambda \in [0,1]$, independently of when that unit of debt was issued. Thus, the average time to maturity of each bond is $\frac{1}{\lambda}$ periods. Each unit of unmatured debt pays a coupon $z \geq 0$ every period. Therefore, the period t budget constraint of a sovereign government that is in good credit standings is given by

$$c_t + \left[\lambda + (1-\lambda) z\right] B_t \leq y_t + q\left(y_t, B_t, B_{t+1}\right)\left[B_{t+1} - (1-\lambda) B_t\right].$$

The left hand side adds expenditures on consumption, coupon payments, and repayment of matured bonds. The right hand side adds incomes from endowment and newly issued debt.

[10]For models with endogenous market re-access see Yue (2010) and Benjamin and Wright (2013).

[11]The government could organize an auction to sell the bonds. Since in this model there is complete and perfect information about valuations of the bonds, the auction could be designed to extract all the surplus from dealers. That is, dealers would be acting as if there is perfect competition among them.

Dealers are risk-neutral. They can access the primary market without any cost and purchase bonds issued by the government at the competitive price, $q(y_t, B_t, B_{t+1})$. There are no frictions in the primary market. Also, I assume that dealers have permanent access to the primary market so that they only purchase bonds when they want to re-sell them to an investor.[12] In other words, the primary market in the model merges the market for newly issued bonds together with an interdealer market where dealers can trade with each other. In addition, dealers have access to a frictional secondary market where they can trade with investors. The secondary market is characterized by directed search. Specifically, there is a continuum of submarkets that are labeled by the transaction fee that dealers charge to investors if they trade. Entry into submarkets is competitive. To enter any submarket, a dealer needs to pay a per-period flow cost, γ.[13] A dealer compensates the entry cost by the bid-ask spread between submarkets. As the intermediaries between the primary and the secondary market, dealers collect the orders from the secondary market and clear the net demand (or supply) in the primary market at the end of each period.

There is a a fixed measure $\bar{I} < \infty$ of foreign investors in the secondary market. They can trade bonds only by meeting a dealer. In order to trade, investors choose a submarket to enter. That is, investor's search for dealers is also directed. For simplicity, I assume that investors can hold either zero or one unit of the bond. I denote an investor's bond holdings $a \in \{0, 1\}$.[14] Thus, the mass $\bar{I}$ represents the maximum available wealth that could be allocated to purchase sovereign bonds. To capture that the sovereign government is from a small open economy, I assume that $\bar{I}$ is large and the amount of debt issued by the government is not constrained by the total wealth of investors.

To parsimoniously generate incentives to trade sovereign bonds in the secondary market, I assume that there are two types of investors, denoted ℓ and h. Type $i \in \{\ell, h\}$ investors have preferences u_i over the bond, with $u_h > u_\ell$, in addition to other consumption c.[15] To

[12] This simplification avoids working with dealers that hold bond inventories and the need to solve additional dynamic optimization problems. Notice that this assumption replaces a dynamic cost for a static cost. The model does not consider the dynamic cost of maintaining inventories and bearing the associated default risk. However, this cost is replaced by the cost of creating enough transactions to re-allocate to investors all bonds purchased by dealers, within each period.

[13] This cost can be interpreted as a constant marginal cost of allocating a dealer into a submarket, for a bank that participates in the primary market.

[14] Although at the investor's level the demand/supply of bonds in the secondary market is a discrete choice, at the aggregate level the investor's net demand for bonds is continuous on the bond's price and default risk. This is because in equilibrium the trading probabilities are continuous in those variables (see lemma 3 in appendix A). Since the model has aggregate shocks and rational expectations, it would be unfeasible to solve the model with divisible bond holdings because it requires keeping track of the distribution of bond holdings across all investors.

[15] As described by Duffie et al. (2005), these simple differences in preferences for holding bonds are a

fix ideas, one can think of type ℓ investors as liquidity constrained individuals that have a lower than average valuation for holding the bonds. Thus, type ℓ investors are the natural sellers of bonds.

Investors enter the economy as type h and without bonds. In equilibrium, type h investors are the natural buyers of sovereign bonds. Once a type h investor acquires a bond, every period the investor is exposed to a transitory preference shock that arrives with probability $\zeta \in (0,1)$, which is *i.i.d.* across periods and investors. The preference shock changes the investor type to ℓ during the current period. In the equilibrium, type ℓ investors wish to sell the bonds in the secondary market. For simplicity, I assume that once an investor sells the bond, the investor leaves the economy and is replaced by a new type h investor without bonds. An investor can get rid the bond in two ways: (i) by selling it to a dealer in the secondary market, or (ii) if the bond matures, which occurs every period with probability λ. If the investor does not get rid of the bond, next period it will be a type h investor, with probability $1-\zeta$, or a type ℓ investor, with probability ζ. Finally, investors have access to a risk-free, perfectly liquid, one period zero-coupon bond that pays an exogenous return $r > 0$.

In each submarket in the secondary market, there is a constant returns to scale order processing (or matching) technology denoted $\mathcal{M}(d,n)$, where d is the number of dealers and n is the number of investors. Each investor's order is equally likely to be executed at any time. The probability of an order being executed is given by $\alpha(\theta) \equiv \frac{\mathcal{M}(d,n)}{n} = \mathcal{M}(\theta,1)$, with $\theta \equiv \frac{d}{n}$. The number of orders executed by a dealer in a period of time is then $\rho(\theta) \equiv \frac{\mathcal{M}(d,n)}{d}$. I assume that $\mathcal{M}(\cdot,\cdot)$ is the same across submarkets and satisfies that $\alpha(0) = 0$, $\alpha(\infty) = 1$, $\rho(\infty) = 0$, and $\alpha(\cdot)$ is strictly increasing and concave.

The timing of actions within each period is as follows:

1. Endowment y_t is observed. The government decides whether or not to default. If the government defaults the bond is not available as a possible investment choice for investors, and investor's and dealer's problems are irrelevant.

2. If the government repays, it chooses next period debt, B_{t+1}, optimally.

3. Investors' preference shock is realized and a fraction ζ of bond holders become type ℓ investors.

4. A fraction λ of B_t matures. Their owners are replaced by h investors without bonds. Principal of matured bonds is paid to current bond owners. Unmatured bonds pay coupon z and yield utility u_i to investors of type $i \in \{\ell, h\}$.

reduced form to capture that investors face different liquidity needs, financing costs, hedging reasons, tax advantages, and/or personal use of the asset.

5. The centralized primary market and the decentralized secondary markets open. In the centralized primary market, the government and dealers trade at a competitive price. Investors and dealers decide optimally which submarket to visit and, those who meet a counterpart, trade in the secondary market.
6. The government and the dealers derives utility from consumption while the investors derive utility from consumption and from their bond position according to their types.

At the beginning of each period, once the endowment shock is realized, the aggregate state of the economy is $x_t \equiv (y_t, B_t)$. By the time investors make their submarket choices in step 5, government's debt choice $B_{t+1}(x_t)$ is already known. Thus, if the government is under good credit standings, the relevant state of the economy for investors is $s_t = (y_t, B_t, B_{t+1})$.

In what follows I formulate the government's, investors', and dealers' problems in recursive form and eliminate time period subindices.

2.1 Government

At the beginning of each period, the government chooses whether to default, $\delta = 1$, or repay, $\delta = 0$, and the optimal debt issuance in case of repayment. For each state $x = (y, B)$, its value function is:

$$V(x) = \max_{\delta \in \{0,1\}} \left\{ (1-\delta) V^R(x) + \delta V^D(y) \right\}, \tag{1}$$

where $V^R(\cdot)$ is the value of repaying debt obligations and $V^D(\cdot)$ is the value of default. The value of defaulting is

$$V^D(y) = U(h(y)) + \beta \mathbb{E}_{y'|y} \left[\phi V\left(y', 0\right) + (1-\phi) V^D\left(y'\right) \right]. \tag{2}$$

That is, if the government decides to default, it does not repay its outstanding debt and consumes the total output of the current period. However, there is an output loss associated to the default decision and today's consumption is given by the function $h(y)$. In addition, the continuation value is a weighted average of the value of re-gaining credit access, which occurs with probability ϕ, and starting next period in default, with probability $(1-\phi)$. If the government re-gains access to credit, it starts with zero outstanding debt, with all investors being type h and holding no bonds. In the case of debt repayment, the government chooses consumption of the domestic household, c, and the new stock of debt, B'. The value of repaying is

$$V^R(x) \quad = \quad \max_{c, B'} \left\{ U(c) + \beta \mathbb{E}_{y'|y} V(x') \right\}, \; s.t.:$$

$$[BC_G] \quad : \quad c + [\lambda + (1-\lambda)\, z]\, B = y + q(x, B') \left[B' - (1-\lambda)\, B\right].$$

The price schedule $q(y, B, B')$ is determined endogenously in the primary debt market and depends on the amount of newly issued government bonds. The government internalizes the effect of changes in the stock of debt on bonds' price, but it takes as given the pricing function. As described in more details in section 2.4, the price schedule has the current stock of debt as a state variable. This is because, in equilibrium, ζB investors are sellers of the bond in the secondary market and reduce the net demand for newly issued bonds. The total amount of bonds sold in the secondary market is determined by the mass of investors that exogenously change types from h to ℓ as well as by the *endogenous* mass of investors that are able to trade bonds in the decentralized market. The government internalizes that by changing the supply of bonds it will affect investors' policy functions and the evolution of trade flows in the secondary markets. Using the budget constraint to substitute for c, the objective function becomes:

$$V^R(x) \quad = \quad \max_{B'} \{U\left(y + q(x, B')\left[B' - (1-\lambda)\, B\right] - [\lambda + (1-\lambda)\, z]\, B\right) + \beta \mathbb{E}_{y'|y} V(x')\}. \quad (3)$$

As it shall be described later, the price of newly issued bonds, q, is determined by the government supply of bonds and the dealers' net demand for bonds in the primary market. Liquidity frictions in the secondary market and the state of economy affect the dealers' net demand for bonds and, thus, q. Therefore, liquidity frictions affect the price of newly issued bonds, the optimal debt issuance, B', the value of repayment, V^R, and eventually the optimal choice of repayment versus default, δ.

2.2 Investors

Investors trade bonds in the secondary market. I denote the set of submarkets as $\bar{F} = [f_{\min}, f_{\max}]$. I let the lower bound be $f_{\min} \equiv \gamma$, since no dealer will be willing to enter a submarket that does not pay enough to cover the entry cost. In addition, the price for a bond that matures with probability λ, with a flow return $(z + u_h)$ every period until maturity, and that never defaults would be $\frac{\lambda + (1-\lambda)(z+u_h)}{\lambda + r}$. Thus, we can define the upper bound $f_{\max} \equiv \frac{\lambda + (1-\lambda)(z+u_h)}{\lambda + r}$ since no investor would be willing to pay a higher intermediation fee, even if the trading probability is equal to one and $q = 0$.

Denote the value functions of an investor holding a units of the bond at the beginning of the period as I^a, with $a \in \{0, 1\}$. Denote J_i^1 the value function for an investor of type $i \in \ell, h$ with bond holdings $a = 1$ after the realization of the preference shock determines the type of the investor in the current period.

2.2.1 Investors without a bond

For each state $s=(y,B,B')$, the value for an investor with $a=0$ is given by

$$\begin{aligned} I^0(s) \quad = \quad & \max_{f\in\bar{F}}\{\alpha\left(\theta(f)\right)\left[-q(s)-f+u_h+\frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta(x')\right]I^1(s')\right] \\ & +\frac{1-\alpha\left(\theta\left(f\right)\right)}{1+r}\mathbb{E}_{y'|y}\left[1-\delta(x')\right]I^0(s')\}. \end{aligned} \tag{4}$$

The investor chooses optimally which submarket to visit (how much transaction fee, f, to pay) in order to purchase a unit of the sovereign bond. In submarket f, the investor will be able to trade with a dealer with probability $\alpha\left(\theta(f)\right)$. Once matched, the investor purchases a unit of the bond after paying its price in the primary market, $q(s)$, plus the transaction fee, f. In addition, holding a bond derives a continuation value of $I^1(s')$ in the next period, provided that the government does not default on the bond when next period starts. The continuation value is discounted at rate r, which is the rate of return on the perfectly liquid, risk-free bond. If the investor is not matched with a dealer, which happens with probability $1-\alpha\left(\theta(f)\right)$, the investor receives the discounted continuation value of not holding a bond at the beginning of the next period, conditional on the government not defaulting. If the government defaults the continuation values are zero.

Notice that the value for an investor is computed for all B', including those $B'\neq B'(x)$ that are off the equilibrium path because they are not optimal choices for the government. This is because the government internalizes how the amount of newly issued debt affects investors incentives to buy and sell bonds.

2.2.2 Investors with a bond

The value of holding a bond at the beginning of the period is,

$$I^1(s) \quad = \quad \lambda+(1-\lambda)\left[z+\zeta J_\ell+(1-\zeta)J_h\right]. \tag{5}$$

The investor obtains the face value, 1, if the bond matures, which occurs with probability λ. If the bond does not mature, the bond pays the coupon z and, depending on their individual realization of the preference shock, they obtain the value $J_i, i\in\{\ell,h\}$, with

$$J_\ell=\max_{f\in\bar{F}}\left\{\alpha\left(\theta(f)\right)\left[q(s)-f\right]+\left[1-\alpha\left(\theta(f)\right)\right]\left[u_\ell+\frac{\mathbb{E}_{y'|y}[1-\delta(x')]I^1(s')}{1+r}\right]\right\} \tag{6}$$

$$J_h = \max_{f \in \bar{F}} \left\{ \alpha\left(\theta(f)\right)\left[q(s) - f\right] + \left[1 - \alpha\left(\theta(f)\right)\right]\left[u_h + \frac{\mathbb{E}_{y'|y}[1-\delta(x')]I^1(s')}{1+r}\right]\right\} \tag{7}$$

The value of holding a bond for a type ℓ investors is the maximized value of participating into the secondary market for sovereign bonds. Thus, investors choose optimally a submarket, f, to sell their bond and trade with associated probability $\alpha\left(\theta(f)\right)$. If they match with a dealer, they sell their bond at the interdealer price $q(s)$ minus the intermediation fee f. If they do not match with a dealer, type ℓ investors receive a flow value $u_\ell < 0$ for holding the bond and the expected discounted continuation value of holding a bond in the next period, $I^1(s')$, conditional on the government deciding to repay its debt obligations.

Similarly, the value of holding a bond for a type h investor is derived from the optimized value of participating in the secondary market for sovereign bonds. In equilibrium, it is usually the case that type h investors do not wish to re-sell their bonds and choose and f consistent with $\alpha\left(\theta(f)\right) = 0$. They obtain the flow value $u_h > 0$ for holding a bond plus the expected continuation value of being a bond holder, $I^1(s')$, conditional on the government not defaulting. However, if the price of sovereign bonds, q, in the interdealer market is high enough, even type h investors may decide to pay a fee $f > \gamma$ and have a positive probability of selling their bond. For a type h investor with $a = 1$ to chose to enter a submarket $f > \gamma$, the price $q(s)$ should be relatively large, as these investors like holding the bond. In addition, since $I^0(s') \geq 0$ and the type h investor's revenue of buying a bond when holding $a = 0$ is lower than the value of keeping a bond for investors with $a = 1$, i.e.,

$$\frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta(x')\right]\left[I^1(s') - I^0(s')\right] < u_h + \frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta(x')\right]I^1(s'),$$

then, whenever type h investors wish to sell their bond, no investor is willing to purchase a bond. Thus, only investors trying to sell will participate in the secondary market, which can only occur in the case that the sovereign government decides to retire a large amount of bonds from the market. That is, bond buybacks are expensive for the government when it needs to induce type h investors to sell.

2.3 Dealers

Dealers participate competitively in debt markets. Each dealer chooses a transaction fee to charge investors for the intermediation service. To enter any given submarket a dealer needs to pay a flow cost $\gamma > 0$. A dealer posts the intermediation fee to maximize expected profits:

$$\Pi = \max_{f \in F} \{\rho(\theta(f)) f - \gamma\}, \tag{8}$$

where $\rho(\cdot)$ represents the probability of being able to execute an order, derived from the matching technology $\mathcal{M}$ described earlier. Competitive entry of dealers implies that

$$\Pi(f) \leq 0 \text{ and } \theta(f) \geq 0, \tag{9}$$

holds with complementary slackness. Whenever expected profits for dealers are negative in a submarket f, the associated market tightness $\theta(f)$ is zero, since no dealers have incentives to enter. On the other hand, whenever the market tightness is positive, a positive mass of dealers enter the submarket until expected profits are zero.

Condition (9) provides a mapping from each submarket intermediation fee, f, to the tightness in that submarket. This mapping is given by

$$\theta(f) = \begin{cases} \rho^{-1}\left(\frac{\gamma}{f}\right) & \text{if } \Pi(f) = 0, \\ 0 & \text{otherwise} \end{cases} . \tag{10}$$

In turn, market tightness determines trading probabilities for investors as a function of intermediation fees, $\alpha(\theta(f))$.

2.4 Market clearing

The primary market is Walrasian and only government and dealers can access it. In each state s, and conditional on the government being in good credit standings, the price $q(s)$ must clear the bonds primary market.

Recall that $\bar{I}$ is the total mass of investors. Each period, after the realization of idiosyncratic preference shocks, let $H_0 = \bar{I} - B$ be the mass of type h investors with $a = 0$, $H_1 = (1-\zeta)B$ the mass of type h investors with $a = 1$, and $L_1 = \zeta B$ the mass of type ℓ investors with $a = 1$. Then, in any given period, $\bar{I} = H_0 + H_1 + L_1$. In addition, since outstanding bonds are held by investors, $B = H_1 + L_1$.[16] Using this notation, the total supply of bonds in the primary market is given by the newly issued debt by the government plus

[16]Notice that knowing the outstanding stock of debt, B, is enough to know the distribution of investors over their types and bond holdings, which significantly reduces the dimensionality of the state spaces and simplifies the computation of the model.

the selling orders received by dealers from investors in the secondary market. That is,

$$\underbrace{\max\left\{B',0\right\}-(1-\lambda)\,B}_{\text{Government's supply}}+\underbrace{\alpha\left(\theta_\ell^1\right)(1-\lambda)\,L_1}_{\text{Sellers' supply}}+\underbrace{\alpha\left(\theta_h^1\right)(1-\lambda)\,H_1}_{\text{Potential type } h \text{ sellers}}.$$

The first term is the government's new bond issuances. The operator $\max\{B',0\}$ captures the possibility that the government chooses $B'<0$, in which case the government can at most demand the $(1-\lambda)\,B$ outstanding bonds in the primary market before turning its net savings positive. The second term is the supply of bonds from "sellers", i.e., bond holders who are hit by the preference shock to become type ℓ investors. They wish to sell their bond holdings. Since a fraction λ of them will see their bond mature, only a fraction $1-\lambda$ of them will attempt to sell the bond to a dealer. Among the sellers, only a fraction $\alpha\left(\theta_\ell^1\right)$ will get matched with a dealer, where θ_ℓ^1 is the tightness in the submarket optimally chosen by type ℓ investors. Finally, the third term is the potential supply of bonds by type h investors with $a=1$, which is positive only if the government is buying back bonds at a sufficiently high price, as discussed in section 2.2.2.

On the other side of the primary market, the demand for bonds, are the buying orders received by dealers from the fraction of type h investors, $\alpha\left(\theta_h^0\right)$:

$$\underbrace{\alpha\left(\theta_h^0\right)H_0}_{\text{Old Buyers' demand}}+\underbrace{\alpha\left(\theta_h^0\right)\lambda B}_{\text{New Buyers' demand}}.$$

The first term represents "old buyers," i.e. type h investors with $a=0$ who are in the market from the last period. The second term, represents the "new buyers," i.e. those type h investors who entered the economy in the current period to replace the investors that left the economy after their unit of the bond matured. The tightness θ_h^0 corresponds to the submarket optimally chosen by type h investors with $a=0$.

Combining the demand and the supply for bonds in the primary market, I define the excess demand function for each state $s=(y,B,B')$ as

$$\begin{aligned} ED(s) \quad \equiv \quad & \underbrace{\alpha\left(\theta_h^0(s)\right)\left[\bar{I}-(1-\lambda)\,B\right]}_{\text{Buyers' demand}}-\underbrace{\left[\max\left\{B',0\right\}-(1-\lambda)\,B\right]}_{\text{Government's supply}} \\ & -\underbrace{\alpha\left(\theta_\ell^1(s)\right)(1-\lambda)\,\zeta B}_{\text{Sellers' supply}}-\underbrace{\alpha\left(\theta_h^1(s)\right)(1-\lambda)\,(1-\zeta)\,B}_{\text{Potential type } h \text{ sellers}}. \end{aligned}$$

Thus, the price schedule in this economy is implicitly defined by the following conditions

$$ED(s;q(s)) \leq 0 \text{ and } q(s) \geq 0, \tag{11}$$

which hold with complementary slackness. As I show in appendix A.1, this excess demand function is consistent with only one price $q(s)$ clearing the market, for each state s. In the appendix, Lemma 3 states that, given a government's default policy function and a choice for next period's debt, the dealers' net demand for bonds in the primary market is decreasing in the price of the bond, and strictly decreasing in most of the cases. The result follows because: (i) the fraction of type h investors without bonds that purchase one from a dealer, $\alpha(\theta_h^0)$, is decreasing in the price of the bond, and (ii) the fraction of investors holding a bond that sell it to a dealer, $\alpha(\theta_\ell^1)$, is increasing in the price of the bond. $\alpha(\theta_h^0)$ is decreasing in q because the dealer charges the investor $q + f_h^0(s)$, and so the investor pays more for the same expected return. Therefore, the investor responds by optimally reducing the intermediation fee $f_h^0(s)$. As a result, dealers earn lower expected profits and there is less entry into the secondary market, which reduces the matching probability of investors trying to purchase bonds. As each investor's trading probability decreases, a smaller mass of them trade. A similar argument explains why the $\alpha(\theta_\ell^1)$ increases with q.

Appendix A.2 shows that there is only one market clearing price $q(s)$ for each s. To be more precise, this uniqueness statement is conditional on a given government's default and bond supply policies, investors value functions, and future expected prices. As it is standard in sovereign default models of long term debt, I cannot guaranteed that the solution of the price schedule is unique in general equilibrium. However, the result highlights the parallelism of the pricing schedule to the standard no-arbitrage condition that maps future prices and a default policy function into current prices. In this sense, solving this model is not harder than other models of sovereign default. Instead of having a closed form expression for the price as in the standard no arbitrage condition, I need to find the price consistent with (11).

2.5 Equilibrium

The equilibrium concept used here is recursive competitive equilibrium.

Definition 1. *A Recursive Competitive Equilibrium (RCE) in this economy consists of a set of value functions $\left\{V, V^R, V^D, I^0, I^1, J_\ell, J_h, \Pi\right\}$, a set of policy functions $\left\{\delta, B', f_h^0, f_h^1, f_\ell^1\right\}$, a tightness function θ, and a pricing function q, such that for all $x = (y, B)$: (i) given functions $q(x, B')$, $f_\ell^1(x, B')$, $\theta(x, B')$, the functions $V(x)$, $V^R(x)$, $V^D(y)$, $\delta(x)$, $B'(x)$, solve the sovereign government's problem in (1)-(3); (ii) for all $s = (x, B')$, given $q(s)$,*

$\delta(x)$, $B'(x)$, $\theta(s)$, the functions $I^0(s)$, $I^1(s)$, $J_\ell(s)$, $J_h(s)$, $f_h^0(s)$, $f_h^1(s)$, $f_\ell^1(s)$ solve the investor's problem in (4), (5), (6) and (7); (iii) the tightness function $\theta(x, B')$ is consistent with free entry of dealers according to (10); and (iv) the function $q(x, B')$ clears the primary market for bonds.

3 Main mechanisms and the role of liquidity

Before turning to a quantitative evaluation of the role of trading frictions on sovereign bond markets, I discuss the most novel qualitative features of the model. First, I provide a simplified example where I can show in closed form how trading frictions affect the bond price schedule and compare it to standard no arbitrage pricing. Second, I provide a two period example that shows how trading frictions can contain or amplify the fall in bond prices when default risk increases.

3.1 The role of trading frictions

To build intuition, I consider a particular case in which I can write a closed form expression for the pricing schedule implicitly defined in (11). In particular, I assume that the order processing technology (matching function) is the "Telephone Line" function, given by

$$\mathcal{M}(n, d) = \frac{n \times d}{n + d}$$

In addition, I assume that high type investors never become low type, $\zeta = 0$. Under these assumptions, the masses of investors buying, keeping and selling their bond are, respectively,

$$H_0 \equiv \left[\overline{I} - (1-\lambda)\, B\right],\ H_1 \equiv (1-\lambda)\, B,\ L_1 \equiv 0.$$

For this example I define $\Delta B \equiv B' - (1-\lambda)B$, the amount of newly issued bonds. After solving for q in the market clearing condition, I can write the price in the primary market as

$$q(s) = \underbrace{u_h + \frac{1}{1+r}\mathbb{E}_{y'|y}\left\{\left[I^1(s') - I^0(s')\right]\right.}_{\text{Value of holding bond}}\underbrace{\left.\left[1 - \delta(y', B')\right]\right\}}_{\text{Default Risk}} - \underbrace{\gamma\left[\frac{1}{1-\frac{\Delta B}{H_0}}\right]^2}_{\text{Liquidity Component}}.$$

Some remarks are in order. The price is divided into three component: (i) investor's expected discounted value of acquiring a bond, (ii) an adjustment for default risk, and (iii)

a liquidity component.[17] The first term in the right hand side corresponds to components (i) and (ii) and is very similar to the standard no arbitrage condition of sovereign default models. The only difference is that $q(s')$ is replaced by the value of becoming bond holder, $\left[I^1(s') - I^0(s')\right]$, and that having possession of the bond gives investors utility u_h. The second term in the right hand side is the liquidity component, which contains the following ingredients. First, γ represents the importance of intermediation frictions. The more dealers have to pay to participate in the secondary market (higher γ), the larger is the price discount from the liquidity component. Second, the ratio $\Delta B/H_0$ (≤ 1), represents the size of new issuances relative to potential investors' demand for bonds. The larger is the amount of newly issued debt the larger is the price discount from the liquidity component. This is because the larger is the debt issuance, the more investors need to be matched with dealers in the secondary market, which is only possible if more dealers enter. Entry is larger only if investors visit a submarket where they pay a larger intermediation fee. Therefore, since the total amount paid by investors is $q+f$, to induce investors to pay a higher intermediation fee and attract more dealers, the price in the primary market has to fall. This term highlights how the flows of bonds traded affect the bond's price in the primary market. Finally, the last ingredient of the liquidity component is the maturity probability λ. Because of the small open economy assumption, $\bar{I} > B'$, it is always true that the longer the maturity of the bond (smaller λ), the lower is the price discount due to the liquidity component. Mechanically, this is because a smaller λ implies a smaller $\Delta B/H_0$ ratio. The reason is that, in order to achieve certain new stock of debt B', a smaller flow of debt issuance is needed when a smaller fraction of bonds mature every period.

3.2 The amplification mechanism

Section 3.1 provides some intuition on how trading frictions affect the level of bond's price. To do so, the example eliminates type ℓ investors' bond re-sales in the secondary market, which compete for buyers with government's newly issued bonds. In the data re-sales usually represent the largest fraction of bonds supplied. I re-incorporate investors' sales in the secondary market to turn to the question of whether the bond price's response is more or less than proportional to changes in default risk. To simplify the exposition, I use a simplified two-period version of the model. Although simple, this example misses part of the dynamic effects of changes in default risk on investor's utility.

[17]Notice that some part of the effects of liquidity are hidden inside the term $\left[I^1(s') - I^0(s')\right]$ which takes into account future liquidity conditions and their effects on the value for holding the bond. The purpose of this example is to build some intuition.

Assume that there are only two periods, $t = \{1, 2\}$. The government has an initial outstanding debt level $B_1 > 0$. $L = \zeta B_1$ of the bonds are held by type ℓ investors while $B_1 - L$ are held by type h investors. In addition, there is a mass H of type h investors that do not hold a bond and could potentially buy a unit. At $t = 2$, if there is no default, low type investors receive a value $U_\ell < 1$ per unit of bond held and high type investors receive a value $U_h > 1$ per unit of bond in their portfolio.

At $t = 1$, the government issues new debt to be paid at the final period, $\Delta B \equiv B_2 - B_1$, where B_2 is the stock of debt at the beginning of $t = 2$. All bonds mature in period 2 and the government defaults with probability $\bar{\delta}$.

Investors decide whether to buy or sell a unit of a bond in the secondary market, and pick a submarket to do so. Type h investors holding no bonds (buyers) maximize

$$I^0(B_1, B_2) = \max_{\theta_h} \alpha(\theta_h) \left\{ -q + \frac{(1-\bar{\delta})U_h}{1+r} \right\} - \gamma \theta_h,$$

where I use the conditions $\rho(\theta) f = \gamma$ and $\frac{\alpha(\theta)}{\rho(\theta)} = \theta$ to solve for θ_h instead of f_h. Similarly, type ℓ investors holding a bond (sellers) maximize

$$J_\ell(B_1, B_2) = \max_{\theta_\ell} \alpha(\theta_\ell) \left\{ q - \frac{(1-\bar{\delta})U_\ell}{1+r} \right\} - \gamma \theta_\ell.$$

At an interior optimum[18], we have that

$$[\theta_h] : \gamma = \alpha'(\theta_h) \left\{ -q + \frac{(1-\bar{\delta})U_h}{1+r} \right\} \implies \theta_h = \alpha'^{-1} \left(\frac{\gamma}{-q + \frac{(1-\bar{\delta})U_h}{1+r}} \right), \text{ and}$$

$$[\theta_\ell] : \gamma = \alpha'(\theta_\ell) \left\{ q - \frac{(1-\bar{\delta})U_\ell}{1+r} \right\} \implies \theta_\ell = \alpha'^{-1} \left(\frac{\gamma}{q - \frac{(1-\bar{\delta})U_\ell}{1+r}} \right).$$

The market clearing condition at $t = 1$ is given by

$$\Delta B = \alpha(\theta_h) H - \alpha(\theta_\ell) L.$$

Assuming the same functional form of the matching function as in section 3.1, and using

[18] Whenever investors have strictly positive gains from trade there is an interior optimum in this example. If gains from trade are weakly negative the optimal tightness is zero and investors do not participate of the secondary market. In this example I focus on interior optima and assume that gains from trade are strictly positive for both types of investors.

investors' optimality conditions, the price of the bond solves

$$\Delta B = H\left\{1-\left[\frac{\gamma}{(-q+\frac{(1-\bar{\delta})U_h}{1+r})}\right]^{1/2}\right\} - L\left\{1-\left[\frac{\gamma}{(q-\frac{(1-\bar{\delta})U_\ell}{1+r})}\right]^{1/2}\right\}$$

I define $M \equiv H - L - \Delta B$, which I assume to be positive, as it will be the case in the calibrated model in section 4.[19] Using this definition and the optimality conditions, I express the market clearing condition as

$$\frac{H}{(-\frac{q}{1-\delta}+\frac{U_h}{1+r})^{1/2}} - \frac{L}{(\frac{q}{1-\delta}-\frac{U_\ell}{1+r})^{1/2}} = \frac{M}{\gamma^{1/2}}(1-\delta)^{1/2}$$

After changes in the default probability, if the price changes in the same proportion as the probability of repayment, the above condition does not hold. This is because the left hand side remains unchanged while the right hand side decreases (because $M > 0$). Therefore, the left hand side must decrease to restore equilibrium. The left hand side decreases if and only if the ratio $\frac{q}{1-\delta}$ falls. That is, to clear the bond market, the price of bonds has to fall more than the probability of repayment. This implies that the price of the bond gets relatively closer to valuation of low type investors. I provide a formal proof in appendix A.3.

Intuitively, when the default probability increases, low valuation investors are willing to pay higher intermediation fees in order to sell their bonds faster. Therefore, more investors sells their bonds. But, since high valuation investors also trade bonds in the secondary market, more buyers have to meet dealers to acquire the larger amount of bonds sold by low type investors. That can only happen if more dealers enter submarkets with high type investors, which requires that high type investors pay higher intermediation fees to attract them. Thus, to induce high type investors to pay higher intermediation fees, the price of the bond in the centralized primary market, has to fall more than what would compensate investors for higher default probability. The mechanism highlights the importance of allowing for endogenous trading probabilities as a determinant of secondary market liquidity in equilibrium.

The small open economy assumption that the mass of potential buyers, H, is large relative to the debt levels is key for this result. It implies that $M > 0$. If $M = 0$ market clears when

[19] In the calibrated model $M > 0$, reflecting the assumption that we are working with an small open economy. Therefore, the worlds' potential demand for sovereign bonds from the small open economy is larger than the newly supplied bonds and the bonds held by low type investors together. The effective demand for bonds, $\alpha(\theta_h)H$, depends on how intensively each investor aims to trade a bond, which in turns is endogenously determined by the expected return of the bond.

the ratio $\frac{q}{1-\delta}$ remains unchanged. If $M < 0$ the result reverses and an increase in the default probability results in a decrease in the price of the bond that is less than proportional to the fall in the probability of repayment.

As a final remark, notice that this simple example assumes that the amount of newly issued bonds ΔB is exogenous and constant. In the full model, the government responds to changes in the state of the economy by adjusting the supply of bonds. If the supply of bonds increases (M decreases) in states in which the probability of default increases the amplification effect is stronger. If the supply of bonds decreases when default risk increases the amplification effect is weaker and can even be reverted.

4 Quantitative analysis

In this section I assess the quantitative importance of trading frictions in the secondary market for sovereign bonds and conduct exercises to measure some of the effects of policy interventions. It is well known that long-term debt models have convergence issues as clearly documented by Chatterjee and Eyigungor (2012). To obtain convergence, I follow the literature and solve an approximate model by introducing preference shocks to the default and next period's debt choices of the government. Preference shocks are as small as needed to get convergence and do not significantly change government optimal choices.[20] The main difference from a standard long-term debt sovereign default model is that the price schedule is determined by market clearing condition (11) instead of the standard no-arbitrage condition. Updating the pricing schedule requires solving for investors' optimal choices and the net demand from investors for each solution of the government problem. Then, given the solution of the government problem and investors' net demand for bonds, I can solve for the market clearing price, $q(s)$, for each $s = (y, B, B')$. Although the solution requires some more steps than solving standard models, it does not significantly increase the computational burden. I describe the model with preference shocks and the solution algorithm in appendix B.

4.1 Interest rate spreads and liquidity measures in the model

The model produces trading probabilities for dealers and investors as well as intermediation fees. In this section I show how trading probabilities and intermediation fees in the model can be mapped to the measures of liquidity observed in the data such as the bid-ask spread, volume traded, and the turnover rate of bonds.

[20] See Dvorkin et al. (2021) and Gordon (2019) for further details.

I first consider the bid-ask spread. The bid-ask spread is defined as the difference of the ask price, q^A, that an investor pays to buy a bond and the bid price, q^B, that an investor gets for selling a bond. This spread is measured as a proportion of the mid price, q^M. In the model, for each $s=(y,B,B')$, I define

$$q^A(s)\equiv q(s)+f_h^0(s),\ q^B(s)\equiv q(s)-f_\ell^1(s),\ \text{and}\ q^M(s)\equiv\frac{q^A(s)+q^B(s)}{2}.$$

So the model counterpart of the bid-ask spread is given by the sum of the intermediation fees divided by the mid-price and multiplied by 10,000 to measure it in basis points, i.e.,

$$S^{B-A}(s)\equiv\frac{q^A(s)-q^B(s)}{q^M(s)}\times 10{,}000=\frac{f_h^0(s)+f_\ell^1(s)}{q^M(s)}\times 10{,}000. \tag{12}$$

Using the model, I also construct the traded volume in secondary markets, given by

$$Vol\,(s)\equiv\alpha\left(\theta\left(f_h^0\,(s)\right)\right)\left[\overline{I}-(1-\lambda)\,B\,(s)\right]+\alpha\left(\theta\left(f_\ell^1\,(s)\right)\right)(1-\lambda)\left[\zeta B\,(s)+(1-\zeta)\,L_1\,(s)\right].$$

Finally, I define the turnover rate for bonds as

$$Turnover\,(s)\equiv\frac{Vol\,(s)}{B\,(s)}. \tag{13}$$

In addition, I compute the interest rate spread of the risky sovereign bond over a perfectly liquid risk-free bond that pays an interest rate r every period. To compute this credit spread, denoted $S^R\,(s)$, I calculate the return rate $r_g\,(s)$ that equates the present discounted value of the promised sequence of future payments on a bond to its price. That is, $q\,(s)=\frac{\lambda+(1-\lambda)z}{\lambda+r_g(s)}$. Then, the credit spread is

$$S^R\,(s)\equiv(1+r\,(s))^4-(1+r)^4=\left[1+\frac{\lambda+(1-\lambda)\,z}{q\,(s)}-\lambda\right]^4-(1+r)^4. \tag{14}$$

The power 4 in (14) it to calculate annualized spreads because I calibrate the model at the quarterly frequency. In subsection 4.2, I use available information on data counterparts for $S^{B-A}\,(s)$, $S^R\,(s)$, and $Turnover\,(s)$, together with standard variables used in the sovereign default literature, to calibrate the parameters of the model.

4.2 Functional forms and parameters

Functional forms. The utility function of the government is a standard CRRA and the output cost of default has the same functional form as in Chatterjee and Eyigungor (2012),

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \text{ and } h(y) = y - \max\left\{0, d_0 y + d_1 y^2\right\}.$$

The stochastic process for the endowment, $F(y_t|y_{t-1})$, is given by

$$y_t - \bar{y} \quad = \quad \rho_y (y_{t-1} - \bar{y}) + \varepsilon_t, \text{ with } \varepsilon_t \sim N(0, \eta_y),$$

where the $\bar{y}$ is the average endowment and is normalized to 1. I use a standard Dagum (1975) matching function, also known as the telephone line matching function, or the Den Haan et al. (2000) matching technology in the labor search literature. Trading probabilities for investors and dealers are given by

$$\alpha(\theta) = \frac{\theta}{1+\theta} \text{ and } \rho(\theta) = \frac{\alpha(\theta)}{\theta},$$

where market tightness $\theta \equiv \frac{d}{n}$. So, if a submarket has measure n of investors orders and tightness θ, then the measure of dealers is θn and the measure of matches is

$$\mathcal{M}(n, \theta n) = \alpha(\theta) n = \frac{n \times (\theta n)}{n + (\theta n)}.$$

Some convenient properties of this matching function are that $\alpha(\theta) \in [0,1]$ and that $\alpha(\cdot)$ is twice continuously differentiable for all $\theta \in \mathbb{R}_+$.

Parameters. The model has in total 17 parameters and I calibrate it at a quarterly frequency using data moments for Greece. To assign parameter values I take some of them from the literature and calibrate some others using quarterly Greek data from 1995Q1 until the default in 2012Q1. Table 1 shows all parameter values. Appendix B describes the solution algorithm and the simulation of moments to calibrate some parameters for Greece. Appendix C describes all data sources to calibrate the model to the Greek economy.

Calibration strategy. Some of the parameters require simulating the model to match some moments in the data. These parameters can be divided into two groups. The first group of parameters consist of $\{\beta, d_0, d_1, z\}$, which are standard to most sovereign default models. To calibrate those parameters, I follow the strategy of Chatterjee and Eyigungor (2012) and target the average credit spread, the volatility of credit spreads, the debt-to-output ratio, and choose the coupon z to target an average price for bonds equal 1, which means that bonds are traded at par value on average. The second group consists of the new parameters that govern trading frictions and activity in the secondary market for sovereign bonds. These parameters are: u_h, u_ℓ, ζ, γ and $\bar{I}$. The entry cost for dealers, γ and the difference between u_h and u_ℓ are key determinants of the bid-ask spread. I calibrate γ to the minimum bid-ask spreads observed in the data and calibrate u_h and u_ℓ to match the average bid ask spread in the data. In addition, the parameters u_h, u_ℓ, are set such that the ex-ante expected flow utility of holding the bond is zero so that the price for a buyer is not artificially distorted by flow utilities. Investors holding a bond become low type with probability ζ. In equilibrium, low type investors have incentives to sell their bonds. Thus, the fraction of bond holders that become low type is a key determinant of the turnover rate for bonds in secondary markets (defined in equation (13)). Therefore, the parameter ζ is the one consistent with the turnover rate in secondary markets. Finally, I set the measure of investors $\bar{I}$ to be significantly larger than the stock of debt issued in equilibrium so that it is never the case that the government would want to choose a debt level in the grid where there are not enough investors in the economy to purchase such amount.

Table 1: Calibration

Symbol	**Description**	**Greece**
σ	Intertemporal elasticity of substitution	2.000
ϕ	Probability of re-gaining market access	0.050
ρ_y	Persistence of endowment process	0.9527
η_y	Variance of endowment innovations	0.0203
λ	Bond's maturity probability	0.039
γ	Dealers' entry cost	0.00025
r	Risk-free rate	0.010
β	Government's discount factor	0.976
d_0	Slope of output cost of default	-0.522
d_1	Curvature of output cost of default	0.650
z	Bond's coupon rate (%)	1.133
u_h	Type h utility for holding bonds	0.001
u_ℓ	Type ℓ utility for holding bonds	-0.160
ζ	Investor's preference shock probability	0.315
$\bar{I}$	Mass of investors in the economy	5.000
σ_{ε_D}	Std. dev. default preference shocks	0.0001
σ_m	Std. dev. issuance preference shocks	0.0001

Note: The table lists all parameters of the models and the assigned values.

Greece. The risk aversion parameter is set to $\sigma = 2$. The probability of regaining access to international credit markets after a default is chosen to be $\phi = 0.05$, which implies an average exclusion length of 5 years, in line with evidence in Cruces and Trebesch (2013) [21] The output process is discretized using Rouwenhorst's method. After discretization, income follows a Markov process with transition probabilities given by $\Pr(y_{t+1} = y_m | y_t = y_n) = \pi^y_{n,m}$, for all $n, m \in \{1, 2, ..., N\}$, I choose $N = 51$. I then calibrate ρ_y and η_y to match the persistence for the AR(1) process for the GDP cycle in Greece for the period $1995Q1 - 2023Q4$, and η_y to match the standard deviation of the residuals. The maturity parameter is set to $\lambda = 0.0385$, which represents an average expected time to maturity of 6.5 years. This corresponds to the average time to maturity of outstanding bonds before the debt restructuring as reported by Mihalache (2020). The parameters $\{\beta, d_0, d_1\}$ are calibrated to target an average spread of 4.43%, a standard deviation of spreads of 4.65% and a debt-to-output ratio of 115% observed on average between $1995Q1 - 2012Q1$. The quarterly coupon rate $z = 1.13\%$ to obtain an average price for bonds of 1. Dealers' entry cost, γ, determines the minimum transaction fee that can arise in the model. Since I do not have direct estimations of the intermediation cost I set $\gamma = 0.00025$, which is 2.5 basis points of the average price. This is consistent with the very low bid-ask spreads of around $5 - 10$ basis points observed around 2006 after Greece joined the Euro and before US subprime crisis. Finally, $r = 0.01$ corresponds to the average interest rates for 3-month German bonds. In the model the risk free rate is assumed to correspond to perfectly liquid bonds. German bonds are almost perfectly liquid with bid-ask spreads below 5 basis points in most of trading dates, including the period of the European debt crisis. I set $\bar{I} = 5$, which is more than two times larger than the stock of debts observed in equilibrium. Appendix D.3 shows that increasing $\bar{I}$ further does not significantly change the results.

4.3 Model fit and implications for business cycle moments

Table 2 shows data moments for Greece, the model fit and its implications for standard business cycle moments that are not targeted in the calibration. The model calibration provides a close fit to the data, except that it produces a samller volatility of spreads. Relative to a model with frictionless secondary markets, trading frictions add more slope to the price schedule as illustrated by the example in section 3.1. As a response the government implements a relatively less volatile debt issuance policy. In net, it the government behavior keeps spreads volatility low.

[21] This number is also within the 2 year median duration calculated by Gelos et al. (2011) since 1990 and the more than 7 year median duration computed by Benjamin and Wright (2013).

Table 2: Targeted and untargeted moments

Targeted moments	Data	Model
Average bond spread (%)	3.43	3.42
Std. dev. bond spread (%)	4.65	2.18
Debt to output (%)	115	124
Average bid-ask spread (bps.)	75	76
Bonds turnover rate (%)	78	78
Untargeted moments	Data	Model
Ratio of std. dev. of consumption and output	0.98	1.07
Corr. between bond spread and trade balance to GDP	0.71	0.58
Corr. between bond spread and consumption	−0.45	−0.77
Corr. between bond spread and output	−0.56	−0.75
Corr. between output and trade balance to GDP	−0.59	−0.43

Note: Data moments for Greece are calculated using data from 1995Q1 until 2012Q1, when Greece defaulted on its sovereign debt. The data sources are described in details in appendix C. The moments from the model are constructed from over $1,000$ simulations of the same length as the data, computing the moment for each simulation and averaging across simulations, as described in appendix B.2.

Table 2 also reports the implications of the model for business cycle moments often analyzed in the sovereign default literature. The model captures well the correlations between the main variables of the model. However, as it is standard in sovereign debt models, consumption is more volatile than output, while in the data output is slightly more volatile than consumption.

4.4 The role of trading frictions and secondary market flows

In this section I explore quantitatively the role trading frictions and net demand flows from investors in the secondary market for sovereign bonds. To quantify the role of trading frictions I compare the baseline model to a counterfactual model with frictionless secondary market. The goal of this exercise is to understand how secondary market frictions affect the financial constraint of the government. Thus, I keep the parameters of the baseline model but I consider the case in which $\gamma = 0$ and the price of bonds is determined by a standard no-arbitrage condition. Table 3 reports the results from the frictionless model in the third column. Compared to the benchmkark model, the frictionless model produces a spread that is 75 basis points lower on average, reducing the borrowing cost for the government. Moreover, the average debt to output ratio becomes 139%, which is 15 percentage points larger than the baseline model. That is, trading frictions significantly tighten government's financial constraint, reducing its borrowing capacity and increasing is borrowing costs.

Table 3: The role of secondary market's trade frictions and flows

Key Parameters	Baseline	Frictionless	Longer holding	Hold to maturity
Low type probability: ζ	0.315	—	0.1575	0.000
Dealer's entry cost: $\gamma \times 100$	0.025	0.000	0.025	0.025
Moments	Baseline	Frictionless	Longer holding	Hold to maturity
Mean bond spread (%)	3.42	2.67	3.03	2.00
Std. dev. bond spread (%)	2.18	2.67	2.81	1.83
Debt to output (%)	124	139	134	137
Mean bid-ask spread (bps.)	76	—	74	71
Bonds turnover rate (%)	78	—	42	6

Note: The table describes the key parameters that differ across counterfactual scenarios and the moments obtained from simulations after changing only the key parameters. Baseline refers to the model under the baseline calibration of 1. Frictionless refers to a counterfactual model in which there are no trading frictions in the secondary markets and bonds are priced by a no-arbitrage condition like the one in equation (15). Longer holding refers to a counterfactual scenario in which the average holding horizon of investors is twice as large as in the baseline model. This is obtained by reducing the probability of becoming ℓ by a half. Hold to maturity refers to the counterfactual scenario in which investors buy a bond and never become ℓ type investors, so they hold the bond until they mature. The turnover rate is defined as in (13) and the bid-ask spread is defined as in (12) and measured in basis points.

To understand the quantitative importance of investors trade flows in the secondary market, I compare the baseline results to two alternative counterfactuals. In these two counterfactuals I keep all the same parameters of the baseline calibration but change the probability ζ of becoming a type ℓ investor. Reducing ζ increases the holding horizon of investors and reduces the supply of bonds in the secondary market. Table 3 shows the results for these two counterfactuals in columns 4 and 5, labeled "longer holding" and "hold to maturity". The longer holding scenario considers a probability ζ that is a half of the baseline parameter while the hold to maturity case eliminates the possibility of becoming ℓ type and investors do not supply bonds in the secondary market.[22] Compared to the baseline model, the longer the holding horizon of the investors the lower is the average credit spread and the larger is the stock of debt to output in equilibrium. In fact, if investors held bond to maturity the average spread would be about 140 basis points smaller than in the baseline and the stock of debt to output would be on average 13 percentage points larger.

In sum, trading frictions and sell flows in the secondary market are important determinants of the financial constraint of a government. Trading frictions in the secondary market may be more of a technological constraint outside the reach of policy but the design of secondary market may help improve in this margin. However, average holding horizons of investors can generate important changes in the financial constraint of the government, which

[22] In the hold to maturity scenario there is still activity in the secondary market because investors buy newly issued bonds from dealers.

can more easily be address by policy. For example, quantitative easing programs of central banks change the lender composition of sovereign governments and affects the average holding horizon. By committing to keep bonds in their balance sheet for long enough, central banks can relax the borrowing constraint of the government. I explore the effects of such interventions in section 5.4. Alternatively, sovereign governments could target lenders who keep bonds for longer horizons in their portfolio to relax their financial constraints.

5 The Greek debt crisis

5.1 Greek time series

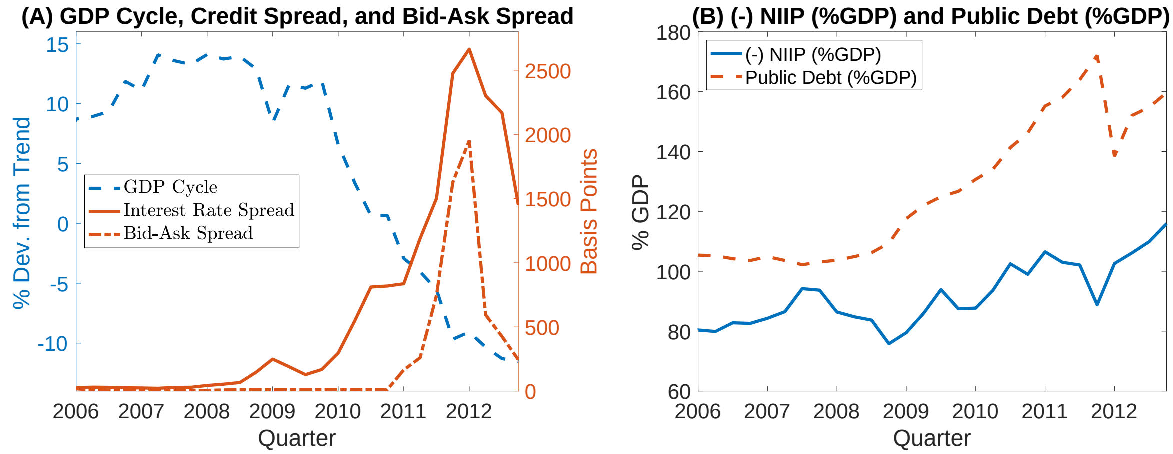
The Greek sovereign debt crisis of 2010-2012 resulted in a debt restructuring in the first quarter of 2012. Figure 4 shows the time series for macroeconomic variables from 2006$Q1$ until 2012$Q4$. The left panel shows Greek GDP cycle (blue dashed line with scale in left axis), the interest rate spread of Greek long-term bonds compared to same maturity German bonds as defined in equation (14) (red solid line with scale in right axis), and the bid-ask spread for Greek bonds in secondary markets as defined in equation (12) (red dashed-dotted line with scale in right axis). The right panel shows the negative of net international investment position (NIIP) as percentage of GDP[23] (blue solid line) together with total public and publicly guaranteed (PPG) debt as percentage of GDP (red dashed line). Appendix C describes the variables and data sources.

Between 2006$Q1$ and 2010$Q4$ Greece experienced sustained economic growth, only temporarily interrupted in 2009$Q1$ during the sub-prime crisis. However, during 2010 and 2011 the GDP gap decreased sharply taking the economy into a deep recession.

Before the subprime crisis, the Greek government was able to take debt at very low interest rates of $20-50$ basis points above German rates. Cheap interest rates allowed Greece to accumulate debt over the years until 2008, when U.S. crisis hit international financial markets and Greek interest rate spreads began to increase, reaching a first spike at 250 basis points in the first quarter of 2009. As interest rate started increasing, Greece began reducing its net international investment positions during 2008 and the beginning of 2009. In 2009 output recovered and interest rate spreads went down to stay around 150 basis points during the entire year. By the end of 2009 there was a fast increase in interest rate spreads of Greek bonds, moving from 170 basis points in 2009$Q4$ to around 900 basis points during 2010$Q2$, amid a political crisis and the revelation that Greece had been understating its debt and

[23] A positive number means a negative NIIP with the rest of the world.

Figure 1: Greek time series $2006Q1 - 2012Q4$.



Note: The figure shows quarterly data for Greece from 2006Q1 to 2012Q4. The left panel shows the evolution of the GDP cycle (blue dashed line) together with the interest rate spread (red solid line) and the bid-ask spread (red dashed-dotted line). The right panel plots the negative of the net international investment position (NIIP) as percentage of GDP (blue solid line) together with total public debt as percentage of GDP. Source: Bloomberg and Eurostats.

deficit figures for years. Attempts to stop the crisis during 2010 were not successful and by the end of 2011 interest rate spreads were above 2500 basis points. In $2012Q1$ Greek debt was restructured involving bond swaps and a 65% haircut on investors' bond holdings. On March of 2012 the International Swaps and Derivatives Association declared a triggering credit event. In other words, a default. During 2012 interest rate spreads decreased due to the debt relief on Greek bond in $2012Q1$ following the restructure but remained high for the rest of the year. In $2012Q4$ Greece bought back a large fraction of the newly issued bonds over the debt restructuring, which increase the market price of bonds on 20%, significantly reducing the interest rate spreads. See Zettelmeyer et al. (2013) and Trebesch and Zettelmeyer (2018) for detailed and clear exposition of events in during Greek debt crisis.

The bid-ask spreads, defined as in (12), remained below 50 basis points during the period $2006Q1 - 2010Q4$. However, bid-ask spreads sharply increase from 165 basis points during $2011Q1$ to about 2000 basis points before the debt restructure of March in 2012, when total interest rate spreads where between $2600 - 3000$ basis points. The model presented in section 2 can be used to assess how the endowment process affects the rest of variables depicted in Figure 4, and determine how much movements in interest rate spreads reflects changes in the probability of default and liquidity frictions.

5.2 Model time series

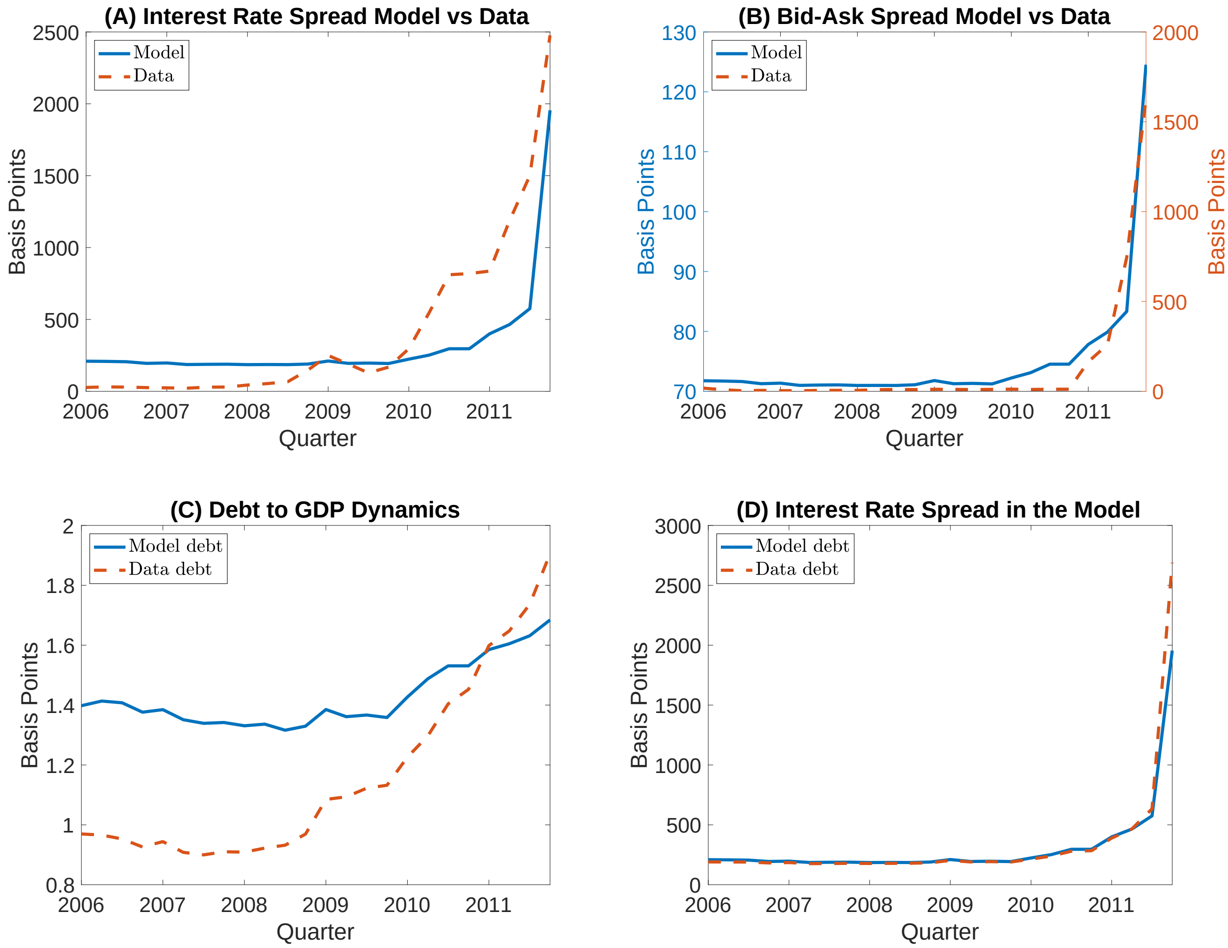
In this section, I simulate the model for Greece. I feed the GDP cycle plotted in the left panel of Figure 4 into the model as the realizations of the endowment process for $t \in \{1995Q1, ..., 2011Q4\}$. I stop the time series in $2011Q4$ because Greece defaulted in $2012Q1$ and the model shuts down the secondary market after a default because the recovery value is zero. I let the government, investors, and dealers react optimally by choosing debt issuances, defaulting or not, and optimal submarket to visit. Using optimal choices I compute the model implied interest rate spreads, S_t^R, and the bid-ask spreads, S_t^{B-A}, as defined in (14) and (12), respectively. Figure 2 shows model's predictions compared to the data. The blue solid lines represent model's predictions while red dashed lines correspond to data.

Panel (A) shows the evolution of credit spreads. The model does a good job to capture the dynamics of spreads and it is able to explain a large fraction of the changes in magnitude in the data. However, it is not able to account for the exact magnitudes. This is not surprising given that the model is calibrated to match the average spreads in Greece in a time period with very large changes. Before the subprime crisis in the US, Greece was able to take debt at German rates. After the subprime crisis interest rate spreads behaved as those ones in emerging economies. In addition, the model abstracts from bailout expectations, time varying changes in investors discount factors and redenomination risk, which have been shown to be important determinants of interest rate spreads in European countries in the period of analysis.[24]

Panel (B) shows the evolution of bid-ask spreads. The figure shows that the model is able to capture the fact that bid-ask spreads increase when the economy gets closer to a default episode and the qualitative dynamics over the business cycle. However, the model (blue solid line with scale in left axis) is not able to account for the extremely large increase in bid-ask spreads in the data (red dashed line with scale in right axis). In the model, the bid-ask spread at the peak of the crisis are around 65% higher than pre-crisis levels while in the data bid-ask spreads at the peak of the crisis are 5000% larger. Again, the calibration of the model targets average bid-ask spreads in the sample. Before the crisis Greek bonds were almost perfect substitutes to German bonds and very liquid. When the crisis hit, Greek bonds became extremely illiquid.

[24] See Bocola and Dovis (2019) for a decomposition of interest spreads that accounts for a time varying investor's discount factor and Dovis and Kirpalani (2018) for the effects of bailout expectations on interest rate spreads dynamics. See Krishnamurthy et al. (2018) for the role of redenomination risk.

Figure 2: Interest Rate Spreads and Bid-Ask Spreads.



Note: In panels (A)-(C) the blue solid lines show model equilibrium outcomes while the red dashed lines show data counterparts for interest rate spreads, bid-ask spreads and debt-to-output ratios, respectively. Panel (D) shows the interest rate spread from the model (blue solid line) as in panel (A) together with the equilibrium interest rate spread that the model would produce if the government issued debt as in the data (red dashed line).

Panel (C) shows the dynamics of debt to GDP in the model (blue solid line) and the dynamics of Public and Publicly Guaranteed (PPG) debt as percentage of GDP in the data (red dashed line). Panel (D) shows the spreads in the model when debt dynamics are those produced by the government's optimal policy in the model (blue solid line) and the spreads that would arise in the model if debt to GDP ratio was as the PPG debt to GDP ratio in the data. Tracking debt stocks in the data generates a spread of 2500 basis points in 2011Q4 as observed in the data in panel (A). However, incorporating actual debt dynamics is still not enough to match the low spreads observed in the data before 2009.

5.3 Spread decomposition

How large is the liquidity premium? To answer this question, I use the structure of the model to decompose the predicted interest rate spreads, S_t^R, into a default risk component and a liquidity component. To decompose interest rate spreads I do the following exercise. I take optimal government policies from the model and calculate the bond's price that would reflect the probability of default in an alternative model in which bonds are perfectly liquid. That, is I solve the following Bellman equation for prices

$$\tilde{q}(y,B) = \frac{1}{1+r}\mathbb{E}_{y'|y}\left\{\left[1-\delta^*\left(y',B'^*\right)\right]\left[\lambda+(1-\lambda)\left(z+\tilde{q}\left(y',B'^*\right)\right)\right]\right\}, \tag{15}$$

where $B'^* \equiv B'(y,B)$ is the optimal policy of the government in the model with liquidity frictions. Using the counterfactual price for bonds we can calculate the interest rate consistent with default risk. For each pair (y,B), this interest rate is given by

$$r^d(y,B) = \frac{\lambda+(1-\lambda)z}{\tilde{q}(y,B)} - \lambda.$$

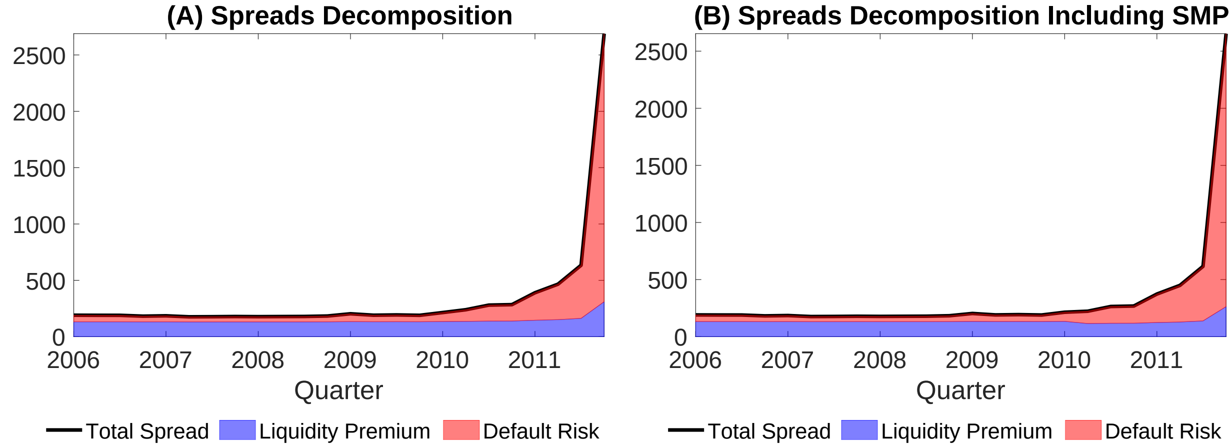
Then, for each state (y,B,L_1) I decompose interest rate spreads, $S^R(y,B,L_1)$, into a default risk component, $S^d(y,B,L_1)$, and a liquidity component, $S^\ell(y,B,L_1)$, which are given by

$$\begin{aligned}
S^R(y,B,L_1) &= S^d(y,B,L_1)+S^\ell(y,B,L_1),\\
S^d(y,B,L_1) &\equiv \left(1+r^d(y,B)\right)^4-(1+r)^4,\\
S^\ell(y,B,L_1) &= S^R(y,B,L_1)-S^d(y,B,L_1).
\end{aligned}$$

As a residual, the liquidity component captures both pure liquidity frictions plus the feedback interactions between liquidity risk and default risk. Figure 3 shows the results of this decomposition. In the left panel, the black solid line is the total interest rate spread generated by the model, S_t^R, the red area represents the amount of the spread representing default risk while the blue area is the liquidity component. The right panel show the shares of total spread that are due to default risk and liquidity in red and blue, respectively.

The model interpretation of the data is that liquidity frictions can significantly contribute to interest rate spreads. According to this decomposition trade frictions in the secondary market add about 140 basis points paid by Greek government bonds.

Figure 3: Interest Rate Spreads and Bid-Ask Spreads.



Note: The left panel shows the interest rate spread predicted by the model assuming that the government issues the amount of debt observed in the data as show in panel (C) of Figure 2 (black line). The total interest rate spread is decomposed into a default risk component (red area) and a liquidity component (blue area). The right panel shows the same decomposition of interest rate spreads but it also includes the effects of the Securities Market Programme (SMP) that the ECB implemented on Greek bonds during 2010.

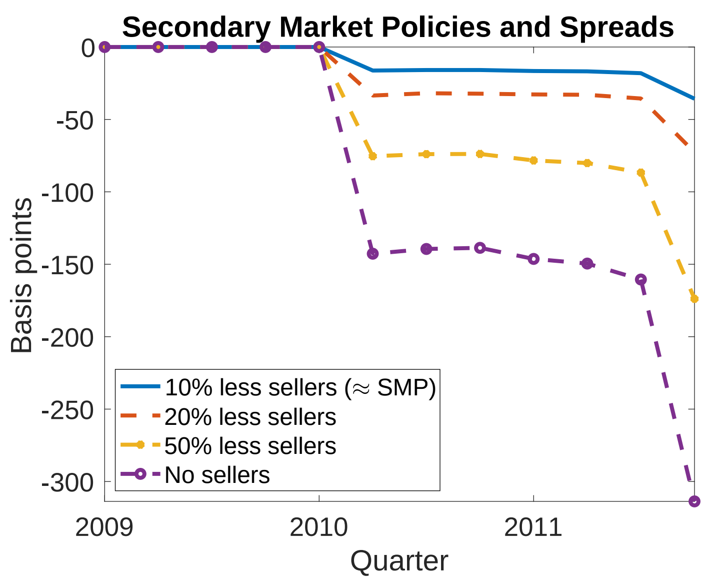
5.4 Liquidity and secondary market interventions

One of the key contributions of this paper is to propose a framework in which default risk and the liquidity of bonds in the secondary market are endogenously determined in equilibrium. This has important implications for policy analysis that do not arise in other frameworks in the literature. In a simple way, the model captures the fact that heterogenous investors have different trading needs. Thus, how bonds are distributed across investors is a key determinant for bond's price because it affects the magnitudes of demand and supply flows. Therefore, any intervention in the economy that affects the distribution of bond holdings across different types of investors would have an impact on the equilibrium bond price. This result relies in the endogeneity of trading probabilities and intermediation fees.

During the European debt crisis, the European Central Bank (ECB) directly purchased sovereign bonds in the secondary market in 2010 under the Securities Market Programme (SMP). Interventions of this kind would affect liquidity if the ECB trading behavior is different from investor's behavior. Interventions in the secondary market can potentially have long lasting consequences in bond prices depending on future trading decisions. In the case of ECB's intervention in 2010, the European Central Bank committed to keep bonds purchased under the SMP in its balance sheet until maturity. Therefore, in addition to an increase in net demand at the time of intervention, there is an increase in future net demand too. This is because ECB's behavior in the secondary market differs from type ℓ investor's behavior. Instead of reselling bonds, the ECB kept the bonds in its balance sheet until maturity reducing the supply of those bonds in the secondary market, relative to what type ℓ investors would have done.

Under the SMP, the ECB purchased about 10% of Greek outstanding debt within the first month of the program, launched in May 2010. Trebesch and Zettelmeyer (2018) study the effects of such intervention on Greek bonds' price. They conduct a difference-in-difference excersise and compare the change in bonds' price before and after the intervention for the group of Greek bonds that the ECB bought in the secondary market relative to the group of Greek bonds that the ECB did not purchase. Consistent with the implications of the model, Trebesch and Zettelmeyer (2018) find that the SMP has a significant and persistent positive impact on the price of bonds purchased by the ECB. Eser and Schwaab (2016) and De-Pooter et al. (2018) reach similar conclusions after evaluating the effects of the SMP in countries targetted by the program.

Figure 4: The effect of secondary market interventions on interest rate spreads.



Note: The figure reports the difference in interest rate spreads between the baseline each counterfactual scenario and the baseline. Each of the counterfactuals considers the effects of a reduction in the probability of becoming ℓ type investors (ζ) from its baseline value $zeta = 0.315$ to 0.02835 in blue solid line, to 0.252 in red dashed line, to 0.1575 in yellow dashed-dotted line, and to zero in purple dashed-circled line.

Using the model, I compute the effects of interventions like the SMP. I model such an intervention as a permanent change in the fraction of bonds that are in hands of type ℓ investors. That is I reduce the probability of becoming type ℓ investors by 10% to capture the fact that the ECB purchased about 10% of the outstanding Greek sovereign debt and kept the bonds until maturity. That is the SMP policy is modeled as an unexpected and permanent fall in the parameter ζ to 0.2835 in the second quarter of 2010. In Figure 4 I also plot the effects of reducing the probability of becoming type ℓ investors by 20% ($\zeta = 0.252$), to a half ($\zeta = 0.1575$) and to zero. In all cases, I keep the debt issuance policy as in the baseline scenario without intervention. That is, all the effects in the spread are purely coming from the amount of sell orders in the secondary market and not from the response of the government to changes in prices.[25]

Panel (B) of Figure 3 shows the decomposition of credit spreads taking into account the effects of the SMP. However, the effects of the SMP on credit spreads can be appreciated more clearly in Figure 4. The blue solid line shows the effects of the policy on Greek interest rate spreads relative to no intervention in the blue solid line. The model predicts that the SMP reduced Greek spreads by around 15 to 35 basis points.

Figure 4 shows the effects of reducing the fraction of sellers by alternative magnitudes. I find that reducing ζ by 20% generates a fall in spreads of 32 to 72 basis points. Reducing ζ to a half of the baseline produces a fall in spreads of 73 to 173 basis points, while bringing ζ to zero reduces the spread by 139 to 313 basis points. In all cases, the maximum reduction in spreads is observed when default risk is the highest, i.e. in the last quarter of 2011.

6 Conclusions

I incorporate endogenous liquidity frictions into a standard quantitative model of sovereign default. To model liquidity I introduce directed search into the secondary market for sovereign bonds, where investors need to meet dealers in order to trade. Since search is directed, investors and dealers face a trade-off between the intermediation fee and the trading probability. For investors, the higher the intermediation fee that they choose to pay, the higher the probability of trading. For dealers, the higher the intermediation fee that they choose to charge, the lower the probability of trading. In addition, the optimal balance of this trade-off varies with the state of the economy. Thus, as trading probabilities and intermediation fees are endogenous and time varying, the liquidity of the secondary market for bonds is also endogenous and time varying over the business cycle.

The model provides a micro-foundation for transactions of bonds in the secondary market that highlights the importance of taking into account the size of trade flows to determine the price of both outstanding and newly issued bonds. I find that trading friction are significantly tighten the financial constraint of the government. After calibrating the model

[25]The results in table 3 incorporate the response of the government.

to the recent debt crisis in Greece, the model based decomposition of interest rates shows that trade frictions in the secondary market significantly contributed to explain credit spreads. Between $2006Q1-2011Q4$, they increased credit spreads by around 140 basis points.

The model presented in this paper is also useful to understand the effects of policy interventions in the secondary market. In section 5.4, I study the effects of the ECB's Securities Market Programme. I model the policy as a permanent 10% fall in the flow of sell orders in the secondary market because the purchased around 10% of the oustanding stock of debt and committed to keep it until maturity. The model predict that the SMP reduced interest rate spreads by 15 to 35 basis points. Policies that reduce the magnitudes of sell volume in larger fractions could achieve significantly stronger falls in credit spreads. For examples, if sell orders fell to zero instead, the credit spreads would have by 139 to 313 basis points.

The analysis can be extended in several ways. One important dimension that is not considered in this framework is maturity choice. There seems to be a trade-off between offering a wide set of maturities that satisfy the needs of different types of investors and the liquidity of each of the alternative bonds issued. In the data, governments tend to offer a wide range of maturities but usually a couple of them are much more liquid than the others. The model also abstracts from the effects of changes in the risk free rate on liquidity conditions of risky bonds due to changes in investors' discount factor. Such changes would affect interest rate spreads by the usual channels studied in the literature[26] but could also generate interesting amplification dynamics in liquidity premium. In addition, the model highlights that different types of investors can generate different demand and supply flows and affect prices. It would fruitful to understand and incorporate richer heterogeneity of investors and their behavior in sovereign debt markets. Finally, the model abstracts from government bonds held by domestic households. As pointed out by Broner et al. (2010), bonds transactions in secondary markets may rule out default episodes. This is because a benevolent government does not want to default on domestic households. Then, in equilibrium foreign investors sell bonds to domestic households before a default is declared. However, my model predicts that secondary markets endogenously become more illiquid in bad times, exactly when foreign investors have incentives to transfer bonds to domestic ones. Thus, the composition of debt holdings across foreign and domestic investors may significantly affect the cost of bonds re-allocation. I leave these extensions for future research.

[26]See Lizarazo (2013) and Bocola and Dovis (2019).

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Appendix (For Online Publication)

A Proofs

A.1 Investors Problems

Definition 1. *I say that for a given bond price q, an investor participates in the secondary market for bonds if there exists an intermediation fee $f \geq \gamma$ at which gains from trade are positive. I denote $\mathcal{Q}_i^a(s)$ the set of prices at which investors with holdings $a \in \{0,1\}$ and type $i \in \{\ell, h\}$ participates in secondary markets when state of the economy is $s \in \overline{S}$.*

For each state $s \in \overline{S}$ I define the expected continuation value of holding a bond as

$$
\begin{aligned}
E^0(s) &\equiv \mathbb{E}_{y'|y}\left[1-\delta\left(y', B'\right)\right]\left[I^1(s') - I^0(s')\right] \\
E^1(s) &\equiv \mathbb{E}_{y'|y}\left[1-\delta\left(y', B'\right)\right]\left[I^1(s')\right],
\end{aligned}
$$

and the gains from trade, before including transaction costs, for each type of investor as

$$
\begin{aligned}
R^0(s) &\equiv -q(s) + u_h + \frac{1}{1+r}E^0(s), \\
R_h^1(s) &\equiv q(s) - u_h - \frac{1}{1+r}E^1(s), \\
R_\ell^1(s) &\equiv q(s) - u_\ell - \frac{1}{1+r}E^1(s).
\end{aligned}
$$

Before proving lemma 3 it is handy to show some intermediate results. First, I show that, for each state $s \in \overline{S}$, we can find price thresholds that determine the set of investors that participate in the secondary market. In addition, I show that there might be a price region in which the secondary market completely shuts down.

Lemma 1. *For any state $s \in \overline{S}$, If the maximum gain for a type ℓ seller is smaller than the maximum gain for a type h buyer, i.e.*

$$
u_\ell + \frac{1}{1+r}E^1(s) + \gamma < u_h + \frac{1}{1+r}E^0(s) - \gamma, \tag{A.1}
$$

there exist three price thresholds $\{\tilde{q}_1(s), \tilde{q}_2(s), \tilde{q}_3(s)\}$ such that

(i) If $q(s) < \tilde{q}_1(s)$, $q(s) \in \mathcal{Q}_h^0(s)$ and $q(s) \notin \mathcal{Q}_\ell^1(s), \mathcal{Q}_h^1(s)$. Only type h investors with $a = 0$ participate in secondary markets;

(ii) If $q(s) \in (\tilde{q}_1(s), \tilde{q}_2(s))$, $q(s) \in \mathcal{Q}_h^0(s), \mathcal{Q}_\ell^1(s)$ and $q(s) \notin \mathcal{Q}_h^1(s)$. Both type h investors with $a = 0$ and type ℓ investors with $a = 1$ participate in secondary markets;

(iii) If $q(s) \in (\tilde{q}_2(s), \tilde{q}_3(s))$, $q(s) \in \mathcal{Q}_\ell^1(s)$ and $q(s) \notin \mathcal{Q}_h^0(s), \mathcal{Q}_h^1(s)$. Only type ℓ investors with $a = 1$ participate in secondary markets; and

(*iv*) *If* $q(s) > \tilde{q}_(s)$*,* $q(s) \in \mathcal{Q}_\ell^1(s), \mathcal{Q}_h^1(s)$ *and* $q(s) \notin \mathcal{Q}_h^0(s)$*. Both type* ℓ *investors with* $a = 1$ *and type* h *investors with* $a = 1$ *participate in secondary markets.*

If the inequality in (A.1) is reversed, there exist three price thresholds $\{\tilde{q}_1(s), \tilde{q}_2(s), \tilde{q}_3(s)\}$ *such that*

(*i*) *If* $q(s) < \tilde{q}_1(s)$*,* $q(s) \in \mathcal{Q}_h^0(s)$ *and* $q(s) \notin \mathcal{Q}_\ell^1(s), \mathcal{Q}_h^1(s)$*. Only type* h *investors with* $a = 0$ *participate in secondary markets;*

(*ii*) *If* $q(s) \in (\tilde{q}_1(s), \tilde{q}_2(s))$*,* $q(s) \notin \mathcal{Q}_h^0(s), \mathcal{Q}_\ell^1(s), \mathcal{Q}_h^1(s)$*. No investor has incentives to participate and secondary markets shut down;*

(*iii*) *If* $q(s) \in (\tilde{q}_2(s), \tilde{q}_3(s))$*,* $q(s) \in \mathcal{Q}_\ell^1(s)$ *and* $q(s) \notin \mathcal{Q}_h^0(s), \mathcal{Q}_h^1(s)$*. Only type* ℓ *investors with* $a = 1$ *participate in secondary markets; and*

(*iv*) *If* $q(s) > \tilde{q}_3(s)$*,* $q(s) \in \mathcal{Q}_\ell^1(s), \mathcal{Q}_h^1(s)$ *and* $q(s) \notin \mathcal{Q}_h^0(s)$*. Both type* ℓ *investors with* $a = 1$ *and type sh investors with* $a = 1$ *participate in secondary markets.*

Proof. Notice that investors with $a = 0$ can only participate in trades in which they purchase a bond and investors with $a = 1$ can only participate in trades where they sell a bond. We start by analyzing the case in which

$$u_\ell + \frac{1}{1+r} E^1(s) + \gamma < u_h + \frac{1}{1+r} E^0(s) - \gamma.$$

Here we define

$$\tilde{q}_1(s) \equiv u_\ell + \frac{1}{1+r} E^1(s) + \gamma, \tag{A.2}$$

$$\tilde{q}_2(s) \equiv u_h + \frac{1}{1+r} E^0(s) - \gamma, \tag{A.3}$$

$$\tilde{q}_3(s) \equiv u_h + \frac{1}{1+r} E^1(s) + \gamma,$$

with $\tilde{q}_1(s) < \tilde{q}_2(s) < \tilde{q}_3(s)$. Notice that if

$$q(s) < \tilde{q}_1(s) < \tilde{q}_2(s),$$

then no type ℓ investor is willing to participate in secondary markets since it will imply a negative gain from trade for all $f \geq \gamma$. In addition, since $R_h^1(s) < R_\ell^1(s)$, also type h investors with $a = 1$ have no incentives to sell their bonds. Finally, if $\tilde{q}_2(s) > q(s)$ implies that $R^0(s) > \gamma$. Therefore, type h investors with $a = 0$ participate on secondary markets.

Next, if the price is such that

$$\tilde{q}_1(s) < q(s) < \tilde{q}_2(s),$$

we get that $R_\ell^1(s), R^0(s) > \gamma$. So, both of these types of investors participate on secondary markets. Also, since $I^0(s') \geq 0$, we have that

$$q(s) < \tilde{q}_2(s) \leq \tilde{q}_3(s),$$

which implies that $R_h^1(s) < \gamma$, and hence type h investors with $a = 1$ do not participate in secondary markets.

Now, lets consider the case in which the price is such that

$$\tilde{q}_2(s) < q(s) < \tilde{q}_3(s).$$

Here we get that $R_\ell^1(s) > \gamma > R_h^1(s), R^0(s)$. Thus, type ℓ investors with $a = 1$ are the only ones that participate in secondary markets. Finally, if $\tilde{q}_3(s) < q(s)$, we get that $R_\ell^1(s) > R_h^1(s) > 0 > R^0(s)$, and all investors with $a = 1$ participate in secondary markets.

It remains to analyze the case in which

$$u_\ell + \frac{1}{1+r} E^1(s) + \gamma \geq u_h + \frac{1}{1+r} E^0(s) - \gamma.$$

In this case $\tilde{q}_2(s) < \tilde{q}_1(s) < \tilde{q}_3(s)$. So, if $q(s) < \tilde{q}_2(s)$, we get that $R^0(s) > \gamma > R_\ell^1(s), R_h^1(s)$, and only type h investors with $a = 0$ participate in secondary markets. If $q(s) \in (\tilde{q}_2(s), \tilde{q}_1(s))$, then we have that $R_h^0(s), R_h^1(s), R_\ell^1(s) < \gamma$. Therefore, secondary markets shut down since no investor has incentives to participate. In a similar way as before, it can be checked that if $q(s) \in (\tilde{q}_1(s), \tilde{q}_3(s))$ we get that $R_\ell^1(s) > \gamma > R_h^0(s), R_h^1(s)$, so only type ℓ investors with $a = 1$ participate in secondary markets. Finally, since the definition of $\tilde{q}_3(s)$ has not change, it is straightforward to see that $q(s) > \tilde{q}_3(s)$ implies that all investors holding a bond participate in secondary markets. This completes the proof. □

Next, I show that whenever investors participate in the secondary market, optimal transaction fee $f_h^0(s)$ is decreasing and $f_\ell^1(s), f_h^1(s)$ are increasing in $q(s)$.

Lemma 2. *Given an aggregate state and a debt issuance choice $s = (y, B, B')$ and taking government policy functions $\delta(y, B)$, $B'(y, B)$ as given:*

(i) The optimal submarket choice $f_h^0(s)$ is unique, continuous, and strictly decreasing in $q(s)$, for all $q(s) \in int\left(\mathcal{Q}_h^0(s)\right)$.

(ii) The optimal submarket choice $f_\ell^1(s)$ is unique, continuous, and strictly increasing in $q(s)$, for all $q(s) \in int\left(\mathcal{Q}_\ell^1(s)\right)$.

(iii) The optimal submarket choice $f_h^1(s)$ is unique, continuous, and strictly increasing in $q(s)$, for all $q(s) \in int\left(\mathcal{Q}_h^1(s)\right)$.

(iv) $f_i^a(s) = 0$ is optimal for all $q(s) \notin int\left(\mathcal{Q}_i^a(s)\right)$, all $i \in \{\ell, h\}$, and all $a \in \{0, 1\}$.

Proof. Let the aggregate state of the economy and debt issuance choice be an arbitrary $s = (y, B, B') \in \overline{S}$. In all cases we focus on the price region in which investors are willing to participate in secondary markets, characterized in proposition 1.

(i) Using the free entry condition (10) we have that in any active submarket $\rho(\theta) f = \gamma$ and, by properties of the matching function we have that $\frac{\alpha(\theta)}{\rho(\theta)} = \theta$. So, we can re-write the problem of a type h with $a=0$ as if the investor chooses θ instead of f. That is,

$$\begin{aligned} I^0(s) &= \max_{\theta} \alpha(\theta)\left[-q(s)+u_h+\frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta\left(y',B'\right)\right]I^1\left(s'\right)\right]-\gamma\theta \\ &+\frac{1-\alpha(\theta)}{1+r}\mathbb{E}_{y'|y}\left[1-\delta\left(y',B'\right)\right]I^0\left(s'\right). \end{aligned}$$

Now, since $\alpha(\cdot)$ is differentiable, we can take first order condition with respect to θ to get

$$[\theta]:\alpha'(\theta)\left\{-q(s)+u_h+\frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta\left(y',B'\right)\right]\left[I^1(s')-I^0\left(s'\right)\right]\right\}=\gamma.$$

We defined

$$R^0(s)\equiv -q(s)+\frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta\left(y',B'\right)\right]\left[I^1(s')-I^0\left(s'\right)\right],$$

which is independent of θ and decreasing in $q(s)$. Remember we focus in the region of prices in which $R^0(s)$ is positive, else investors would prefer not to purchase the asset and optimal tightness with be zero. Thus, we have that the optimal choice of θ in state s is given by

$$\theta_h^0(s)=\alpha'^{-1}\left(\frac{\gamma}{R^0(s)}\right).$$

Next, notice that since $\alpha(\cdot)$ is strictly concave, $\alpha'(\cdot)$ is strictly decreasing in its argument. Therefore, its inverse is also strictly decreasing. So, since $R^0(s)$ is decreasing in $q(s)$, $\gamma/R^0(s)>0$, and $\alpha'^{-1}(\cdot)$ is strictly decreasing, we have that $\theta_h^0(s)$ is decreasing in $q(s)$. Finally, since in any open submarket $\rho(\theta)f=\gamma$ and $\rho(\cdot)$ is strictly decreasing, we have that $f_h^0(s)$ is strictly decreasing in $q(s)$. Continuity follows from continuity of $R^0(s)$ on $q(s)$ and by continuity of $\alpha'(\cdot)$.

(ii) Similarly, we can write the first order condition for a type ℓ investors with $a=1$ as

$$[\theta]:\alpha'(\theta)\left\{q(s)-u_\ell-\frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta\left(y',B'\right)\right]I^1\left(s'\right)\right\}=\gamma.$$

So, defining

$$R_\ell^1(s)\equiv q(s)-u_\ell-\frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta\left(y',B'\right)\right]I^1\left(s'\right),$$

which is positive and increasing in $q(s)$, we can find that the optimal submarket choice is given by

$$\theta_\ell^1(s)=\alpha'^{-1}\left(\frac{\gamma}{R_\ell^1(s)}\right).$$

Using similar arguments than in (i) since $R_\ell^1(s)$ is strictly increasing in $q(s)$ we get that $\theta_\ell^1(s)$ is strictly increasing in $q(s)$ and so is $f_\ell^1(s)$. Continuity follows from continuity of $R_\ell^1(s)$ on $q(s)$ and by continuity of $\alpha'(\cdot)$.

(*iii*) Finally, using the definition

$$R_h^1(s) \equiv q(s) - u_h - \frac{1}{1+r}\mathbb{E}_{y'|y}\left[1-\delta\left(y',B'\right)\right] I^1(s'),$$

we can write the first order condition for a type h investors with $a=1$ as

$$[\theta] : \alpha'(\theta) R_h^1(s) = \gamma.$$

So, we have that

$$\theta_h^1(s) = \alpha'^{-1}\left(\frac{\gamma}{R_h^1(s)}\right),$$

and noticing that $R_h^1(s)$ is strictly increasing in $q(s)$ and similar arguments as before we get the proposed result.

□

Proof for Lemma 3

Lemma 3 is an important (partial equilibrium) intermediate result that characterises the slope of the inverstor's net demand for bonds as a function of the bond price, for given government policy functions. This result allows me to establish the existence of a unique market clearing price for each state of the economy and for given government policies in proposition 1. These results show we can find the bond price schedule face by the government from condition (11).

For each s, I define investors' aggregate net demand for bonds as

$$\begin{aligned} ND(s) \equiv & \underbrace{\alpha\left(\theta_h^0(s)\right)\left[\overline{I}-(1-\lambda)B\right]}_{\text{Buyers' demand}} - \underbrace{\alpha\left(\theta_\ell^1(s)\right)(1-\lambda)\zeta B}_{\text{Sellers' supply}} \\ & \underbrace{-\alpha\left(\theta_h^1(s)\right)(1-\lambda)(1-\zeta)B}_{\text{Potential type } h \text{ sellers}}. \end{aligned} \tag{A.4}$$

I now show that it is decreasing and continuous in $q(s)$, for all $s \in \overline{S}$. The functions $\tilde{q}_1(s)$ and $\tilde{q}_2(s)$ are defined as in (A.2) and (A.3), respectively.

Lemma 3. *For any $s \in \overline{S}$, and given a government's default policy function $\delta(x)$, investors' aggregate net demand defined in (A.4) is continuous and decreasing in $q(s)$. Moreover, if $\tilde{q}_1(s) < \tilde{q}_2(s)$ it is strictly decreasing for all $q(s) \in \mathbb{R}_+$. And, if $\tilde{q}_1(s) \geq \tilde{q}_2(s)$ it is constant for all $q(s) \in [\tilde{q}_2(s), \tilde{q}_1(s)]$ and strictly decreasing for all $q(s)$ in the complement of this set in $\mathbb{R}_+$.*

Proof. First, notice that for any given $s=(y,B,B')$, $ND(s)$ is continuous since $\alpha(\cdot)$ is continuous by assumption and by proposition 2 we know that $\theta_h^0(s)$ and $\theta_\ell^1(s)$ are also continuous in $q(s)$. In addition, notice in the case in which

$$\tilde{q}_1(s) \equiv u_\ell + \frac{1}{1+r}E^1(s) + \gamma < u_h + \frac{1}{1+r}E^0(s) - \gamma \equiv \tilde{q}_2(s),$$

from proposition 1 there is always at least one type of investors participating in secondary markets, so from proposition 2 it follows that $ND(s)$ is strictly decreasing in $q(s)$. This is because $\alpha(\cdot)$ is a strictly increasing function, θ_h^0 is strictly decreasing in $q(s)$ so buyers' demand is strictly decreasing in $q(s)$, and because $\theta_\ell^1(s)$ and $\theta_h^1(s)$ are strictly increasing in $q(s)$ so then the negative of sellers' supply is strictly decreasing. In the case in which

$$\tilde{q}_1(s) \geq \tilde{q}_2(s),$$

there is always at least one type of investor participating in secondary markets as long as $q(s) \notin [\tilde{q}_2(s), \tilde{q}_1(s)]$, so from proposition 2 it follows that $ND(s)$ is strictly decreasing in $q(s) \notin [\tilde{q}_2(s), \tilde{q}_1(s)]$. When, $q(s) \in [\tilde{q}_2(s), \tilde{q}_1(s)]$, from proposition 1 we know that there are no investors participating in the secondary market. Therefore, $ND(s) = 0$ and constant for the whole interval. □

A.2 Market Clearing Price

I use lemma 3 to show that for each s and given a government's default policy function $\delta(y,B)$, there is a unique price that is consistent with market clearing. Then, I characterize the pricing schedule faced by the government. Throughout this subsection I denote $B(s)$ and $B'(s)$ the second and third components of $s = (y, B, B')$, respectively.

For each s and any given price q, define the excess demand function for bonds in the primary market as

$$ED(s;q) \equiv ND(s;q) - \left[\max\left\{B'(s), 0\right\} - (1-\lambda)B(s)\right], \tag{A.5}$$

with $ND(s;q)$ defined as in (A.4). Define $\tilde{q}_1(s)$ and $\tilde{q}_2(s)$ as in (A.2) and (A.3).

Proposition 1. *For any policy function $\delta(x)$ and any $s \in \overline{S}$ such that $B'(s) > 0$, if $\tilde{q}_1(s) < \tilde{q}_2(s)$ there is a unique price $q(s) \in \mathbb{R}_+$ consistent with*

$$q(s)ED(s;q(s)) = 0. \tag{A.6}$$

Moreover, either $q(s) > 0$ and $ED(s;q(s)) = 0$, or $q(s) = 0$ and $ED(s;q(s)) \leq 0$. In addition, when $\tilde{q}_1(s) \geq \tilde{q}_2(s)$ the result still holds except when $B'(s) = (1-\lambda)B(s)$, in which case any price within $[\tilde{q}_2(s), \tilde{q}_1(s)]$, is consistent with $q(s)ED(s;q(s)) = 0$.

Proof. Consider first the case in which s is such that, $ND(s;0) \leq B'(s) - (1-\lambda)B(s)$. Then, since $q \geq 0$ and from lemma 3 $ND(s;q)$ is decreasing in q, either $ND(s;0) = \max\{B'(s), 0\} - (1-\lambda)B(s)$ or there is no price such that $ND(s,q) = B'(s) - (1-\lambda)B(s)$. Therefore, for any q there is an excess supply of bonds in the primary market (i.e. $ED(s;q) \leq 0$). Thus, the unique price consistent with (A.6) is $q(s) = 0$.

Next, consider the case in which s is such that $ND(s;0) > B'(s) - (1-\lambda)B(s)$. Here we have two cases. First, if

$$\tilde{q}_1(s) < \tilde{q}_2(s),$$

from proposition 3 we know that $ND(s;q)$ is strictly decreasing in q, for all $q \in \mathbb{R}_+$. Then we just need to increase the price until $ND(s;q) = B'(s) - (1-\lambda)B(s)$. The second case, is the case in which

$$\tilde{q}_1(s) \geq \tilde{q}_2(s).$$

Now, if $ND(s,0) > B'(s) - (1-\lambda)B(s) > 0$, from proposition 1 we know that for any $q \in [0, \tilde{q}_2(s)]$ $ND(s)$ is strictly decreasing and continuous, and also we know that $ND(s, \tilde{q}_2(s)) = 0$, since above that price no type h investor with $a = 0$ is participating in secondary markets. Thus, by the intermediate value theorem, there must exist a price between 0 and $\tilde{q}_2(s)$ such that $ED(s, q(s)) = 0$. A similar argument applies if $B'(s) - (1-\lambda)B(s) < 0$. In this case we will find a price above $\tilde{q}_1(s)$ such that $ED(s, q(s)) = 0$. The only case that is a little more subtle is the case in which $B'(s) = (1-\lambda)B(s)$. Here, we have that government supply is zero. In addition, we know that for any $q(s) \in [\tilde{q}_2(s), \tilde{q}_1(s)]$, secondary markets shut down, so $ND(s, q(s)) = 0$. In this case, $ED(s, q(s)) = 0$ for any $q(s) \in [\tilde{q}_2(s), \tilde{q}_1(s)]$. This multiplicity arises because the government is not trying to sell or buy bonds, and investors have no incentives to participate in secondary markets. □

The price schedule faced by a government, conditional on a given policy function $\delta(x)$, can be charaterized as in Corollary 1.

Corollary 1. *The price schedule faced by a government conditional on a given policy function $\delta(x)$ is given by*

$$q(s) = \begin{cases} \{p \in \mathbb{R}_+ : ED(s;p) = 0\} & \text{if } ND(s;0) > B'(s) - (1-\lambda)B(s) \\ 0 & \text{if } ND(s;0) \leq B'(s) - (1-\lambda)B(s) \end{cases}. \tag{A.7}$$

Proof. Directly follows from the previous results. □

The price schedule in (*A.7*) replaces the standard no-arbitrage condition usually found in the literature of sovereign default. Proposition 1 states that the price that clears the primary market for sovereign bonds is unique, except for the knive edge case in which neither the government nor the dealers participate in the primary market. To be more precise, this uniqueness statement is conditional on a given government's default policy function, investors value functions, and future expected prices. However, the result highlights the parallelism of the pricing schedule to the standard no-arbitrage condition that maps future prices and a default policy function into current prices. In this sense, solving this model is not harder than other models of sovereign default. Instead of having a closed form expression for the price as in the standard no arbitrage condition, I need to find the price consistent with (*A.6*).

A.3 Amplification in two periods example

Proposition 2. *Under the assumptions of section 3.2 the bond price elasticity with respect to $1-\delta$ is larger than 1.*

Proof. I proceed using the implicit function theorem on the market clearing condition for bonds defined by $F = 0$, with defined as:

$$F = \frac{H}{\left(-\frac{q}{1-\delta} + \frac{U_h}{1+r}\right)^{1/2}} - \frac{L}{\left(\frac{q}{1-\delta} - \frac{U_\ell}{1+r}\right)^{1/2}} - \frac{M}{\gamma^{1/2}}(1-\delta)^{1/2}$$

Then taking the derivative of F withrespect to $(1-\delta)$ I obtain:

$$\frac{\partial F}{\partial(1-\delta)} = \frac{-H\frac{q}{(1-\delta)^2}}{2\left(-\frac{q}{1-\delta}+\frac{U_h}{1+r}\right)^{3/2}} + \frac{L\frac{q}{(1-\delta)^2}}{2\left(\frac{q}{1-\delta}-\frac{U_\ell}{1+r}\right)^{3/2}} - \frac{M}{2\gamma^{1/2}}(1-\delta)^{-3/2}$$

Similarly taking the derivative of F with respect to q I obtain:

$$\frac{\partial F}{\partial q} = \frac{H\frac{1}{(1-\delta)}}{2\left(-\frac{q}{1-\delta}+\frac{U_h}{1+r}\right)^{3/2}} - \frac{L\frac{1}{(1-\delta)}}{2\left(\frac{q}{1-\delta}-\frac{U_\ell}{1+r}\right)^{3/2}}$$

Therefore by the implicit function theorem it is the case that:

$$\frac{\partial q}{\partial(1-\delta)} = -\frac{\frac{\partial F}{\partial(1-\delta)}}{\frac{\partial F}{\partial q}} = -\frac{\frac{-H\frac{q}{(1-\delta)^2}}{2\left(-\frac{q}{1-\delta}+\frac{U_h}{1+r}\right)^{3/2}} + \frac{L\frac{q}{(1-\delta)^2}}{2\left(\frac{q}{1-\delta}-\frac{U_\ell}{1+r}\right)^{3/2}}}{\frac{H\frac{1}{(1-\delta)}}{2\left(-\frac{q}{1-\delta}+\frac{U_h}{1+r}\right)^{3/2}} - \frac{L\frac{1}{(1-\delta)}}{2\left(\frac{q}{1-\delta}-\frac{U_\ell}{1+r}\right)^{3/2}}} + \frac{\frac{M}{2\gamma^{1/2}}(1-\delta)^{-3/2}}{\frac{H\frac{1}{(1-\delta)}}{2\left(-\frac{q}{1-\delta}+\frac{U_h}{1+r}\right)^{3/2}} - \frac{L\frac{1}{(1-\delta)}}{2\left(\frac{q}{1-\delta}-\frac{U_\ell}{1+r}\right)^{3/2}}}$$

This expression simplifies to :

$$\frac{\partial q}{\partial(1-\delta)} = \frac{q}{(1-\delta)} + \frac{\frac{M}{2\gamma^{1/2}}(1-\delta)^{-3/2}}{\frac{H\frac{1}{(1-\delta)}}{2\left(-\frac{q}{1-\delta}+\frac{U_h}{1+r}\right)^{3/2}} - \frac{L\frac{1}{(1-\delta)}}{2\left(\frac{q}{1-\delta}-\frac{U_\ell}{1+r}\right)^{3/2}}}$$

Defining

$$Z \equiv \frac{H\frac{1}{(1-\delta)}}{2\left(-\frac{q}{1-\delta}+\frac{U_h}{1+r}\right)^{3/2}} - \frac{L\frac{1}{(1-\delta)}}{2\left(\frac{q}{1-\delta}-\frac{U_\ell}{1+r}\right)^{3/2}} > 0,$$

and multiplying both sides by $\frac{1-\delta}{q}$ we can get the expression in terms of the elasticity:

$$\frac{1-\delta}{q}\frac{\partial q}{\partial(1-\delta)} = 1 + \frac{\frac{M}{2\gamma^{1/2}}(1-\delta)^{-1/2}}{Zq}$$

Since $\frac{\frac{M}{2\gamma^{1/2}}(1-\delta)^{-1/2}}{Zq} > 0$ it is the case that:

$$\frac{1-\delta}{q}\frac{\partial q}{\partial(1-\delta)} > 1$$

Which concludes the proof. $\square$

B Solution Algorithm

It is well known that long-term debt models have convergence issues. To obtain convergence, I follow the literature and solve an approximate model that adds preference shocks to the default debt issue choices of the government as in Dvorkin et al. (2021) and Gordon (2019). Preference shocks are as small as needed to get convergence and do not significantly change government optimal choices. I describe the approximate model and the solution algorithm in appendix B.1. Appendix B.2 describes how simulations are computed.

B.1 The Approximate Model

I first show the equations of the approximate model that I solve. As in Dvorkin et al. (2021) and Gordon (2019), the problem of the government consist on a discrete choice model with preference shocks for default and repayment, as well as preference shocks for each choice of debt for next period B'. Conditional on repayment, the sovereign chooses a level of debt for next period B' from a discrete set with N_b options $\mathcal{B} \equiv \{B_1, B_2, ..., B_{N_b}\}$. The problem of the government is given by

$$
\begin{aligned}
V(y,B,\varepsilon) &= \max_{\delta \in \{0,1\}} \Big\{ (1-\delta) V^R(y,B,\varepsilon) + \delta V^D(y,\varepsilon_D) \Big\} \\
V^R(y,B,\varepsilon) &= \max_{B' \in \mathcal{B}} u(c) + \varepsilon_{n_{B'}} + \beta \mathbb{E}_{y'|y} \mathbb{E}_{\varepsilon'} \Big[V(y',B',\varepsilon') \Big] \\
s.t.: c &= y - [\lambda + (1-\lambda) z] B + q(y,B,B') \Big[B' - (1-\lambda) B \Big] \\
V^D(y,\varepsilon_D) &= u(h(y)) + \varepsilon_D + \beta \mathbb{E}_{y'|y} \mathbb{E}_{\varepsilon'} \Big\{ (1-\theta) \Big[V^D(y',\varepsilon'_D) \Big] + \theta \Big[V(y',0,\varepsilon') \Big], \Big\}
\end{aligned}
$$

where $\varepsilon \equiv \{\varepsilon_1, \varepsilon_2, ..., \varepsilon_{N_b}, \varepsilon_D\}$ is a vector with $N_b + 1$ specifying a utility value for defaulting, and for each choice of next period debt B'. It is assumed that ε is $i.i.d$ over time and has the following CDF

$$
F(\varepsilon) = \exp\left[- \left(\sum_{n_{B'}=1}^{N_b} \exp\left(-\frac{\varepsilon_{n_{B'}} - \mu}{\sigma} \right) \right) - \exp\left(-\frac{\varepsilon_D - \mu}{\sigma} \right) \right],
$$

where μ is the mean of the shocks and σ is the variance of the shocks. This is the Generalized Extreme Value distribution pioneered by McFadden (1978). What this means, is that if the government needs to choose a level of debt for the next period it would choose element $n^* \in 1, ..., N_b$ if the utility derived from B'_{n^*} is the largest among all choices.

For a given price schedule, I can determine the optimal bond supply from the government and default decisions. To compute the price of the bond we need to compute the net demand of bonds from investors that dealers channelize to the primary market. To compute investors' demand and supply of bonds we need to solve their respective problems. For each state $x = (y, B)$ at the beginning of the period and $s = (x, B')$, which is the information needed by an investor[27], the value of an investor without bonds is

$$
\begin{aligned}
I^0(s) &= \max_{f\in\bar{F}}\{\alpha(\theta(f))\left[-q(s)-f+u_h+\frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta(y',B',\varepsilon')\right]I^1(s')\right] \\
&\quad+\frac{1-\alpha(\theta(f))}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta(y',B',\varepsilon')\right]I^0(s')\}.
\end{aligned}
$$

The value of holding a bond at the beginning of the period is,

$$
I^1(s) = \lambda+(1-\lambda)\left[z+\zeta J_\ell+(1-\zeta)J_h\right].
$$

where the values $J_i, i\in\{\ell,h\}$ are given by

$$
\begin{aligned}
J_\ell &= \max_{f\in\bar{F}}\left\{\alpha(\theta(f))\left[q(s)-f\right]+\left[1-\alpha(\theta(f))\right]\left[u_\ell+\frac{\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta(y',B',\varepsilon')\right]I^1(s')}{1+r}\right]\right\} \\
J_h &= \max_{f\in\bar{F}}\left\{\alpha(\theta(f))\left[q(s)-f\right]+\left[1-\alpha(\theta(f))\right]\left[u_h+\frac{\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta(y',B',\varepsilon')\right]I^1(s')}{1+r}\right]\right\}
\end{aligned}
$$

where $s'=(y',B',B''(y',B',\varepsilon'))$.

In each s, the optimal submarket choices for each type of investor are given by

$$
\begin{aligned}
\theta^0(s) &= \alpha'^{-1}\left(\frac{\gamma}{-q(s)+u_h+\frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta(y',B',\varepsilon')\right]I^1(s')}\right)\times\mathbb{I}^0\{active\}, \\
\theta^1_\ell(s) &= \alpha'^{-1}\left(\frac{\gamma}{q(s)-u_\ell-\frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta(y',B',\varepsilon')\right]I^1(s')}\right)\times\mathbb{I}^0_\ell\{active\} \\
\theta^1_h(s) &= \alpha'^{-1}\left(\frac{\gamma}{q(s)-u_h-\frac{1}{1+r}\mathbb{E}_{y',\varepsilon'|y}\left[1-\delta(y',B',\varepsilon')\right]I^1(s')}\right)\times\mathbb{I}^0_h\{active\},
\end{aligned}
$$

where $\mathbb{I}^a_i\{active\}$ is an indicator function that takes value equal one when that type of investor is active in the secondary market. Investors are active in the secondary market whenever the gains from trade can at least compensate the entry cost for dealers, γ. Finally, knowing the optimal submarket choices of investors, we can determine the fraction of each type of investor that meets a dealer in the secondary market and compute the excess demand function in the primary market as

$$
\begin{aligned}
ED(s) \equiv{}& \alpha\left(\theta^0_h(s)\right)\left[\bar{I}-(1-\lambda)B\right]-\left[\max\left\{B',0\right\}-(1-\lambda)B\right] \\
&-\alpha\left(\theta^1_\ell(s)\right)(1-\lambda)\zeta B-\alpha\left(\theta^1_h(s)\right)(1-\lambda)(1-\zeta)B.
\end{aligned}
$$

I can pin down the unique price that is consistent with $q(s)ED(s)=0$. These steps provide me with an algorithm to solve the model. This algorithm begins with a guess of the price schedule, updates the policy functions of the government, solves for investors optimal sub-market choice as a function of price $q(s)$ in each state $s\in\overline{S}$, and finally finds the price $q(s)$

[27] Note that investors do not need to know the preference shock because they observe the choices of the government and the preference shocks for the government are *i.i.d.* over time.

consistent with zero excess demand in primary markets, for each $s \in \overline{S}$. Then, I used the resulting price schedule as a new guess and iterate until all equilibrium objects converge.

B.2 Model Simulation

I simulate the model over $T = 500,000$ periods. Then, I burn the $T_1 = 1,000$ initial periods. Find all episodes of length $T = 69$ periods where the 69th period is a default episode and none of the previous 99 periods are default periods. I discard the first $T_0 = 30$ periods and keep 69 periods before default. Length T is chosen to be 69 because I use data on Greek GDP from 1995Q1 and default happens 68 quarters later in $2012Q1$. Since after re-gaining access to international financial markets the government re-enters with $B = 0$, I choose to discard T_0 periods before the beginning of each replica of the economy so that I let the model reach the ergodic set. Then, I compute the summary statistics for each of them. Finally, I average over these episodes and report the averages of the moments.

C Data Description

I collect the following time series for Greek economy.

National Accounts. Quarterly time series from 1995Q1 - 2012Q4 for consumption, exports, imports, and GDP are obtained from Eurostats. I use seasonal adjusted and calendar adjusted chain-linked (2010) in million euros.

Debt and Investment Position. Data on net international investment position (as % of GDP) for the period 2003Q4 - 2012Q4 is obtained from Eurostat. Public and Publicly Granted debt (as % of GDP) for the period 2000Q1 - 2012Q4 is obtained from the World Development Indicators database from the World Bank.

Interest Rates and Spreads. Interest rates data is collected from Eurostats and Bloomberg. Interest rate spreads is calculated as the difference in the annual interest rates between Greek and German long term government yields in Eurostats. Long term debt yields are composed from central government bonds with residual maturity of around 10 years. Computing interest rate spreads using generic central government bonds from Greece and Germany collected from bloomberg results in almost identical time series. Daily bid and ask prices are collected from Bloomberg using generic central government bonds. I compute quarterly time series using average of active days in each quarter. I use time series for bid and ask prices for 10 year bonds. Daily bid and ask prices for 5 year bonds have some missing values but quarterly time series results in almost identical bid-ask spreads as for 10-year bonds.

Secondary Market Volumes and Turnover Rates. Information on secondary market trade volumes is obtained from the electronic secondary securities market (HDAT) available at the Bank of Greece website.[28] Monthly traded volumes is available from January 2001 to December 2012. Quarterly time series are calculated as the sum of monthly traded volumes.

[28]https://www.bankofgreece.gr/Pages/en/Markets/HDAT/statistics.aspx

D Details on the Calibration

D.1 Calculating utility from holding bonds

I calibrate preferences bonds u_h and u_ℓ such that they satisfy to conditions: (i) that ex-ante expected utility from holding the asset is zero for a given ζ, and (ii) such that $u_h - u_\ell$ targets the average bid-ask spread in the data. Condition (i) imposes the following restriction

$$\begin{aligned} 0 &= u_h + \sum_{j=1}^{\infty} \left\{ \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)(1-\zeta) \right]^j u_h + \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)\zeta(1-\alpha(\theta_{\ell,j})) \right]^j u_\ell \right\} \\ &= \sum_{j=0}^{\infty} \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)(1-\zeta) \right]^j u_h + \sum_{j=1}^{\infty} \left[\frac{\Pr\{\delta_j = 0\}}{1+r} (1-\lambda)\zeta(1-\alpha(\theta_{\ell,j})) \right]^j u_\ell. \end{aligned}$$

Although I use the constraint above to calibrate the model, we can develop further intuition assuming that the probability of default is more or less constant around the targeted average default probability $\bar{\delta}$. In such case, the previous expression is approximately equal to

$$\begin{aligned} 0 &\approx \sum_{j=0}^{\infty} \left[\frac{1-\bar{\delta}}{1+r} (1-\lambda)(1-\zeta) \right]^j u_h + \sum_{j=1}^{\infty} \left[\frac{1-\bar{\delta}}{1+r} (1-\lambda)\zeta(1-\bar{\alpha}) \right]^j u_\ell \\ &= \frac{(1+r)\, u_h}{1+r-(1-\bar{\delta})(1-\lambda)(1-\zeta)} + \frac{(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})u_\ell}{1+r-(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})} \\ \Longrightarrow & \frac{(1+r)}{1+r-(1-\bar{\delta})(1-\lambda)(1-\zeta)} u_h \approx -\frac{(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})}{1+r-(1-\bar{\delta})(1-\lambda)\zeta(1-\bar{\alpha})} u_\ell, \end{aligned}$$

where $\bar{\alpha} \equiv \alpha(\theta_{\ell,j} | \Pr[\delta = 1] = \bar{\delta})$ is the probability of matching for a low type investor conditional on a default probability equal to $\bar{\delta}$, which would also be a constant if default risk is constant.

D.2 Computing Turnover Rates

The turnover rate is 78% in HDAT in Greece is per quarter. This includes transactions between dealers and investors as well as interdealer transactions. To calculate the turnover rate in the model we have to compute all the transactions that happen in secondary markets between dealers and investors and also interdealers transactions in primary markets. This is not exactly what happens in reality as some dealers hold inventories and do not need to trade with other dealers and some other trades occur through a long chain of dealer to dealer transactions. We will assume that the number of transactions in the model approximates the amount of transactions in the data. In the model, we can calculate the amounts of transactions in both the primary and secondary market as follows. In the primary market, whenever the government issues debt it is purchased by a dealer. Then, the number of transactions in the primary market is given by

$$\max\left\{B', 0\right\} - (1-\lambda) B.$$

Now, in the secondary markets, the following trades occur between a dealer and an investor:

$$\begin{aligned}
\ell-type \text{ investors to dealer} &: \alpha\left(\theta_\ell^1\right)\zeta\left(1-\lambda\right)B \\
\text{Potential } h-type \text{ investors to dealer} &: \alpha\left(\theta_h^1\right)\left(1-\lambda\right)\left(1-\zeta\right)B \\
\text{Dealer to old buyer} &: \alpha\left(\theta^0\right)\left(I-B\right) \\
\text{Dealer to new buyer} &: \alpha\left(\theta^0\right)\lambda B.
\end{aligned}$$

Since we do not model the amounts of trades in the interdealers market this is the minimum amount of transactions in secondary markets. The maximum amount of transactions adds to these transactions the maximum amount of possible trades between dealers in the interdealers market, so we have to clean the turnover rate in the data to remove those trades. This is the sum of the following trades.

$$\begin{aligned}
\text{Primary buyers to selling dealers} &: \max\left\{B',0\right\}-\left(1-\lambda\right)B \\
\text{Secondary buyers to selling dealers} &: \alpha\left(\theta_\ell^1\right)\zeta\left(1-\lambda\right)H_1+\alpha\left(\theta_\ell^1\right)\left(1-\lambda\right)L_1\alpha\left(\theta_h^1\right)\left(1-\lambda\right)\left(1-\zeta\right)H_1.
\end{aligned}$$

Since primary market clears we can simplify this calculation using total purchases by investors,

$$\text{Total investors' purchases}: \alpha\left(\theta_h^0\right)H_0+\alpha\left(\theta_h^0\right)\lambda B=\alpha\left(\theta_h^0\right)\left[\bar{I}-\left(1-\lambda\right)B\right].$$

Then, the maximum amount of transactions in secondary markets, given that interdealers market is competitive, is given by

$$X^{\max}=2\left[\alpha\left(\theta_h^0\right)H_0+\alpha\left(\theta_h^0\right)\lambda B\right]+\alpha\left(\theta_\ell^1\right)\zeta\left(1-\lambda\right)H_1+\alpha\left(\theta_\ell^1\right)\left(1-\lambda\right)L_1+\alpha\left(\theta_h^1\right)\left(1-\lambda\right)\left(1-\zeta\right)H_1.$$

In reality the minimum amount of trades in secondary markets is given by

$$X^{\min}=\alpha\left(\theta_h^0\right)H_0+\alpha\left(\theta_h^0\right)\lambda B+\alpha\left(\theta_\ell^1\right)\zeta\left(1-\lambda\right)H_1+\alpha\left(\theta_\ell^1\right)\left(1-\lambda\right)L_1+\alpha\left(\theta_h^1\right)\left(1-\lambda\right)\left(1-\zeta\right)H_1.$$

This happens when the same dealer is connecting both the investor selling and the investor buying and just acting as a bridge. As mentioned before, the number of transactions in reality could be lower or higher than X^{SM} as long chains of dealers would be require to transfer one bond from an investor to another one. Li and Schurhoff (2018) find that for municipal bonds in United States, the average chain involves 1.5 dealers. So, we can compute an intermediate amount of trades in secondary markets as

$$X^{mean}=1.5\left[\alpha\left(\theta_h^0\right)H_0+\alpha\left(\theta_h^0\right)\lambda B\right]+\alpha\left(\theta_\ell^1\right)\zeta\left(1-\lambda\right)H_1+\alpha\left(\theta_\ell^1\right)\left(1-\lambda\right)L_1+\alpha\left(\theta_h^1\right)\left(1-\lambda\right)\left(1-\zeta\right)H_1.$$

We will use X^{mean} to compute the model's implied turnover rate as

$$\text{Turnover rate}=\frac{X^{mean}}{B}.$$

D.3 Alternative values for $\bar{I}$

This appendix shows that increasing the parameter $\bar{I}$ does not significantly change the results compared to the baseline calibration. This is because the amount of potential demand $\bar{I}$ is sufficiently large in the baseline calibration and it does not bind in any significant way the amount of debti issued by the government in equilibrium. If course, reducing the parameter $\bar{I}$ can affect the results as debt issuance might start being bound by the potencial world demand for sovereign bonds. Table D.1 shows how the main moments change as we change $\bar{I}$. For larger values of $\bar{I}$ a small re-calibration of the model would restore the baseline moments.

Table D.1: The role of secondary market's trade frictions and flows

Moments	Baseline ($\bar{I} = 5$)	$\bar{I} = 7.5$	$\bar{I} = 10$	$\bar{I} = 15$
Mean bond spread (%)	3.42	3.38	3.37	3.37
Std. dev. bond spread (%)	2.18	2.17	2.17	2.18
Debt to output (%)	124	125	126	126
Mean bid-ask spread (bps.)	76	76	76	75
Bonds turnover rate (%)	78	78	78	78

Note: The table shows the values for the targeted moments as we increase the parameter determining the potential demand for sovereign bonds $\bar{I}$.