Interest Rate Shocks and the Composition of Sovereign Debt

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Abstract

There has been a growing concern about the vulnerability of emerging countries to fluctuations in international interest rates. Here, I show that the ability of countries to issue debt domestically can mitigate this vulnerability. The model integrates a domestic banking sector into a sovereign default model where governments can issue domestic and external debt and selectively default. Issuing domestic debt crowds out investment, but as financial markets develop, the share of domestic debt increases. The quantitative analysis is consistent with two novel facts: less financially developed countries issue relatively more external debt and are more vulnerable to external shocks.

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1 Introduction

A large body of empirical evidence shows that emerging countries suffer significant output drops when international interest rates increase.[1] This paper asks why do countries choose to expose themselves to such suffering by borrowing large amounts of external debt. By doing so, an increase in the international interest rate raises the amount of debt that these countries owe to their foreign creditors which in turn has negative effects on their economies. An obvious alternative is to eschew such foreign debt and instead borrow from domestic agents. The thesis of the paper is that this alternative has costs because domestic debt crowds out investment. Hence, governments choose to expose themselves to international interest rate fluctuations in a way that optimally balances the costs from such exposure from external debt against the crowding-out costs of domestic debt.

These trade-offs imply that the vulnerability of countries to fluctuations in the international interest rate depends critically on the government's choice of domestic versus external debt and on how easily they can raise funds domestically. Since the literature mostly abstracts from the choice of domestic versus foreign debt, I fill this void by developing a model that focuses on the decision of how a country finances its debt, show that it can account for the key patterns of the data, and use it to conduct counterfactual exercises. The main result of the paper is that it shows that financial development is crucial in explaining the different degrees of vulnerability to external shocks that we see in the data among emerging countries. In particular, an important feature of the data documented here is that less-financially developed countries tend to both finance a large fraction of their debt with foreigners and suffer larger losses in output for a given increase in international interest rates. The model is able to capture this crucial pattern of the data and is used to understand the forces behind it.

To analyze how governments balance the costs and benefits of domestic and external debt, the model has to include two key ingredients that highlight the costs and benefits of doing so. The first one is that it introduces a crowding-out effect of domestic debt stemming from a friction in the way that financial intermediaries, namely banks, can raise funds from domestic agents. This creates a cost of domestic debt relative to external debt. Intuitively, the degree of such frictions in the economy determines the level of financial development of the country: the larger these frictions are, the more difficult it is for countries to raise funds domestically. Given the scarce amount of funds that banks can borrow from domestic consumers when governments increase their sales of domestic bonds, which are held precisely by those banks, investment in domestic capital by banks is crowded out. The second feature of the model is that, as in the data, governments can default on their debt. Moreover, as shown in Reinhart and Rogoff (2011) they often do so *selectively*, that is, they discriminate between domestic and external debt when they choose to default. In the sample of emerging countries analyzed here, defaults only on external debt represent around 70% of all

[1] See, for instance, Neumeyer and Perri (2005), Canova (2005), Uribe and Yue (2006), Maćkowiak (2007), Banerjee, Devereux, and Lombardo (2016), Dedola, Rivolta, and Stracca (2017) , Iacoviello and Navarro (2019) for papers that study the effects of increases in the U.S. interest rates on other developed countries and on emerging countries.

the default events. Despite this evidence, existing models in the literature either abstract from the distinction between domestic and foreign debt or assume non-discriminatory default. In the model proposed in this paper, I allow governments to decide every period, not only whether to default or repay as is standard in sovereign default models, but also on what type of debt they choose to default. Governments take into account that defaulting on domestic debt directly hurts their own citizens so, for similar amounts of debt, they endogenously prefer to default on external rather than on domestic debt. This, in turn, gives an advantage to domestic debt relative to foreign, as it lowers the risk premium governments have to pay on debt.

The quantitative results show that the model is successful in accounting for the drop in output in emerging countries after a shock that increases international interest rates as in the data. In addition, by changing a parameter in the model that controls the degree of financial development of the economy, the model can account for the heterogeneity in the share of domestic sovereign debt across emerging countries as well as for the differences in vulnerability to external shocks in such countries. An additional result from the model is that, by allowing selective default, it generates the pattern of default observed in the data, that is, countries tend to default most of the time only on external debt.

In order to understand the vulnerability of emerging countries to shocks to the world interest rate, the first thing that the model has to rationalize is why emerging countries suffer output losses after an increase in the world interest rates. In the model, this happens due to the increase in the cost of financing government expenditures. Every period, governments must decide how to finance their exogenously given stream of expenditures: issuing external debt, domestic debt, or using taxes on labor income. After an increase in the international interest rate, external debt becomes more expensive. In this situation, the government could do any of the following. First, it could keep the same amount of borrowing and taxes, but given the increase in interest rate, that would imply higher taxes in the future to repay the debt, which through its distortionary effect on labor would decrease future output. Second, it could substitute the now more expensive external borrowing with higher taxes. Similarly, this would have a negative effect on today's output. And, third, it could substitute external borrowing with domestic borrowing. However, domestic debt reduces the resources that domestic banks have to invest in capital, that is it crowds out investment. This turns into lower future capital and thus lower future output. In any of these cases, regardless of what the government chooses to do, the economy suffers a decrease in output after the increase in international interest rate.

These forces explain why emerging countries experience output losses due to external shocks. But what determines the degree of vulnerability to these external shocks? I show that financial development is key in explaining this vulnerability. In the model, a higher level of financial development allows governments to finance a larger share of their expenditures using domestic debt. This implies that when an interest rate shock hits the economy and makes external debt more expensive, countries with higher financial development are less exposed to these changes. First, because they do not rely so heavily on external debt, and second, because the crowding out effects of substituting external debt with domestic debt are not as large as in countries with a low level of financial development.

I document two new empirical findings for a panel of 14 emerging countries that motivate the main exercise in the paper. I first show a significant positive relationship between the share of domestic debt in a country and its financial development measured as the ratio of liquid liabilities over GDP. This measure of financial development quantifies the degree to which financial institutions in a country can raise liquid funds in the economy. I find that a higher level of financial development is associated with a larger share of domestic sovereign debt. Intuitively, higher financial development decreases the crowding out costs and allows governments to issue a higher fraction of their debt domestically. Second, following this argument, it should be true that when countries are less financially developed, they find it more costly to substitute external debt with domestic debt, which makes them more vulnerable to a shock that increases the world interest rate. In the data, I show that this is indeed what happens: after a shock that increases the U.S. interest rate by 100 basis points, countries that are less financially developed and, therefore, have a larger share of external debt suffer on average a 0.6 percent decrease in output. However, when countries are more financially developed, their output drop is only 0.2 percent.

Now, I turn to elaborate on the main forces in the model. Two ingredients are crucial for explaining the results of the model. First, the model needs to incorporate a clear concept of financial development, in the sense that domestic banks that act as the financial intermediaries in the economy are constrained on the resources that they can obtain. I model these constraints in the tradition of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) in that banks have limited access to households' savings due to an agency problem. I assume that the level of these frictions, captured by a parameter in the collateral constraint reflecting the enforceability of contracts, represents the financial development of the country. The idea is that in less financially developed countries, the ability to enforce contracts is weak in that banks suffer only a small punishment when they break financial contracts such as repaying their depositors. Hence, in such countries, consumers are willing to lend to banks relatively small amounts of resources, the ratio of deposits to output is relatively low, and banks can fund relatively small amounts of investment. As countries become more developed in their financial markets, these costs are lowered, which translates into an increase in the amount of resources that banks can borrow. Banks then use these resources to invest in capital and government bonds.[2]

Second, in the model, consistent with the data, governments can default selectively between domestic and foreign creditors. Differences in the costs of default between these types of debt affect the relative probability of default and cost of debt. In the model, the costs of default are a combination of the standard exogenous costs of default, adopted from the sovereign default literature and, criti-

[2]This mechanism is in line with empirical evidence that shows the relation between domestic debt held by banks and a crowding-out effect on corporate lending from these banks. See, for instance, Becker and Ivashina (2018).

cally, the endogenous cost of defaulting on domestic debt in a model with financial intermediation. These exogenous costs are that upon default on either type of debt, countries experience an output loss and enter an autarky state in which they do not have access to that type of debt market for a random number of periods. To focus attention on the differential endogenous cost of defaulting on domestic and external debt, these exogenous costs are assumed equal across the two types of default. The differential endogenous cost naturally arises because when the government defaults on domestic debt, it defaults on its citizens rather than on foreigners. In particular, a default on domestic debt reduces the net worth of domestic banks and, hence, the amount of resources that they have for investment, which then reduces future output in the economy. This endogenously larger ex-post cost for domestic default, in turn, leads domestic agents to expect that, for the same amounts of domestic and external debt, the government has less of an incentive to default on domestic debt. Therefore, all else equal, this endogenous force tends to lower expected default rates and thus the premium on domestic debt over the world rate relative to that on foreign debt.[3]

Then, I use the model to quantitatively assess the role of domestic debt and financial development in mitigating shocks to international interest rates. Specifically, I assume that external creditors have access to an international risk-free asset, the return on which evolves stochastically, and that is calibrated to match the features of the U.S. Federal funds rate. Critically, the model is able to generate the main features of the data in emerging countries in terms of the characteristics of sovereign debt and default in these countries, and its relation with their level of financial development.

When there is an unexpected increase in the international risk-free rate, the model generates a drop in output similar to what we observe in the data. How does domestic debt mitigate the drop in output? First, I show that in a counterfactual economy where only external debt is allowed, output decreases almost 0.2 percentage points more after an increase in world interest rates than in the baseline economy where the government has access to both, domestic and external debt. This is so because the government responds to the interest rate shock by increasing the share of domestic debt. In the absence of domestic debt, the government needs to raise taxes further to meet its payments. Because labor income taxes are distortionary, they push output to decrease even more. Second, the model predicts that more financially developed economies suffer less from external shocks in line with what happens in the data. I model a country's increase in financial development as an increase in its ability to enforce contracts which manifests itself as a parameter in the collateral constraint that implies a better ability by the government to borrow from domestic residents. As a country's ability to enforce contracts increases, the balance of the relative benefits and relative costs of domestic debt shifts, and the government endogenously chooses to issue a higher share of domestic debt. The quantitative model captures the same patterns as in the data: more financially developed countries tend to have higher shares of domestic debt.

[3] The decrease in net worth and private lending from banks after default is observed in the data as shown in Gennaioli et al. (2018) and Baskaya and Kalemli-Ozcan (2016), which provide evidence that after a domestic default, banks holding government debt experience a large decrease in their net worth and aggregate lending.

Interestingly, the model has three additional implications that are consistent with the evidence in the data that it was not explicitly designed to address. The first feature is that the ratio of domestic to external debt is countercyclical in the data and model. This occurs because, on the margin, banks must be indifferent between investing a unit of their resources in domestic debt and investing a unit in physical capital and lending it to firms. When domestic productivity is high so is the return on capital and, hence, governments must pay a relatively high return on domestic debt. This discourages issuing domestic debt in periods of high productivity. The second feature is related to the frequency of the different types of default. In the data, we observe that most of the time governments default only on external debt, sometimes on both domestic and external, and almost never default only on domestic debt. The quantitative model is able to reproduce these patterns: due to the endogenously different cost of default, default only on external debt is more frequent than default on both types of debt.

The third feature is a pattern of discriminatory default in the data, that I refer to as the *pecking order of default*: in moderate recessions, countries tend to default only on external debt, and only in severe recessions do they default on both domestic and external debt. That the model also generates this feature is connected to the model's implied countercyclicality of the domestic debt share. If a country starts in a relatively productive state, it tends to have a relatively high share of external debt. If following such a state the economy experiences a drop in productivity, given that defaulting on domestic debt is more costly than on external, the government chooses to default only on external debt. If instead, the adverse shock happens during a period when the economy is already experiencing low to moderate productivity, the government tends to have a much higher share of domestic debt. Hence, even if it defaults on external debt it does not generate much revenue from it and, hence, it chooses to default on both types of debt.

Finally, to keep the analysis of the baseline model relatively simple, I abstracted from real exchange rates as another external shock in addition to international interest rates. I show that the model mechanisms and quantitative predictions are robust to the introduction of real exchange rates. This is important because one might worry that the co-movements of international interest rates and real exchange rates affect the attractiveness of each type of debt if we consider that usually domestic debt is denominated in local currency and external debt in foreign currency. Although in the data analysis in this paper, domestic debt is considered as debt issued under domestic jurisdiction and external debt as issued under foreign jurisdiction, in general domestic-law debt tends to be denominated in local currency, and foreign-law debt tends to be denominated in foreign currency. That is why the decision on the type of debt to issue might be influenced by shocks to the real exchange rate, as this would affect the cost of borrowing externally. Moreover, as shown for instance in Eichenbaum and Evans (1995), shocks to the U.S. interest rate change the real exchange rate, and so the effect on output in emerging countries after such a shock depends on the evolution of the real exchange rate.

To assess the role of real exchange rates both in the decision of the government about what type of debt to issue and on the effect of an increase in the international interest rate, I extend the model to incorporate an exogenous process for the real exchange rate, possibly correlated to the process for the international interest rate. I use the data to estimate these processes and show that, once those are incorporated into the model, the main mechanisms hold and are quantitatively similar to the baseline model. I find that a shock that increases interest rates has a positive effect on future real exchange rates defined as local over foreign units of goods. In the model, this makes repayment of foreign credit more expensive, and so the same mechanisms apply by which the government substitutes external debt with domestic debt and taxes, implying, therefore, a decrease in total output.

Related Literature. This paper combines elements of the sovereign default and the financial intermediation literature to study the government's decision on whether to borrow domestically or externally in an economy that is subject to international interest rate shocks. It contributes to the standard sovereign default literature, such as Aguiar and Gopinath (2007) and Arellano (2008), by introducing two types of debt, external and domestic, on which the government can selectively default.

The model in this paper is related to Gennaioli, Martin, and Rossi (2014), Sosa-Padilla (2018), and Bocola (2016). They incorporate financial intermediation and domestic debt to analyze the effect of sovereign default on domestic banks' balance sheets. However, they assume either that there is only domestic debt, or that default is non-discriminatory. I argue, based on the evidence in Reinhart and Rogoff (2011), Erce, Mallucci, and Picarelli (2022), and other literature that focuses on the legal aspects of sovereign default, such as Gelpern and Setser (2003), that governments can, and actually do, discriminate between domestic and foreign creditors. More similar to this paper are Mallucci (2022) and Perez (2015) which include both domestic and external debt in the decision of the government. However, they assume that governments cannot selectively default on domestic and external debt, which is at odds with what we observe in the data. Therefore, this paper contributes to the sovereign default literature by introducing a new framework where governments can optimally choose their debt issuance on both domestic and external debt and can default selectively. Allowing for selective default as in the data is key in explaining the trade-offs the government faces when deciding between domestic and external debt. This is due to the different costs of default between each type of debt that arise endogenously in the model, which will affect the price of debt.

Arellano, Atkeson, and Wright (2016) include both types of debt, domestic and external, and the possibility of selective default in a two-period theoretical model. Assuming exogenous costs of default in each type of debt, they then study how public debt crises may have spillover effects on the private sector in scenarios with differences in the level of government's interference in private contracts and fiscal flexibility. Broner, Erce, Martin, and Ventura (2014) consider the role of domestic debt in a model where debt has a crowding-out effect on domestic banks and allow governments to also selectively default on creditors. This paper differs from theirs mainly in two respects: first, the default decision is essentially exogenous in their case, whereas the model presented here considers the endogenous decision of default with endogenous default costs on domestic debt. Second, their model only considers one type of debt that can be held both by domestic or international creditors, whereas I consider a segmented market between domestic and international creditors. The data support this assumption, as the characteristics of debt issued under domestic law are different from those of debt issued under foreign law—which is the definition used here of domestic and foreign debt—especially regarding default probabilities.[4]

This paper also contributes to the growing literature on the spillover effects that financial conditions in developed countries have on the rest of the world. On the theoretical side, Bai et al. (2023) develop a model which can account for the co-movements of emerging countries spreads with those in develop economies, and hence co-move with a global financial cycle. On the empirical side, Rey (2015), Miranda-Agrippino and Rey (2015), and Bruno and Shin (2015) provide evidence on the existence of a global financial cycle and argue that this cycle is largely driven by monetary policies in the United States, which is unrelated to the economic conditions in developing countries. Other papers that find a strong response of emerging countries to interest rate shocks are Dedola, Rivolta, and Stracca (2017), and Maćkowiak (2007). Here, I show that the spillover effects in emerging countries are related to their level of financial development. My results are in line with the empirical literature that indicates that an important mechanism of transmission of international shocks is the financial channel (e.g., Canova, 2005 and Georgiadis, 2016), and that countries differ in their response depending on how vulnerable they are to external conditions as shown in Iacoviello and Navarro (2019). I add to this empirical literature by providing a new mechanism of transmission that relates to the level of financial development of the country and the borrowing decisions of its government. In my framework, these two factors capture endogenously the level of vulnerability of countries to external shocks. Banerjee, Devereux, and Lombardo (2016) also associate financial intermediation with the response of countries to external shocks. However, they ignore the role of the government and focus mostly on the transactions of domestic banks with international banks.

The effect of international interest rates on emerging countries is studied in Neumeyer and Perri (2005). In relation to this literature, recent papers have also explored the role of international interest rate shocks in emerging countries through its effect on sovereign debt. Almeida, Esquivel, Kehoe, and Nicolini (2019) study the impact of changes in the international rate on the default probability and renegotiation costs in emerging countries. Similarly, Johri, Khan, and Sosa-Padilla (2022) also introduces fluctuations in the international interest rate in a sovereign default model, which in addition features an economy with stochastic volatility. However, these papers only include external debt, and therefore abstract from the decision of governments between issuing domestic and external debt, and the role of domestic debt in mitigating such fluctuations in the international interest rate.

[4]Using data from Arslanalp and Tsuda (2014), I show in Section E.1 that indeed debt issued under domestic law largely coincides with debt held by domestic residents. This reinforces the idea that debt markets are segmented. Unfortunately, the data on who holds the debt is only available for recent years, so it is not suitable for the analysis on this paper.

Finally, this paper complements the literature that studies the implications of an increasing use of domestic debt in developing countries. Bua, Pradelli, and Presbitero (2014) documents this trend in low-income countries, and show that this increase has been associated with a decrease in borrowing costs. Panizza (2008) finds similar trends and studies the potential trade-offs that this switch to domestic debt may have in their economies. I contribute to this literature by providing a new framework to analyze the costs and the benefits of domestic debt, and to understand the incentives of governments that led them to issue more domestic debt over the last years.

2 Empirical Evidence

In this section, I provide the evidence that motivates this paper on the vulnerability of countries to interest rate shocks, and on the composition of sovereign debt. This section also shows evidence of selective default on sovereign debt that is used to discipline the model. In particular, there are three main facts documented in this section for a sample of 14 emerging countries. First, countries often default in a discriminatory way against external debt. In more than 70% of the default events during the years analyzed here, countries defaulted only on external debt and not on domestic debt. Second, countries issue on average 54% of their sovereign debt domestically, and this share of domestic debt increases with financial development. And, third, emerging countries suffer output drops after a shock that increases the U.S. interest rate, and this effect is bigger for less financially developed countries.

The main data analysis focuses attention on 14 developing countries and the choice of countries is given by the availability of data. In the main analysis, I restrict attention to the period 1969-1996. This is because the identification of the interest rate shock is known to be consistent for this period. However, I show that all results are robust to extending the period to 1960-2007. Next, the sources and definitions for the main data variables used in this paper are discussed.

Domestic Debt. There are different definitions that can be used for domestic debt. In the model, domestic debt refers to debt held by residents of the country as opposed to foreigners. However, there is no long-time series data available for this definition. I use data on domestic debt from Reinhart and Rogoff (2011) which uses the definition of domestic debt based on the market where it is issued: under the own country's jurisdiction, or under foreign jurisdiction. In general, in developing countries, the debt issued in domestic debt is held by residents of the country, and the debt issued abroad by foreigners. For instance, the World Bank and IMF (see Arslanalp and Tsuda (2014)) have recently released new data on domestic debt where they can differentiate between debt based on whether holders are residents or foreigners. While this time series is too short to be used in my analysis, I find it reassuring that these series display similar levels and trends to those based on the market of issuance.[5]

[5] Appendix E.1 shows a comparison of these series for the time period where there is data on both.

Default Events. The list of historical default events across developing countries is also taken from Reinhart and Rogoff (2011). Moreover, I use their classification between defaults that involve only external debt and those that involve both, domestic and external debt. They classify all default episodes between those involving external debt only, and overt defaults that involve both domestic and external defaults. Their classification of overt defaults includes cases where the government forced conversions of deposits, bank deposits were frozen, imposed lower coupon rates, there was a unilateral reduction of principal, or suspension of payments.

Financial Development. I use data on liquid liabilities to GDP from the World Bank Financial Structure and Development Dataset as a measure of financial development. In particular, liquid liabilities consist of a broad definition of money including demand and interest-bearing liabilities, such as deposits, at domestic financial institutions. This has been one of the main measures of financial development in the literature (see for example, King and Levine (1993)). This variable measures the size of the financial sector relative to economic activity, which is known as financial depth. Intuitively, when liquidity is high, it is easier to trade among agents in the economy. Low liquidity may indicate the presence of financial frictions that prevent agents from transforming their assets, or that prevent resources from flowing between savers and borrowers.

Other Variables. The main analysis also uses data on real GDP in each country. For this, I use data from Iacoviello and Navarro (2019) which extends annual data from the World Bank's World Development Indicators. Other variables used in the analysis are U.S. interest rates, which are taken as the Fed Funds Rate, and the bilateral real exchange rates, for which I use data from the U.S. Department of Agriculture.

2.1 Selective Default

Table 1 shows descriptive statistics for each country on the sample. It documents that based on the data from Reinhart and Rogoff (2011) these countries experienced a total of 22 defaults over the period 1960 to 1996 which represents a 4% annual frequency. Of those default events, 73% of them were only in external debt. In their database, none of the defaults are labeled as "only domestic debt" defaults. Therefore, those defaults that are not external debt-only are assumed to be defaults on both external and domestic debt.

Countries not only default selectively on their debt, but also the economic conditions around an external default are different from those around a default on both types of debt. For instance, taking data since the year 1960 up to 2007, the average real GDP in the sample of emerging countries used here in the year before default is 0.8% below trend if the default was only on external debt and 1.9% below trend if the default was on both types of debt. Similarly, in the year of default, real GDP was 0.7% below its trend during external-only defaults, whereas it was 4.2% below trend when

Table 1: Descriptive Statistics

	Liquid liabilities	Domestic debt share	Total defaults	External defaults
Argentina	0.172	0.449	2	0
Brazil	0.172	0.848	2	0
Chile	0.267	0.489	1	1
China	0.636	0.359	0	0
Colombia	0.189	0.398	0	0
Ecuador	0.177	0.418	1	1
India	0.297	0.761	2	2
Indonesia	0.194	0.034	1	1
Malaysia	0.632	0.762	0	0
Mexico	0.222	0.562	1	0
Peru	0.186	0.186	6	5
South Africa	0.514	0.958	3	3
Thailand	0.474	0.752	0	0
Turkey	0.195	0.526	2	2
Total	0.312	0.540	22	16

Sources: World Bank Financial Structure and Development, and Reinhart and Rogoff (2011).
Notes: The last row shows the average liquid liabilities to GDP and domestic debt share for the countries in the sample. *Total defaults* refers to the number of years a given country defaulted on either type of debt, and *External defaults* refers to the number of years a country defaulted only on external debt.

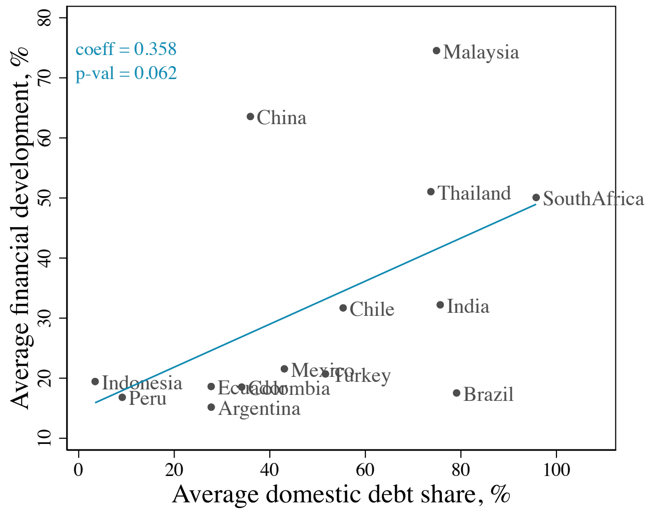
the default was on both types of debt.[6] I refer to this difference in the patterns around default as a *pecking order of default* in that we observe moderate output drops if countries default only on external debt but severe drops if countries default on both types of debt.

2.2 The Composition of Sovereign Debt and Financial Development

I turn now to describing the empirical patterns related to the share of domestic sovereign debt and its relationship with financial development. As shown in Table 1, domestic debt accounts on average for 54 percent of the total amount of sovereign debt in these countries, although there is some variation across countries: Peru and Indonesia have shares of domestic debt of only 18% and 3% respectively, whereas other countries such as India and South Africa have shares over 75%. At the same time, countries also display heterogeneity in their level of financial development. On average liquid liabilities are about 30 percent of their GDP with a large dispersion across countries.

[6] Here, the data used is extended to year 2007 in order to include more default observations, as the number of defaults on both types of debt in the baseline period is only 6, and therefore the statistics on these could have significant noise. However, the patterns remain the same when focusing on the baseline period only. Deviations from trend are measured using the Hodrick-Prescott filter on quarterly GDP, and then taking the annual average.

Figure 1: Average Domestic Debt and Financial Development



Next, I explore the relationship between these two variables: domestic debt share and financial development.

Table 2: Financial Development and Domestic Debt: Data

	(1)	(2)	(3)
Financial development	0.466***	0.487***	0.389***
	(0.070)	(0.078)	(0.088)
Total debt to GDP	-0.250***	-0.297***	-0.263***
	(0.059)	(0.035)	(0.043)
Log GDP per capita	0.754	-31.573***	-16.273***
	(1.581)	(3.399)	(5.956)
Country Effects	No	Yes	Yes
Year Effects	No	No	Yes
Observations	365	365	365

Notes: Driscoll-Kraay standard errors in parentheses.
*p<0.1, **p<0.05, ***p<0.01.

Figure 1 shows the relationship between average financial development and share of domestic debt across countries. There is a strong positive relationship between these variables, that is, countries with higher levels of financial development tend to have on average higher shares of domestic debt. This positive link between financial development and domestic debt might be driven by many factors, such as the level of income per capita or total amount of debt that these countries are able to issue. To control for these factors, I consider the following regression

$$\text{Domestic Debt}_{it} = \alpha_i + \delta_t + \beta \text{FD}_{it-1} + \Gamma X_{it-1} + \varepsilon_{it} \tag{1}$$

where the dependent variable is the share of domestic debt in the country i at time t. The regressors include the main variable of interest, namely financial development measured as the ratio of liquid liabilities to GDP, which is lagged by one period to avoid endogeneity, as well as other regressors in X_{it-1} including the total amount of public debt and log GDP. In this regression, I also allow for country and time-fixed effects. Results are reported in Table 2 for countries in our sample. On average, an increase of 10 points in the ratio of liquid liabilities to GDP, that is, the measure of financial development, is associated with an increase in the share of domestic debt of around 4 percentage points. Moreover, this positive relationship is statistically significant.

2.3 Effect of Interest Rate Shocks on Emerging Countries

Here, I study how shocks to world interest rates affect economic outcomes in developing countries. First, I show that after a shock that increases the U.S. interest rate, emerging countries experience a drop in their real GDP. Second, I show how this drop depends crucially on the level of financial development of these countries, where financial development is measured by the amount of liquid liabilities in the country.

Baseline Specification. In the baseline specification, shocks to the U.S. interest rates are taken from Romer and Romer (2004) on their original sample which covers 1969 to 1996. They identify monetary policy shocks using a narrative approach. This identification strategy has been used in the most recent literature and is known for capturing well the main features of responses to monetary policy shocks.[7] Then, to see the effect of these shocks on the the output of emerging countries, I use the local projection method proposed in Jordà (2005). In particular, consider the following set of regressions

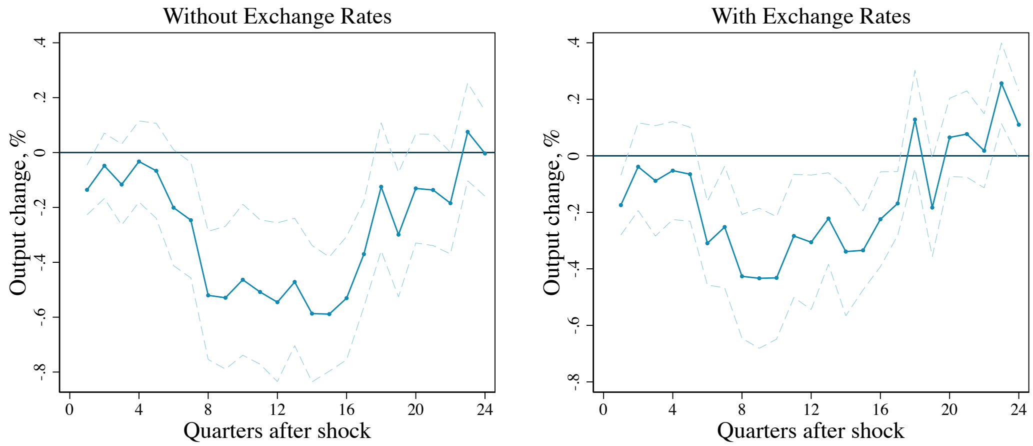
$$y_{it+h} = \beta_h u_t + \sum_{j=1}^{4} \gamma_{1ijh} y_{it-j} + \gamma_{2ih} e_{it-1} + \gamma_{3ih} e_{it+h} + \rho_{ih} R^*_{t-1} + \delta^1_{ih} t_i + \delta^2_{ih} t_i^2 + \alpha_i + \varepsilon_{it+h} \quad (2)$$

for $h = \{0, ..., H\}$, where, y_{it+h} is the real GDP in country i in quarter $t+h$ and u_t is the Romer-Romer monetary policy shock. The parameter of interest in this regression is β which estimates the effect of a monetary policy shock that increases the U.S. interest rate on the GDP of emerging countries. This specification also controls for the effect that real exchange rates of the emerging country with respect to the U.S. in the periods before the shock, $e_{i,t}$, has on output growth, as well as output in previous quarters. Parameters δ_j capture a linear and quadratic time trend. I allow these trends to be country-specific, as the different countries in my sample experienced different growth trends. Other controls included in the regression are lagged values of GDP, lagged value of the level of interest rate in the United States, R^*_{t-1}, as well as a country fixed effect captured by α_i.

Figure 2 shows the estimated coefficient $\widehat{\beta}_h$ for each quarter after the shock. The left panel shows the specification in (2) without real exchange rates and the right panel with the same specification but adding exchange rates as a control. Dotted lines represent the confidence interval of one standard

[7] See Ramey (2016) for a discussion on these methods.

Figure 2: Output Response to an Increase in US Interest Rates



Notes: This figure shows the coefficient β_h from regression (2). The left panel corresponds to the coefficient when exchange rates are not included as regressors in (2), and the right panel includes exchange rates. Dashed lines are 68% confident intervals. Standard errors are computed using the Driscoll-Kraay method.

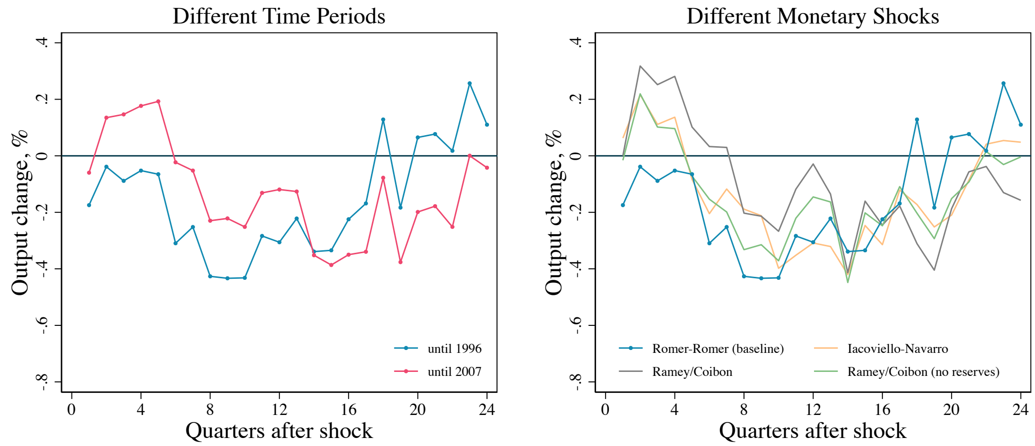
deviation, computed using the Driscoll and Kraay (1998) estimation that allows to control for heteroskedastic, autocorrelated, and possibly correlated errors across countries. The drop in output in emerging countries after a shock that increases interest rates by 100 basis points is about 0.6 percent when exchange rates are not considered and 0.4 percent once we include real exchange rates in the estimation. The peak effect happens 8 quarters after the shock. The delay in the response to this type of shock is standard in the literature. Note that the specification without exchange rates (left panel) shows an even larger delay in the response, possibly due to a delayed transmission of exchange rates themselves.[8]

Alternative Specifications. The original series of monetary shocks developed by Romer and Romer (2004) is estimated until 1996. This is the series that we use in the baseline specifications. More recently, Wieland and Yang (2020) extended this series to more current dates using the same methodology. The left panel of Figure 3 compares results using the two series, where I consider periods up to 2007 to avoid the Great Recession period where interest rates were at the zero lower bound. Results are very similar: it shows a similar fall in output of around 0.4%.

The right panel of Figure 3 compares the results of the baseline specification with those that result from using different monetary policy shocks. I assume that the US interest rate depends on output, inflation, and other real and nominal variables from the US economy, as in a Taylor rule specification, and consider the residuals from a regression on these variables the unexplained variation in the US interest rate, that is a monetary policy shock. In particular, I estimate by OLS

[8] Other papers in the literature have analyzed the spillover effects of U.S. monetary policy using different methods. See for instance: Banerjee et al. (2016), Canova (2005), Dedola et al. (2017), Iacoviello and Navarro (2019), and Maćkowiak (2007). The results shown here on the effect on output of an external shock align with those reported in the literature.

Figure 3: Output Response to US Interest Rates: Alternative Specifications



Notes: The left panel of this figure shows the coefficient β_h from regression (2) computed for two different time periods: baseline (blue line) and extended time period until 2007 (red line). The right panel compares the baseline (blue line) with alternative specifications as explained in the main text.

the following regression

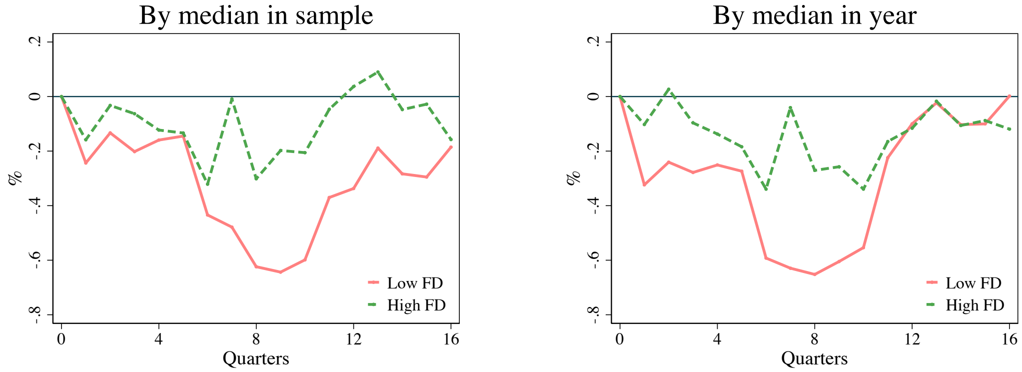
$$r_t^{US} = \sum_{j=1}^{4} \beta_i^r r_{t-j}^{US} + \sum_{j=1}^{4} \beta_i^y y_{t-j}^{US} + \sum_{j=1}^{4} \beta_i^\pi \pi_{t-j}^{US} + \gamma X_{t-1} + u_t,$$

and consider $\hat{u}_t$ as the monetary policy shock that I then use in equation (2). The right panel of Figure 3 shows the results of using these shocks with different control variables included in X_t. Following Iacoviello and Navarro (2019), one of the specifications includes as controls corporate spreads in the US and a measure of the world GDP. I also consider the specification in Ramey (2016) and Coibion (2012) which includes as controls the unemployment rate, commodity prices, money and reserves. Finally, I also show results with the Ramey/Coibon setup excluding reserves from the regression. The baseline specification is similar to what one would obtain if using other shocks. The output response of emerging countries to a Romer and Romer monetary policy shocks lies in between the response obtained by monetary shocks obtained from different Taylor rules specifications.

Other robustness exercises are provided in Appendix E.3, which includes robustness on the sample of countries considered (Latin American or East Asian countries). I also show that the drop in output is not driven by the Volcker period which was a time in which both the interest rates in the United States were high and many emerging countries experienced years of severe recessions (what is known as the *lost decade* in Latin America).

External Vulnerability and Financial Development. The magnitude of the output drop after an increase in the U.S. interest rate varies with the level of financial development in the country. To see this, first I show the results of running the same regressions as in (2) but for two separate samples

Figure 4: Output Response by Level of Financial Development



Notes: This figure shows the coefficient β_h from regression (2) computed for two different samples: observations for which the level of financial development is above the median (dashed green line) and those below the median (solid red line). The median used as a threshold to divide observations is computed in the whole sample in the left panel and year by year in the right panel.

of observations: *high* and *low* financially developed countries. Another way of providing evidence of the difference in response by financial development is to add an interaction term in (2) between the monetary policy shock and the level of financial development. I show that both methods imply a negative relationship between the level of financial development and the vulnerability to external shocks.

First, observations are divided into two groups depending on their level of financial development. Here, I consider two possible classifications: 1) a country-year observation is high financial development if its level is above the median level in the sample, and low financial development otherwise, and 2) a country-year observation is high financial development if its level is above the median level in that year, and low financial development otherwise. Note that with these definitions a country might be low-financially developed in some years, but switch to high-financially developed in other years. The second definition takes into account that the level of financial development changes over time. Then, regression (2) is run separately in each of these groups so that we obtain the response of output for high, $\widehat{\beta}^{\text{high}}$, and for low financially developed, $\widehat{\beta}^{\text{low}}$. Results are shown in Figure 4. When the country's level of financial development is low, output drops by more than 0.6 percent, but countries with high financial development are barely affected, with a drop in output of about 0.2 percent.

To further analyze the role of financial development, consider now the same local projections as in (2), but adding an interaction between the monetary policy shock and the financial development

Table 3: Output Response and Financial Development

	Interaction coefficient, γ_h					
Quarters after shock:	4	6	8	10	12	16
	0.008	0.012	0.018*	0.020**	0.022**	0.000
	(0.006)	(0.008)	(0.010)	(0.009)	(0.010)	(0.008)

Notes: Driscoll-Kraay standard errors in parentheses. *p<0.1, **p<0.05, ***p<0.01.

measure, that is,

$$\begin{aligned} y_{it+h} =& \beta_h u_t + \gamma_h u_t \times \mathrm{FD}_{it-1} + \sum_{j=1}^{4} \gamma_{1ijh} y_{it-j} + \gamma_{2ih} e_{it-1} + \gamma_{3ih} e_{it+h} + \rho_{ih} R^*_{t-1} \\ &+ \delta^1_{ih} t_i + \delta^2_{ih} t_i^2 + \alpha_i + \varepsilon_{it+h}, \end{aligned} \tag{3}$$

where, FD_{it-1} is our measure of financial development. Therefore, our object of interest is the estimate $\hat{\gamma}_h$. If positive, it represents how much an increase in the level of financial development mitigates the drop in output from an external monetary policy shock. Table 3 shows the estimates for the coefficient on the interaction between financial development and the U.S. monetary policy shock, $\hat{\gamma}$. The coefficient is estimated to be positive and significantly different from zero. This implies that higher financial development decreases the output drop after an expansionary monetary policy shock in the United States.[9]

In summary, the main empirical finding of the paper is that emerging countries suffer large output losses from international interest rate shocks, especially so if they are less financially developed. Moreover, it is precisely the less financially developed countries the ones that tend to have higher shares of their government debt held by foreign creditors rather than domestic creditors. In the following section, I present a general equilibrium model that captures the main empirical findings and is then used to analyze the decision of governments on their sovereign debt composition as well as the effect of a shock to international interest rates.

3 Model

I consider a small open economy model with infinitely lived consumers. The model incorporates a banking sector along the lines of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) into a sovereign default model with discriminatory default on domestic and foreign debt. The economy is composed of a representative household whose members can be workers or bankers, a representative firm, international creditors, and a government. Households consume and save by holding deposits at banks. Banks are the financial intermediaries in the economy: they use deposits from households

[9] Appendix E.4 shows that this finding is robust to the use of different samples, both relative to the countries and to the time periods considered in the analysis.

to invest in capital and government domestic bonds, but they are constrained on how much they can invest. Firms use capital supplied by banks, and labor supplied by workers to produce a single good, and are subject to an aggregate productivity shock. The government finances a constant amount of public expenditures using taxes on labor income, and issuing domestic bonds that are bought by banks, and external bonds that are bought by international creditors. The government can decide whether to default on each type of debt separately.

Households. There is a representative household composed of a measure 1 of workers and a measure 1 of bankers. Households maximize their utility function defined over consumption, C_t, and labor L_t, and discount the future at rate β.[10] Their preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \left[C_t - v(L_t)\right], \tag{4}$$

where $v(L_t)$ represents the disutility of working and is assumed to be such that $v' > 0$ and $v'' > 0$.[11] Households can only save by using deposits at banks at price q_t^D, that is, they deposit $q_t^D D_{t+1}$ units today and get back D_{t+1} units tomorrow. A measure $1-\sigma$ of workers randomly become bankers every period. The household transfers $\bar{n}$ to these new bankers so that they can start their activity at the bank. Households receive dividends, X_t from existing banks, and maximize their utility by choosing how much to consume, C_t, how much to save in deposits, D_{t+1}, and how many hours to work, L_{t+1}, at wage w_t, subject to their budget constraint

$$C_t + q_t^D D_{t+1} \leq (1-\tau_t) w_t L_t + D_t + X_t - (1-\sigma)\bar{n}, \tag{5}$$

where τ_t is the labor income tax.[12]

In order for the household to be willing to supply deposits to the bank, it has to be that the price of deposits is at least as large as the rate at which they discount the future, so that, in equilibrium, it must be that $q_t^D = \beta$. Moreover, households choose their supply of labor such that the following condition holds

$$v'(L_t) = (1-\tau_t) w_t. \tag{6}$$

[10] Alternatively, we could include the consumption of public goods, G, in the utility of the households as $U_t = C_t + G_t + v(L_t)$. Notice, however, that given that public goods are assumed to be an exogenous constant in the model, that would not imply any difference in the model.

[11] Here, I assume that households have linear utility in consumption for tractability. However, as it will be clear below, the government still wants to smooth consumption due to the curvature in the $v(.)$ function. In particular, the incentives of the government to smooth consumption are given by its incentives to smooth tax distortions that affect the amount of labor supplied. Therefore, the introduction of risk aversion on private consumption would not change the main mechanisms of the model.

[12] An implicit assumption in the problem of the households is that they cannot hold domestic debt. This allows us to focus on the role of financial development. One of the key elements in the model is the need for financial intermediaries to capture the concept of financial development and how changes in financial development in a given country can affect government decisions regarding the denomination of debt. That is why in this economy all domestic financial transactions are intermediated through domestic banks and households are purposely left out of the domestic debt market. Using data from Arslanalp and Tsuda (2014), banks' holdings of domestic debt account for more than 35% of all domestic debt in emerging countries.

Banks. At the beginning of the period, the aggregate productivity shock, z_t, is realized, and bankers collect returns from their last period investments. Each period bankers learn whether they will continue their activity as a banker in the following period or not. In particular, with probability σ they learn that they will continue as bankers next period and with probability $1-\sigma$ they learn that they will return to the household as a worker. In this latter case, bankers transfer all their net worth to the household. Notice that $1-\sigma<1$ is required because otherwise, banks would build up enough net worth to make financial constraints not binding, and therefore, irrelevant. Every period, a measure $1-\sigma$ of new bankers replace the banks that do not survive.

Bankers that survive, decide on their new investments by choosing investment in capital, k_{t+1}, with return R^K_{t+1}, and government bonds b_{t+1} at price q_t. Government bonds are long-term bonds, and it is assumed that payments on debt decay at a geometric rate in case of no default: a bank that buys one unit of debt today will receive, λ units tomorrow, $\lambda(1-\lambda)$ in two periods, $\lambda(1-\lambda)^2$ in three periods, and so on.[13] The government can default in its debt, in which case it returns nothing to the bank. The repayment decision on domestic debt is denoted by δ_t, which takes value 1 if it repays and 0 otherwise.[14]

Let the value of an existing bank be denoted by $V^b_t(n_t)$, where n_t is the bank's current net worth, $n_t = R^K_t k_t + \delta_t\left[\lambda + (1-\lambda)q_t\right] b_t - d_t$. Hence, current net worth is the sum of the returns on capital and the returns from government bonds, net of the payments to the household from their deposits, d_t. When a banker returns to the household, it transfers its current net worth to the household. Then, the objective of a bank is to maximize the value that it will transfer to the household. We can write the value of a bank as

$$V^b_t(n_t) = \max_{\{b_{s+1},k_{s+1},d_{s+1},x_s\}} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t+1}\left[\sigma^{s-t}x_s + \sigma^{s-t}(1-\sigma)n_s\right], \tag{7}$$

where x_t are the dividends that surviving banks can transfer to households every period. In equilibrium, we can show that $x_t = 0$ in every period, so from now on, we impose this equilibrium outcome. The intuition for this result is that, by paying low dividends, banks can finance more capital and bonds issuing fewer deposits, thus relaxing the collateral constraint. Surviving banks choose their

[13] This follows the way of modeling long-term debt as in Hatchondo and Martinez (2009), which retains tractability as it does not require to keep track of the whole sequence of debt issued in each period: a bank that had b_t amounts of bonds from the previous period, today receives λb_t from the government and will carry on as claim to the next period the remaining $(1-\lambda)b_t$ units. Then, the bank buys new debt this period by an amount of $\tilde{b}_{t+1} = b_{t+1} - (1-\lambda)b_t$ units, so that it starts period $t+1$ with a total claim on b_{t+1}.

[14] In this model domestic banks can only buy domestic bonds and have no access to external bonds and all the external resources have to come in through the government. One way to rationalize this is to assume that foreign creditors do not trust private domestic banks enough to lend them resources, due to their inability to properly monitor their activities. Following this argument, relaxing this assumption and allowing banks to borrow externally would require the inclusion of an additional collateral constraint on the resources that they can get from abroad (similar to the collateral constraint on domestic deposits). Therefore, in this alternative scenario, the banks would still be constrained on how much they can borrow—so, the crowding out effect would still be in place— and changes in the international interest rate will now directly affect them, and thus, the effect of a shock to the interest rate will prevail or could even be amplified.

capital investment, bond purchases, and how much to borrow from households using deposits, d_{t+1}, subject to their budget constraint

$$k_{t+1} + q_t b_{t+1} - q_t^D d_{t+1} \leq n_t. \tag{8}$$

The key financial constraint is a collateral constraint. Banks are constrained on how much they can borrow using deposits from households. In particular, they can not borrow more than a fraction θ of their current net worth, that is

$$q_t^D d_{t+1} \leq \theta n_t. \tag{9}$$

This constraint arises endogenously from an agency problem between banks and households, in which bankers can divert a fraction of their assets. Assume that the bank, after receiving the deposits from households can run away with a fraction $1/(1+\theta)$ of its total assets at that point, $n_t + q_t^D d_{t+1}$, which will return to its household, and start as a new banker with that given new net worth. In order for the banker to not have incentives to divert, it must be the case that $V_t^b(n_t) \geq V_t^b\left(\frac{1}{1+\theta}(n_t + q_t^D d_{t+1})\right)$. Given monotonicity of V_t^b, this implies $n_t \geq \frac{1}{1+\theta}(n_t + q_t^D d_{t+1})$, which gives rise to collateral constraint (9).

The problem for the bank in recursive formulation is then

$$V_t^b(n_t) = \max_{b_{t+1}, k_{t+1}, d_{t+1}} \mathbb{E}_t \beta \left[\sigma V_{t+1}^b(n_{t+1}) + (1-\sigma) n_t\right], \tag{10}$$

subject to the evolution of net worth,

$$n_{t+1} = R_{t+1}^K k_{t+1} + \delta_{t+1} \left[\lambda + (1-\lambda) q_t\right] b_{t+1} - d_{t+1}, \tag{11}$$

the budget constraint (8), and the collateral constraint (9).

Firms. Firms rent capital from banks at rate $R_{K,t}$, and demand labor from workers at price w_t. Following Neumeyer and Perri (2005), we assume that, within a period, firms need to borrow to pay for a fraction κ of workers' wage, that is, they need to borrow working capital. Then, the problem of the firm is

$$\max_{K_t, L_t} z_t F(K_t, L_t) - R_{K,t} K_t - w_t L_t - r_t^* \kappa w_t L_t,$$

so the first-order conditions are

$$R_{K,t} = z_t F_{K,t}(K_t, L_t)$$

$$w_t \left(1 + r_t^* \kappa\right) = z_t F_{L,t}(K_t, L_t).$$

I assume that the intra-period borrowing market is frictionless, and firms always repay at the end of the period. Moreover, the rate at which they borrow is given by the international interest rate, r_t^*. This assumption allows the model to capture a way in which the private sector is also directly affected by changes in the world interest rate, not only indirectly through its effect on the cost of borrowing of the government, which affects only the quantitative results of the model.[15]

International Creditors. The government borrows from risk-neutral international creditors whose one-period risk free rate is r_t^*. There are external shocks that can affect the interest rate on the world risk-free asset, which is characterized by the following stochastic process:

$$r_t^* = (1-\rho_r)\mu_r + \rho_r r_{t-1}^* + \epsilon_{r,t} \qquad \text{where} \quad \epsilon_{rt} \sim \mathcal{N}(0, \sigma_r). \tag{12}$$

The problem of international creditors is to maximize their expected stream of discounted consumption, $\mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} c_{t+s}^*$ subject to

$$c_t^* + \frac{1}{R_t^*} a_{t+1}^* + q_t(b_{t+1}^* - (1-\lambda)b_t^*) = y + a_t^* + \lambda b_t^*,$$

where y is their income, a^* is the one-period international asset which is bought at price $1/R_t^*$ and returns one unit in the next period, where $R_t^* = 1 + r_t^*$ denotes the gross interest rate. It is assumed that the endowment y is large enough to cover all possible supply of bonds, that is, international creditors are assumed to have *deep pockets*.

In equilibrium, international creditors make zero profits. Therefore, the expected return on bonds has to be equal to the return on the international asset, that is:

$$\frac{\mathbb{E}_t \left[\delta_{t+1}^* (\lambda + (1-\lambda) q_{t+1}^*) \right]}{q_t^*} = R_t^*. \tag{13}$$

This equation defines the bond price schedule that the government faces. Conditionally on not defaulting in period $t+1$, the government repays λ on each unit of external debt that it borrows in period t. The value of each unit of the remaining debt is then $(1-\lambda)q_{t+1}^*$, where q_{t+1}^* refers to the bond price schedule given the government's borrowing decision in period $t+1$, which in equilibrium is a function of today's borrowing decisions and the aggregate state of the economy.

Government. The government starts each period with a given amount of outstanding external and domestic debt, $\{B^*, B\}$, on which it owes a fraction λ. It can decide whether to repay each type of debt or to default on it, and it can do so in a discriminatory way. The difference between defaulting on domestic debt and defaulting on external debt arises because domestic debt is held by residents of the country: when the government defaults on domestic debt, it defaults on its own citizens. In order to emphasize this feature, I assume that all exogenous costs of defaulting on debt are identical between defaulting on domestic and external debt. The only difference comes from the endogenous effect that defaulting on domestic debt has on the economy. In particular, domestic

[15] In Section 6.2, I show that the main results hold when there is no working capital in the model, and therefore firms are not directly affected by fluctuations in the world interest rate.

default affects banks' investment decisions because it decreases their net worth, and therefore, it will affect the total output of the country.

Defaulting on either debt implies immediate exclusion from that credit market. Once the government is excluded from a given market, it regains access to it with probability γ every period. I refer to these periods as autarky periods, and, given that the government can default separately on domestic and external debt, there are three possible autarky states in which a country can be: *domestic autarky*, when it defaults on domestic debt, and has access only to external debt, *external autarky*, when it defaults on external debt and has only access to domestic debt, and *autarky in both markets*, that is, when the government has defaulted on both debts and has no access to borrowing. In all these autarky periods, the economy experiences an exogenous productivity loss, such that productivity in these periods is $h(z) \leq z$.

The government has to provide a constant amount of public goods G every period t, and it chooses labor income taxes, τ_t, and new debt to issue (if it has access to credit). Therefore, the government budget constraint when the country is not in autarky is

$$G + \delta_t \lambda B_t + \delta_t^* \lambda B_t^* = \tau_t w_t L_t + \delta_t q_t \left(B_{t+1} + (1-\lambda) B_t\right) + \delta_t^* q_t \left(B_{t+1}^* + (1-\lambda) B_t^*\right), \tag{14}$$

where, δ_t and δ_t^* are the current period default decisions on domestic and external debt respectively. When the country is in one of the three autarky states defined before, the budget constraint is similarly defined, but it does not include the market to which the country does not have access.

In this model, the government has incentives to issue debt to smooth taxes over the cycle. When productivity is low, labor is reduced, and therefore, the tax revenues that the government can get from taxing labor income at a given tax rate are low. That is, if there was no debt, the government would have to raise taxes significantly during downturns in order to finance its expenditures G. Given that taxes are distortionary, the government wants to avoid sharp increases in taxes, and thus, when productivity is low it prefers to issue debt instead, which it will repay during booms.

Finally, the country-level budget constraint for the economy in normal times, defined before government default decisions, is

$$\begin{aligned} & C_t + G_t + K_{t+1} - (1-\delta_K) K_t + \delta_t^* \left(\lambda B_t^* - q_{t+1} \left(B_{t+1}^* + (1-\lambda) B_t^*\right)\right) + \frac{r_t^*}{1+r_t^*} \kappa z_t F_L(K_t, L_t) L_t \\ = \quad & \delta_t^* \delta_t z_t F(K_t, L_t) + (1 - \delta_t^* \delta_t)\, h(z_t) F(K_t, L_t). \end{aligned} \tag{15}$$

This constraint aggregates the budget constraint of households, banks, and the government, such that aggregate consumption in private and public goods, and investment, plus net exports—measured as, payments to foreigners minus transfers from foreigners—must be equal to the production in the economy.[16] If the government defaults on either debt, production is affected by the default produc-

[16]See Appendix B for the derivation of the country-level budget constraint.

tivity cost, $h(z)$.

Next, I turn to defining the Markov equilibrium for this economy. For that, we need first to define the equilibrium of the private agents of this economy, who take as given a set of government policies.

Competitive Equilibrium

Let $S = (B^*, B, K, D, z, a, R^*)$ denote the aggregate state of the economy, where a is an indicator of the current financial state of the country, that is, $a = \{n, d, e, b\}$ indicates non-autarky, domestic autarky, external autarky, and autarky in both markets, respectively. Given a government policy $\pi(S) = (\delta^*, \delta, B^{*\prime}, B', \tau)$, a competitive equilibrium is an allocation $Y(S, \pi) = (C, L, B^{*\prime}, B', K', D')$, households and banks' value function $V^h(d; S)$, $V^b(n; S)$, and pricing functions $P(S, \pi) = \big(q, q^*, q^D, R_K, w\big)$, such that

i. given prices and government policy, households' allocations, (C, l, d'), and value function, $V^h(d; S)$, solve the household problem;

ii. given prices and government policy, banks' allocations, (b', k', d'), and value function, $V^b(n; S)$, solve the bank problem;

iii. given prices and government policy, demand for capital, and labor, solve the firm problem;

iv. international lenders break-even condition is satisfied;

v. government policy satisfies the government budget constraint;

vi. allocation is feasible: it satisfies the country-level budget constraint.

Lemma 1 (Characterization of the Bank's Problem). The value function of banks is linear in net worth

$$V^b(n; S) = \nu(S)n$$

where

$$\nu(S) = \beta \mathbb{E}\left\{\left[1 - \sigma + \sigma\nu(S')\right]\left[(1+\theta)R_K(S') - \theta/q^D\right]\right\}. \tag{16}$$

Moreover, banks prefer not to pay dividends, $x = 0$ if they continue as bankers. They only transfer their net worth to the household when they have to switch back to being workers.

The proof for this lemma can be found in Appendix A and follows Gertler and Kiyotaki (2010). Intuitively, all the choices for banks enter linearly in the problem, due to linearity of the budget constraint and the collateral constraint in n. An implication of this lemma is that the aggregate state S does not need to include the measure over the state variables of individual banks but rather only the aggregate net worth.

The function $\nu(S)$ is interpreted as the marginal value of an additional unit of net worth at the bank. The intuition is the following: if the constraint binds, an extra unit of net worth increases deposits by θ/q^D and investment by $1+\theta$. The net return on this investment is then $R_K(S')(1+\theta) - \theta/q^D$, which is the second term in brackets in (16). When banks collect their returns from the investment in the previous period, with probability $1-\sigma$ they have to return to the household, and they give to their household their entire net worth. With probability σ they continue as bankers, and their value per unit of net worth is $\nu(S')$. Then, the effective discount rate of banks is $m(S') \equiv \beta\mathbb{E}(1-\sigma+\sigma\nu(S'))$, which is the first term in brackets in equation (16).

I now characterize the set of allocations that constitute a competitive equilibrium. Define the productivity in the economy as a function of the current autarky state of the economy as $\tilde{z}(S) = z$ if $\delta(S) = \delta^*(S) = 1$ and $a = n$, that is, normal times, and $\tilde{z} = h(z)$ otherwise. Also, for notation convenience, let $\delta(\cdot, a = \{d, b\}) = 0$ and $\delta^*(\cdot, a = \{e, b\}) = 0$, which just indicates that there is no repayment of the debt if the government is already in an autarky situation for that type of market.

From the bank's first order condition, and substituting the R_K from the firm problem, the schedule of prices that the government offers to the bank must be such that for any choices $A' \equiv (B^{*\prime}, B', K', D')$ and given current productivity, z, and autarky state, a, banks' expected value of the return on bonds must be equal to the expected value of the marginal product of capital:

$$\frac{\mathbb{E}\left[m(S')\delta(S')(\lambda+(1-\lambda)q(A'';S'))\middle|S\right]}{q(A';S)} = \mathbb{E}\left[m(S')R^K(S')\middle|S\right] \tag{17}$$

where $m(S') = \beta\mathbb{E}(1-\sigma+\sigma\nu(S'))$, and $R^K(S) = \tilde{z}(S)F_K(S) + 1 - \delta_K$. That is, for banks to be willing to hold government debt they have to be indifferent between investing in bonds and in capital. Notice from this condition that the interest rate the government has to offer to banks is positively related to the returns that they get from investing in capital. Therefore, in periods when returns on capital are high, it is more costly for the government to borrow from banks.

Given that the value of a unit of net worth at the bank is always higher than the value of that unit for the households, banks always borrow to the maximum from households, that is, $q^D D' = \theta N$. Then, aggregating budget constraints of newborn and continuing banks, and substituting the binding collateral constraint, we get that the aggregate budget constraint of the bank is:

$$K' + \delta(S)q(S)B' = (1+\theta)\left[\sigma\left(\tilde{z}(S)F_K(S)K + (1-\delta_K)K + \delta(S)B - D\right) + (1-\sigma)\overline{n}\right]. \tag{18}$$

International creditors' break-even condition implies that the bond price schedule that the government offers is such that

$$\frac{\mathbb{E}\left[\delta^*(S')\left(\lambda+(1-\lambda)q^*(A'';S')\right)\middle|S\right]}{q^*(A';S)} = R^* \tag{19}$$

Finally, from the labor supply condition of workers, we can write total tax revenues of the government as $T(S) = (\tilde{z}(S)F_L(S) - v'(L))L$. Therefore, substituting this condition in the government budget constraint we get:

$$
\begin{aligned}
&\delta(S)q(S)(B' - (1-\lambda)B) + \delta^*(S)q^*(S)(B^{*\prime} - (1-\lambda)B) \\
&\qquad = G + \delta(S)\lambda B + \delta^*(S)\lambda B^* - \left(\tilde{z}(S)F_L(S) - v'(L)\right) L
\end{aligned} \tag{20}
$$

We refer to the resource constraint (15) together with constraints (17)—(20) as the implementability constraints.

Lemma 2 (Characterization of Competitive Equilibrium).
An allocation $Y = (C, L, B^{*\prime}, B', K', D')$ constitute a competitive equilibrium if and only if it satisfies the implementability constraints.

Given that the implementability constraints were derived using the equilibrium conditions of banks, households, and firms, and that they satisfy the government, bank, and country-level budget constraints, it is immediate that if Y is a competitive equilibrium then it has to satisfy the implementability constraints. To proof sufficiency we can construct taxes such that $\tau = 1 - v'(L)/zF_L$, and prices, $w = zF_L$, $R_K = zF_K + 1 - \delta_K$, $q_D = \beta$, so if Y satisfies the implementability constraints, then Y is part of the competitive equilibrium for these prices and tax rates.

Markov Equilibrium

A recursive Markov equilibrium is policy functions $\pi(S;a) = (\delta, \delta^*, B^{*\prime}, B', \tau)$, a set of allocation rules $Y(S,\pi;a) = (C, L, B^{*\prime}, B', K', D')$, and pricing functions $q(S,\pi;a)$, $q^*(S,\pi;a)$ such that

i. the associated outcomes constitute a competitive equilibrium for all S and π, and autarky state a;

ii. given S, and taking as given future policy functions, allocation rules, and pricing rules, the current policy $\pi(S;a)$ is optimal for the government.

Let $V(S)$ be the value function of the government when the country is not in autarky, and let $W_a(S_a)$ be the value when it is in one of the three possible autarky states, $a = \{d, e, b\}$. The *primal Markov problem* is to choose current allocations $Y = (C, L, B^{*\prime}, B', K', D')$, and current policies δ, δ^*, taking as given future policy functions $\delta(S')$,$\delta^*(S')$, pricing functions $q(S')$, value function of the bank, $\nu(S')$, and value functions of the government, $V(S'), W_a(S'_a)$, with the objective of maximizing the utility of the representative household. That is, the value of the government in normal times is

$$
\begin{aligned}
V(S) = \max_{Y,\delta,\delta^*} \Big\{ & C + v(L) \\
& + \beta \mathbb{E}\left[\delta\delta^* V(S') + \delta(1-\delta^*)W_e(S') + (1-\delta)\delta^* W_d(S') + (1-\delta)(1-\delta^*)W_b(S')\right] \Big\}
\end{aligned} \tag{21}
$$

subject to the implementability constraints (15), (17)–(20) conditional on $a = n$.[17] When the country is in domestic or external autarky it can only issue and default on one type of debt. Let δ_a denote the relevant default decision for each type of autarky state. Then, the value of the government in either external autarky or domestic autarky, $a = \{d, e\}$, is

$$W_a(S) = \max_{Y,\delta_a} \Big\{ C + v(L) \\ + \beta \mathbb{E} \left[\delta_a \left(\gamma V(S') + (1-\gamma) W_a(S') \right) + (1-\delta_a) W_b(S') \right] \Big\} \tag{22}$$

subject to the implementability constraints, where, γ is the probability of exiting the autarky state. Finally, when the government is in complete autarky, there are no default or debt-issuing decisions, so the problem for the government is

$$W_b(S) = \max_{Y,\delta_a} \Big\{ C + v(L) + \beta \mathbb{E} \left[\gamma V(S') + (1-\gamma) W_b(S') \right] \Big\} \tag{23}$$

subject to the implementability constraints.

4 Model Forces

Before turning to the quantitative results, it is useful to explain here the main forces in the model that determine the composition of sovereign debt, and the effect on output of an increase in the international interest rate.

4.1 Sovereign Debt Composition

In the decision of what type of debt to issue, the government balances the costs and benefits of domestic debt relative to external debt. On one hand, the main cost of issuing domestic debt is that it crowds out investment in capital, whereas external debt does not have that negative effect on domestic capital. On the other hand, the relative benefit of domestic debt is that, due to lower ex-post default incentives, it becomes relatively cheaper than external debt, especially in bad times. This generates a trade-off for the government between domestic and external debt, which I illustrate in this section using the government's first-order conditions.

When choosing how much to borrow, the government equates the revenues of issuing an additional unit of each type of debt to the costs. As in standard models of external sovereign debt, the benefit of an additional unit of debt is that it increases the revenues for the government by $\partial(q^* B^{*\prime})/\partial B^{*\prime}$ units, and thus relaxes its budget constraint. The cost of issuing an additional unit

[17] To solve the model computationally, it is assumed that the government receives every period idiosyncratic preference shocks on the amount of borrowing $(B^{*\prime}, B')$ that it chooses. This assumption is standard in the sovereign debt with long-term bonds literature, and it facilitates the stability and convergence of the model. For simplicity of exposition, here I present the model without such shocks. The details of the model with preference shocks can be found in Appendix D.

of debt is that it has to be repaid in the future, and therefore tightens the future government budget constraint.

In addition to these standard effects of borrowing, in this model, there are two additional factors. First, an increase in one type of debt affects the price of the alternative debt. That is, increasing external debt results in a higher external default probability and so decreases its price, but it also generally increases the probability of defaulting on domestic debt, so it also decreases the price of domestic debt. Therefore, there is a *cross-price effect* given by $\partial q/\partial B^{*\prime}$ in the case of external borrowing. Using a similar logic, an increase in domestic debt changes the price of external debt by $\partial q^*/\partial B'$. Second, because in this economy there are domestic financial intermediaries holding government bonds, changes in domestic borrowing will directly affect the investment decisions on capital, generating a *crowding-out* effect when domestic debt increases. However, the effect on capital is indirect in the case of external borrowing as it is present only through the cross-price effect $\partial q/\partial B^{*\prime}$.

To simplify the exposition of the government first-order conditions, the case with one-period debt, $\lambda = 1$, full capital depreciation, $\delta_K = 1$, and no working capital, $\kappa = 0$, is considered here. Under those simplifications, the problem of the government when it is not in autarky as in (21) is

$$V^R(S) = \max zF(K,L) + q^* B^{*\prime} - G - K' - B^* + v(L) + \beta \mathbb{E}\left\{\tilde{\delta}(S')V^R(S') + \Gamma^D(S')\right\}$$

subject to

$$\begin{aligned} \mu: &\quad G + B + B^* - qB' - q^* B^{*\prime} - \hat{T}(L)L \leq 0 \\ \rho: &\quad K' + qB' - \sigma\left(zF_K K + B - D\right) - \beta D' - (1-\sigma)\bar{n} \leq 0 \\ \eta: &\quad \beta D' - \theta\sigma\left(zF_K K + B - D\right) - \theta(1-\sigma)\bar{n} \leq 0 \end{aligned}$$

and the pricing equations. Here, $\hat{T} = F_L - v'(L)$ is the tax revenue per unit of labor. Let μ, ρ, and η be the Lagrange multipliers with respect to the government budget constraint, the bank's budget constraint, and the bank's collateral constraint, respectively. Assuming that the price functions, q and q^*, are differentiable, and letting $\tilde{\delta}{=}\delta\delta^*$ denote the joint repayment of both types of debt, the first-order conditions of the government with respect to external debt is

$$\underbrace{\mu\left[q^* + \frac{\partial q^*}{\partial B^{*\prime}}B^{*\prime} + \frac{\partial q}{\partial B^{*\prime}}B'\right] + q^* + \frac{\partial q^*}{\partial B^{*\prime}}B^{*\prime}}_{\text{revenue effect}} = \underbrace{\beta\mathbb{E}\left[\tilde{\delta}'\left(\mu'+1\right)\right]}_{\text{repayment effect}} + \underbrace{\rho\frac{\partial q}{\partial B^{*\prime}}B'}_{\text{investment effect}} + X^*, \tag{24}$$

and, the first-order condition for domestic debt is

$$\underbrace{\mu\left[q + \frac{\partial q}{\partial B'}B' + \frac{\partial q^*}{\partial B'}B^{*\prime}\right] + \frac{\partial q^*}{\partial B'}B^{*\prime}}_{\text{revenue effect}} = \underbrace{\beta\mathbb{E}\left[\tilde{\delta}'\left(\mu - \sigma\left(\rho' + \theta\eta'\right)\right)\right]}_{\text{repayment effect}} + \underbrace{\rho\left(q + \frac{\partial q}{\partial B'}B'\right)}_{\text{investment effect}} + X, \tag{25}$$

where X^* and X capture additional terms corresponding to the non-repayment continuation value for the government.[18]

We can classify the terms in both of the first-order conditions into three groups: a *revenue effect*, a *repayment effect*, and an *investment effect* of increasing borrowing. On the left-hand side, there is the *revenue effect* from borrowing today, which comes from a relaxation of the budget constraint, μ, and an increase in consumption. The first term reflects that each additional unit of borrowing increases resources by $\partial(qB'+q^*B^{*\prime})/\partial B^{*\prime}$ if external, and $\partial(qB'+q^*B^{*\prime})/\partial B'$ if domestic borrowing, and so the government budget constraint is relaxed, as captured by the term multiplying μ. In the case of external debt, issuing debt has a direct positive effect on consumption due to additional external resources, $\partial(q^*B^{*\prime})/\partial B^{*\prime}$, coming into the economy from abroad, whereas this effect is only indirect through the change in external prices from increasing domestic debt $\partial(q^*B^{*\prime})/\partial B'$. This is captured by the second terms outside the brackets on the *revenue effect*.

Then, the government equates this marginal revenue from borrowing to the marginal cost of repaying tomorrow and from a change in the investment in capital, that is, the right-hand side of equations (24) and (25). Repayment of either external or domestic debt implies a tightening of tomorrow's government budget constraint, μ'. Moreover, external debt is repaid to external creditors, and therefore implies a direct decrease in consumption due to resources flowing out of the economy, whereas domestic debt is repaid to domestic banks, which increases their net worth. These terms correspond to what is labeled as the *repayment effect*. Finally, increasing borrowing has an *investment effect* captured by the term $\rho\frac{\partial q}{\partial B^{*\prime}}B'$ in the case of external debt, and $\rho\left(q+\frac{\partial q}{\partial B'}B'\right)$ in the case of domestic debt. This corresponds to an important distinction between external and domestic debt, which will be analyzed below.

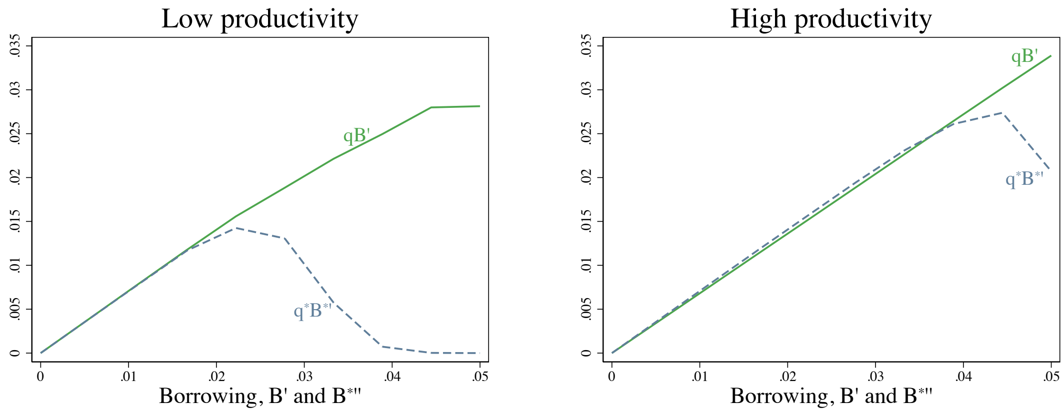
There are two main differences in the first-order conditions (24) and (25) that affect the composition of debt. First, the sensitivity of prices to an increase in debt, $\partial q^*/\partial B^{*\prime}$ and $\partial q/\partial B'$, is generally higher for external debt than for domestic debt. This corresponds to the relative benefit of domestic debt. Second, the effect on capital from increasing debt is positive for external borrowing and negative for domestic borrowing. This represents the relative costs of domestic debt. Importantly, these differences depend on the business cycle, as they are affected by the current productivity in the economy. I analyze these differences in detail in what follows.

Benefit of Domestic Debt. First, we want to analyze the benefit of domestic debt relative to that of external debt. For this, we focus on the left-hand-side of equations (24) and (25), that is, on the *revenue effects*. Then, by dividing the revenue effect of external debt by the revenue effect of domestic debt, we get

$$\text{Relative marginal revenue} = \frac{\mu\left(q^* + \frac{\partial q^*}{\partial B^{*\prime}}B^{*\prime} + \frac{\partial q}{\partial B^{*\prime}}B'\right) + \frac{\partial q^*}{\partial B^{*\prime}}B^{*\prime} + q^*}{\mu\left(q + \frac{\partial q}{\partial B'}B' + \frac{\partial q^*}{\partial B'}B^{*\prime}\right) + \frac{\partial q^*}{\partial B'}B^{*\prime}}. \tag{26}$$

[18]Details about the derivation of this problem and the equations that solve μ, ρ, and η can be found in Appendix C.

Figure 5: Laffer Curve for Borrowing



Notes: This figure plots qB' (solid green line) and $q^*B^{*\prime}$ from the numerical solution of the model explained in Section 5. The values q and q^* are for a chosen asset state space, $(B^{*\prime}, B', K', D')$, the interest rate R^* is set to 1, and productivity z is set to be below the mean in the left-panel, and above the mean in the right-panel. In the x-axis, domestic borrowing B' is used for the qB' line, and external borrowing $B^{*\prime}$ is used for the $q^*B^{*\prime}$ line.

From (26) it is clear that what determines the relative marginal revenue is the shape of the price functions of each type of debt, q and q^*. In particular, it is important to understand the differences in the sensitivity of each price to changes in the amount of borrowing, that is, the derivative of prices with respect to borrowing, which captures changes in the default probability on that debt. Then, changes in default probability affect the price at which the government issues an extra unit of borrowing, which in turn affects the marginal revenue that the government gets from issuing that extra unit of borrowing. But incentives to default on external debt are different from incentives to default on domestic debt, and thus the shape of the price function of external and domestic debt can be different. When the government defaults on external debt, the only costs that it faces are the exogenous decrease in productivity and autarky from external markets for an exogenous amount of time. In contrast, when the government defaults on domestic debt, it faces an additional cost because it defaults on its own domestic banks. In particular, aggregate banks' net worth is given by

$$N = \sigma \left[zF_K(K, L) + \delta B - D \right] + (1 - \sigma)\bar{n}, \tag{27}$$

so upon default, net worth decreases by σ, which is the survival rate of banks. This additional endogenous effect is not present when defaulting on external debt, and thus makes the government more reluctant to default on domestic than on external debt, which in turn makes the price of domestic debt less sensitive than the external debt price for the same amount of debt. This implies that for similar levels of prices, q and q^*, the increase in available resources from an increase in domestic debt is in general bigger than from an increase in external debt.

This effect can be seen in Figure 5, which plots the total resources obtained qB' and $q^*B^{*\prime}$ from borrowing B' and $B^{*\prime}$ units of bonds respectively, that is, the Laffer curve for borrowing on each type of debt. This figure uses the numerical results obtained in the quantitative analysis of the model that will be shown below. As external borrowing increases, q^* quickly decreases, especially in periods of low productivity, z (left panel), and so the increase in total resources $q^* B^{*\prime}$ becomes lower.[19] This happens at a much lower rate in the case of domestic borrowing. Due to the higher sensitivity of external debt prices to increases in debt, the increase in the amount of resources, $q^* + \partial q^* / \partial B^{*\prime}$, is lower for external than for domestic debt, and so the relative marginal gain decreases for high levels of external debt.

Cost of Domestic Debt. The second main difference between the decision of domestic and external borrowing comes from the *investment effect*: domestic debt crowds out investment, and external debt has, in general, a positive effect on investment. In the first order conditions (24) and (25) these effects are captured by the term $\rho \frac{\partial q}{\partial B^{*\prime}} B'$ for external borrowing, and $\rho \left(q + \frac{\partial q}{\partial B'} B' \right)$ for domestic borrowing, where ρ is the multiplier on the banks budget constraint. That is, ρ captures the positive effect of capital investment in the economy, and mainly represents the increase in output in future periods due to an increase in future capital.[20] Importantly, the sign of these effects is different for external and domestic debt: an increase in external borrowing increases investment in capital by $-\frac{\partial q}{\partial B^{*\prime}} B'$, which in general is positive because more external debt tends to increase the default probability on both types of debt. In the case of domestic borrowing, an increase in borrowing changes investment by $-q - \frac{\partial q}{\partial B'} B'$, which is negative, reflecting thus the crowding out of investment in capital.

These effects can be understood from the aggregate banks' budget constraint: $K' = (1 + \theta) N - qB'$. Domestic debt is held by domestic banks, so increasing B' directly decreases the resources available to the bank to invest in capital. The effect of external borrowing on investment is given through the cross-price effect: increasing external debt generally decreases the price of domestic bonds, $\partial q / \partial B^{*\prime} < 0$, due to a higher probability of default. Therefore, for a given amount of domestic debt, it increases the resources available to the bank to invest in capital.

Cyclicality of Domestic Debt. So far we have analyzed the relative benefits and costs of each type of debt for a given state of the economy. However, the state of the domestic economy—the aggregate productivity level in the country—also affects the decision on what type of debt to issue.

In the model, when productivity is low, the share of domestic debt will tend to increase. This is because when productivity is low, expected returns to capital, R_K, are also low. Therefore, first, the price of domestic debt, aside from default risk, is higher as banks' incentives to invest in capital decrease, and second, the cost of crowding out capital is lower, as the increase in output from using capital is lower when productivity is low, making then domestic debt more attractive. Moreover, in low productivity periods, the incentives to default on external debt increase—and, it does so

[19] Note that the government will never optimally choose to borrow on the decreasing side of the Laffer curve of $q^* B^{*\prime}$, as it can get the same amount of resources by borrowing less, as shown in Arellano (2008).

[20] In particular, $\rho = \frac{\partial q^*}{\partial K'} B^{*\prime} - 1 + \beta \mathbb{E} \left[z' F'_K + \mu' \hat{T}'_K L' + [\rho' + \theta \eta'] \sigma z' \left(F'_{KK} K' + F'_K \right) \right] + \mu \left(\frac{\partial q^*}{\partial K'} B^{*\prime} + \frac{\partial q}{\partial K'} B' \right)$, where $\hat{T}$ are tax revenues per unit of labor.

relatively more than for domestic debt—which, therefore, decreases the price of external debt. These two factors, more incentives to issue domestic debt and higher default risk on external debt during periods of low productivity, are what make the share of domestic debt to be countercyclical.

4.2 The Effects of a Shock to the Interest Rate

An increase in the international interest rate increases the cost of borrowing externally. Here, I provide intuition on how the increase in the cost of borrowing translates into a decrease in output, and how the government chooses to substitute the relatively costlier external debt with higher taxes or higher domestic debt to mitigate the drop in output.

Consider the case where the government only has access to external debt and bonds are one-period bonds. Intuitively, the increase in interest rates increases the cost of borrowing externally as it decreases q^*, and therefore increases the cost for the government of financing its expenditures. To see this consider the government budget constraint under repayment,

$$g + B^* = T(L) + q^* B^{*\prime},$$

where $T(L) = (zF_L - v'(L))L$ are total tax revenues. The left-hand side of the budget constraint is given at the beginning of the period and therefore, the government has no further choice on it. Suppose, for the sake of argument, that after the increase in R^*—that is, a decrease in q^*—the government decides not to change its external borrowing $B^{*\prime}$. Then, an increase in the international interest rate implies that, for the same amount of debt issued, the government gets a lower amount of resources $q^*B^{*\prime}$, and therefore has to increase tax revenues in order to make its payments $g + B^*$. The total effect on labor is then,

$$\frac{\partial L}{\partial R^*} = -\frac{\partial q^*}{\partial R^*} B^{*\prime} \left[\left(zF_{LL} - v''(L) \right) L + T(L)/L \right]^{-1} < 0. \tag{28}$$

The first term $\partial q^* / \partial R^* B^{*\prime}$ represents the change in total external resources from increasing R^*. As can be seen from the external creditors pricing equation (13), the change in prices due to an increase in the interest rate captures both, the direct decrease in q^* from the better outside option of external creditors, and from a change in the probability of default. The second term in square brackets represents how much total tax revenues decrease given an increase in labor, $\partial T / \partial L < 0$.

Then the effect on output after an increase in R^* is given by $zF_L \partial L / \partial R^*$. After the initial impact on output, the effect carries on to future periods through the net worth of banks. This is because the decrease in labor affects negatively the returns on capital, and hence decreases the total net worth of banks. This implies that banks have fewer resources to invest in capital, so capital available to produce in future periods decreases, and so does output. This example illustrates how the increase in interest rates is propagated to output in emerging countries, but it is an extreme case where the government does not optimally change its decision on how much to borrow externally, so $B^{*\prime}$ is fixed. In equilibrium, the government will generally prefer to lower the amount of external debt due to its higher costs, up to the point where it equates the marginal gain of external debt to its marginal cost of repaying it. To see this, notice that the relative marginal revenue of external debt in equation (26) is now affected by a decrease in q^* which, for a given amount of debt, decreases the marginal revenue of external debt relative to domestic, so that, in general, the government would prefer to substitute away from external borrowing and into domestic borrowing.

In fact, domestic debt can mitigate the drop in output after an increase in external interest rates. To avoid the output cost of increasing taxes as seen in equation (28), the government can substitute external debt for domestic debt if the crowding out cost of domestic debt is not larger than the higher costs of borrowing externally. This can be seen from the relative marginal gain of issuing external debt in equation (26). In equilibrium, the government equates the relative marginal gain of external debt to the relative marginal cost of repayment net of the effect on capital, that is, the right-hand side of (24) relative to the right-hand side of (25). These forces are analyzed quantitatively in the following section.

5 Quantitative Analysis

This section presents the numerical solution of the model in which data on a sample of emerging countries is used to set the parameters of the model. The numerical solution is then used to evaluate the predictions of the model regarding the effects of shocks to the international interest rate on emerging countries, the role of sovereign debt composition, and the patterns of default. Appendix D contains the details on the computational algorithm used to solve the equilibrium.

5.1 Parameterization

A period in the model is one quarter. I assume that the disutility of working for households takes the following form: $v(L) = \xi \frac{L^{1+\phi}}{1+\phi}$, where ϕ is the inverse of the Frisch elasticity, and the production function is Cobb-Douglas, $F(K, L) = K^\alpha L^{1-\alpha}$ where α is the capital share of production. In normal times, the total output produced at the firm is then $zK^\alpha L^{1-\alpha}$, where z is the aggregate productivity shock. When the economy is in autarky, however, productivity becomes $h(z) < z$. As standard in the literature literature on sovereign default, I assume that this productivity cost is nonlinear and is higher for high realizations of z. Specifically, following Chatterjee and Eyigungor (2012), I assume $h(z) = z - \max\{\zeta_0 z + \zeta_1 z^2, 0\}$, with $\zeta_0 < 0$ and $\zeta_1 > 0$, which implies that defaulting is relatively more costly at higher levels of productivity.

Assigned Parameters. The parameters used in the baseline model are reported in Table 4. Some parameters, $\Theta_1 = \{\mu_R, \rho_R, \sigma_R, \rho_z, \alpha, \delta_K, \lambda, \gamma, \phi, \kappa\}$ are taken directly from the data or from the literature, and the rest of parameters are set using a moment matching procedure so that the model is able to reproduce crucial targets in the data. Most of the parameters assigned from the data or literature are standard. For the parameters regarding the international interest rate process (12),

Table 4: Parameter Values

Assigned parameters		Source
Average world risk free rate	$\mu_R = 0.017$	Average US interest rate (quarterly rate)
Risk free rate autocorrelation	$\rho_R = 0.955$	AR(1) on US interest rate
Risk-free rate standard dev.	$\sigma_R = 0.003$	AR(1) on US interest rate
Productivity autocorrelation	$\rho_z = 0.95$	Neumeyer and Perri (2005)
Capital share	$\alpha = 0.3$	Standard capital share
Capital depreciation	$\delta_K = 0.05$	Standard capital depreciation
Debt decay rate	$\lambda = 0.05$	Average maturity
Autarky duration	$\gamma = 0.063$	Gelos et al. (2011)
Inverse Frisch elasticity	$\phi = 0.5$	Keane and Rogerson (2012)
Working capital	$\kappa = 0.26$	Neumeyer and Perri (2005)

Parameters from matching moments		Moment matched
Discount factor	$\beta = 0.986$	Default probability
Banks survival rate	$\sigma = 0.92$	Deposits to GDP
Collateral constraint	$\theta = 0.46$	Share of domestic debt
Banks initial net worth	$\bar{n} = 0.70$	Returns on equity
Gov. expenditures	$G = 0.035$	Government expenditures to GDP
Disutility of working	$\xi = 2.15$	Hours worked
Productivity standard dev.	$\sigma_z = 0.009$	Volatility of GDP
Productivity cost of default	$\zeta_0 = -0.182$	Debt to GDP
Productivity cost of default	$\zeta_1 = 0.195$	Standard deviation spread

I estimate an AR(1) process using data on the U.S. Federal Funds rate. The process is persistent, with the autocorrelation coefficient being $\rho_R = 0.955$, and a standard deviation of the error term $\sigma_R = 0.003$. As for the productivity process, I follow the same strategy as in Neumeyer and Perri (2005). Due to a lack of available data, it is not possible to construct a reliable series of productivity from the Solow residual. Because of this, I simply fix the autocorrelation parameter ρ_z to 0.95 as in the U.S. data, and choose σ_z so as to match the average volatility of the HP-filtered log output in the countries in the sample, which is 3.08%. Regarding the production technology of firms, the capital share is set to 0.3 and the depreciation rate of capital to 0.05. The parameter λ refers in the model to the debt decay rate and therefore is related to the average maturity of long-term debt. This parameter is set to match an average maturity of 4 years, in line with the standard maturity used in the literature (see Chatterjee and Eyigungor, 2012). The period of autarky after default is assumed to last on average almost 4 years, to match the estimates in Gelos et al. (2011). Finally, the Frisch elasticity is set equal to 2, so $\phi = 0.5$, which is in the range of the macroeconomic estimates (see for instance Keane and Rogerson, 2012).

Parameters From Moment Matching. The remaining eight parameters from the model, $\Theta_2 = \{\beta, \sigma, \theta, \bar{n}, G, \xi, \sigma_z, \zeta\}$, are set to match eight moments from the data. Some of them are standard in the sovereign default literature: average default probability, total debt to GDP, and average

Table 5: Model Fit

	Data	Model
Default probability, annual %	4.51	1.05
Debt to GDP, %	32.5	30.9
Deposits to GDP	0.32	0.33
Share of domestic debt	0.54	0.52
Return on equity, %	12.5	16.5
Government expenditures to GDP	0.14	0.13
Hours worked	0.22	0.22
Output volatility	3.08	3.11
Spread volatility	2.45	1.55

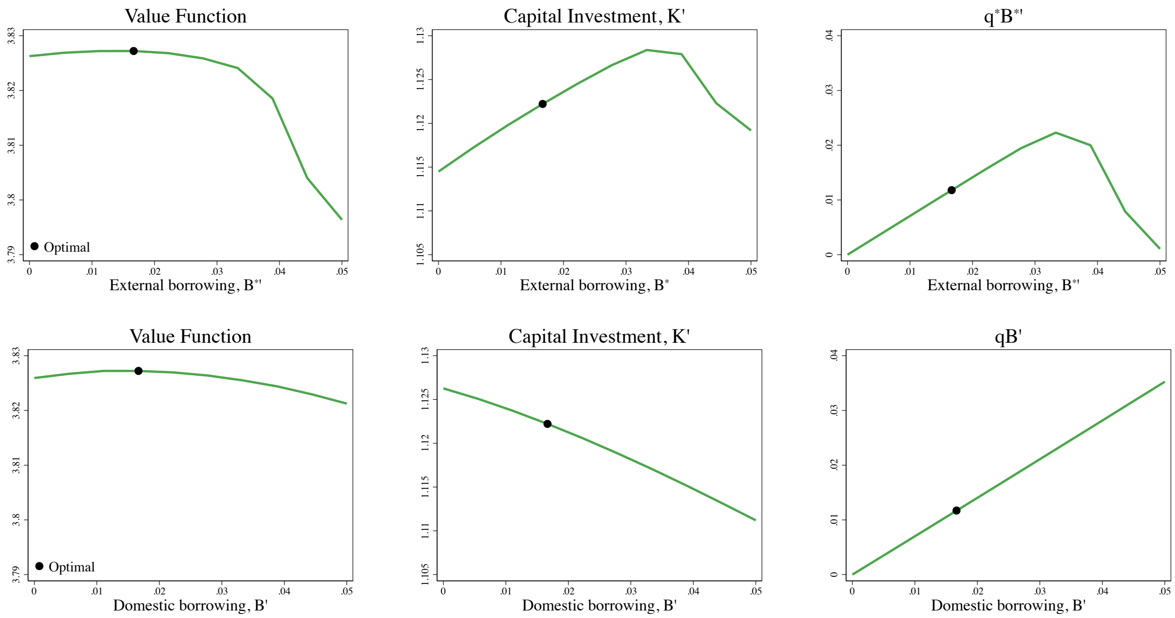
spread on external debt.[21] To assign the value of government expenditures, G, I target the average government expenditures to GDP in the data, and for the disutility of working, ξ, I target the average total hours worked. Three parameters are left that are related to the bank's problem, $\theta, \sigma, \bar{n}$. The fraction of net worth that the banks can borrow from households, θ, is directly related to the amount of deposits or, more broadly liquidity, in the economy. For this reason, I use the average ratio of liquid liabilities to GDP as a target. The remaining two parameters are the survival rate of banks, σ, and the net worth transferred from households to newborn banks, $\bar{n}$. To set these parameters, I use two moments: the share of domestic debt over total debt, and the average return on equity for banks. Intuitively, the average duration of banks affects the decision of the government on how much domestic debt to issue, as repayment of the debt will last for longer at the bank, and will allow for a higher accumulation of capital. The initial net worth at the bank, $\bar{n}$, affects the overall resources of banks and therefore the total capital in the economy, which in turn affects the returns to capital. With this calibration, the value of θ is then set to 0.46, which means that, if the bank were to run away with the deposits of the households, it would get a fraction of 0.68 of its current net worth, that is, $1/(1+\theta)$. The value of σ is set to 0.92, that is, banks survive on average 12.5 years.

5.2 Results in the Baseline Economy

Table 5 compares the targeted moments in the data and in the model. Overall the model can reproduce the main features of the data. Next, I show the trade-off of the government between the benefit of domestic debt, that is, its lower default risk, and the cost of domestic debt, that is its crowding out effect on investment, in the calibrated model. Let the value for the government of choosing a given combination of external and domestic borrowing $(B^{*\prime}, B^{\prime})$ be denoted by $\tilde{V}(S; B^{*\prime}, B^{\prime})$, such that, the problem for the government is $V(S; B^{*\prime}, B^{\prime}) = \max_{B^{*\prime}, B^{\prime}} \tilde{V}(S; B^{*\prime}, B^{\prime})$. Figure 6 shows these values as a function of $B^{*\prime}$ (top panels), and $B^{\prime}$ (bottom panels) together with the implied new capital in the economy, $K^{\prime}$, and total resources from debt that the government obtains from

[21] The average default probability is computed using the default events documented in Reinhart and Rogoff (2011). I compute the average default probability for each country in the sample (including countries that never defaulted), and take the average across developing countries.

Figure 6: Government Borrowing Decisions



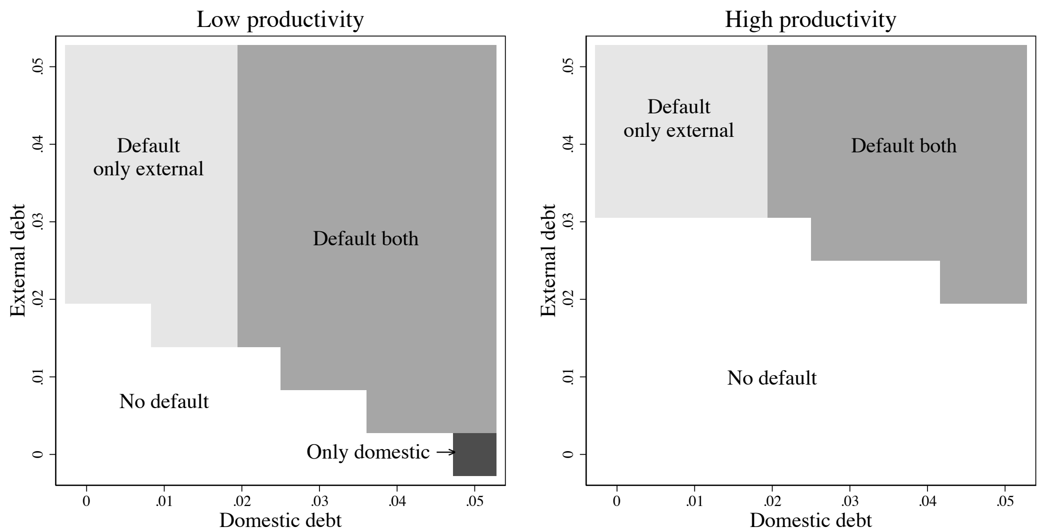
Notes: The upper panels of this figure show the implied value function for the government (right-panel), new capital (middle-panel), and external resources obtain from borrowing $q^*B^{*\prime}$ as a function of new external borrowing $B^{*\prime}$. This is computed for a given state space (B, B^*, K, D, z, R) and for a given amount of domestic borrowing B', which is set to be the optimal one. The black dot in each figure indicates the optimal amount of external borrowing $B^{*\prime}$, that is, the one that maximizes the value function. The bottom panels show the equivalent plots as a function of new domestic borrowing B'.

borrowing such quantities qB'. The optimal decision is indicated in the plots with a black dot. For the chosen level of the state of the economy, S, the optimal amount of domestic and external borrowing is the same, such that, $B' = B^{*\prime} = 0.016$.

The crowding-out cost of domestic debt can be seen in the middle panels of Figure 6: as the government increases domestic debt the amount of capital available for the next period K' decreases. This is the opposite in the case of external debt: increasing external borrowing increases future capital, except for values of debt that are higher than is ever optimal for the government. This is why the value for the government of issuing more domestic debt sharply decreases for high amounts of domestic debt even if qB', that is, the revenue from borrowing more, does not decrease. On the other hand, the relative benefit of domestic debt can be seen precisely from the difference in the total amount of resources that the government gets from borrowing domestically, qB, and externally, $q^*B^{*\prime}$. As will be clear below, external prices are more sensitive to an increase in borrowing than domestic borrowing, as default risk increases more rapidly. As a consequence, the slope of $q^*B^{*\prime}$ starts decreasing for low amounts of external borrowing. This contrasts with the slope of qB', which is always increasing in the amount of domestic borrowing. Therefore, as the amount of external borrowing increases the relative marginal revenue defined in equation (26) starts decreasing.

To explain the difference in the sensitivity of prices to changes in borrowing, Figure 7 plots the

Figure 7: Ex-post Default Decisions

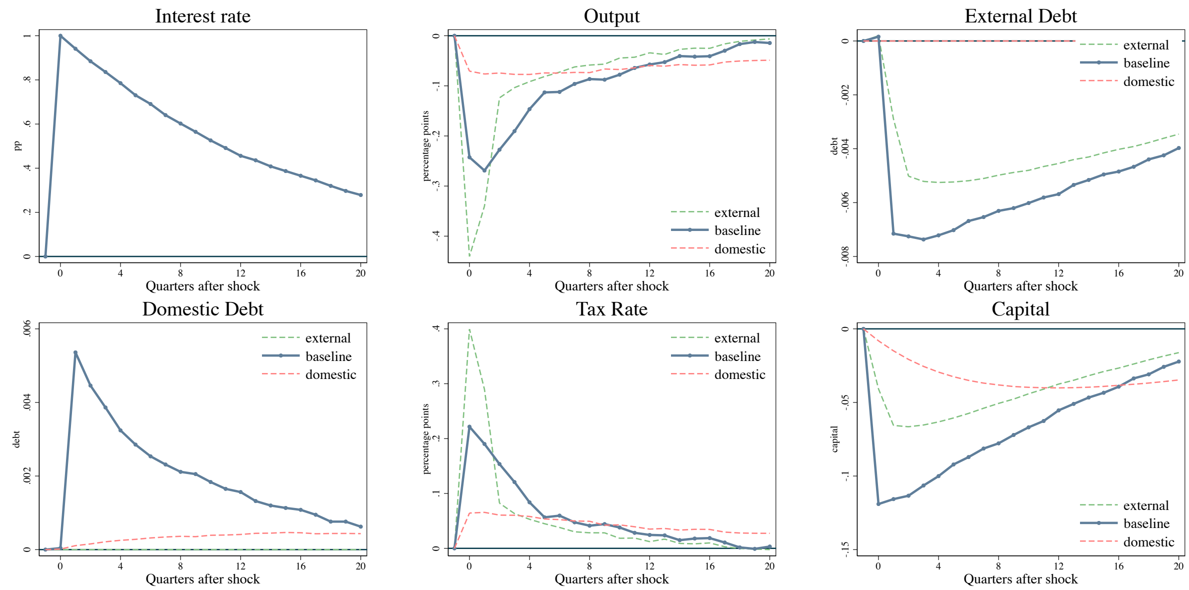


default decisions of the government as a function of (B, B^*). Defaulting on domestic involves an additional endogenous cost because it directly decreases the net worth of domestic banks, as can be seen in equation (27). This implies that, for the same amount of debt, the government will have more incentives to default on external than on domestic debt. When domestic debt is relatively low, the government defaults only on external debt when B^* is high enough. Similarly, when external debt is very low, the government defaults only on domestic debt when domestic debt, B, is very large. However, it is clear from the graph that the government is more tolerant of domestic debt: the region of only external default is larger both in terms of the minimum B^* for which it defaults, and because it requires a higher B for defaulting on both. As debt becomes larger for both types of debt, the government has more incentives to default on both. As productivity increases, the default set shrinks and the no-default area becomes larger, as can be seen in the right panel of Figure 7. Moreover, when productivity is high, there is no region (B, B^*) on which the government defaults only on domestic debt. In this case, the incentives of the government to repay domestic debt increase because, by repaying, it increases the total net worth of the banks, which then can be used to invest in high-productivity capital. Section 5.5 elaborates on the patterns of default and the differences between the conditions under which the government chooses to default on domestic or external debt. Moreover, it shows that these patterns are consistent with what we observe in the data.

5.3 Vulnerability to External Shocks

This section shows the effects of an increase in the international interest rate in output, and how the decisions of the government change after such a shock. In the model, international interest rates are assumed to follow a stochastic process as in equation (12). Figure 8 plots the response

Figure 8: Response to a Positive Interest Rate Shock



Notes: The solid blue lines in this figure show the response of each of the variables in the baseline economy, that is when the government has access to both domestic and external borrowing. The dashed green lines show the case when the government can only borrow externally, and the dashed red lines show the case when the government can only borrow domestically.

to a shock that increases the international interest rate by 100 annual basis points. Output in the baseline economy (solid blue line) drops about 0.3 percentage points, close to what we observe in the data which is around 0.4 percentage points. Notice that none of the parameters of the model are set to match the output drop as in the data. The drop in output after an increase in interest rates is a consequence of higher costs of external borrowing. Given higher interest rates, it is now more costly for the government to finance its expenditures, especially if it finances it using external debt. As shown in Figure 8, in order to balance the higher cost of external debt, the government chooses to decrease its external borrowing and increase domestic debt. In addition, it also needs to increase tax rates to compensate for the decrease in external borrowing. Higher taxes, through its distortionary nature, and higher domestic debt, through its crowding out effect, will decrease both labor and capital in this economy, and thus output drops.

Domestic debt helps mitigate the drop in output. To see this, consider the case where there is only external debt in the economy. In this exercise, I keep the same parameters as in the baseline economy and solve the equilibrium of the model where I constrain the government to have no access to domestic debt. The output response in the economy with only external debt is shown in Figure 8 represented by the dashed green lines. In this case, output drops more than in the baseline economy where the government has access to both types of debt. When the government does not have access to domestic debt, most of the substitution of external debt after the increase in the interest rates is done via an increase in taxes. Therefore, even though there is no crowding out effect and capital does not decrease as much, the large increase in taxes implies a significant decrease in

Table 6: Financial Development and Domestic Debt

	Data		Model	
	Low θ	High θ	Low θ	High θ
Deposits to GDP	0.20	0.45	0.22	0.42
Domestic debt share	0.36	0.68	0.18	0.71

labor. This causes a larger decrease in output in the economy with only external debt. The effect of an increase in interest rates in the economy with only domestic debt comes only from the working capital mechanism, as in this case the cost of borrowing is not directly affected by international prices. Therefore, the drop in output is relatively small in the case of only domestic debt.

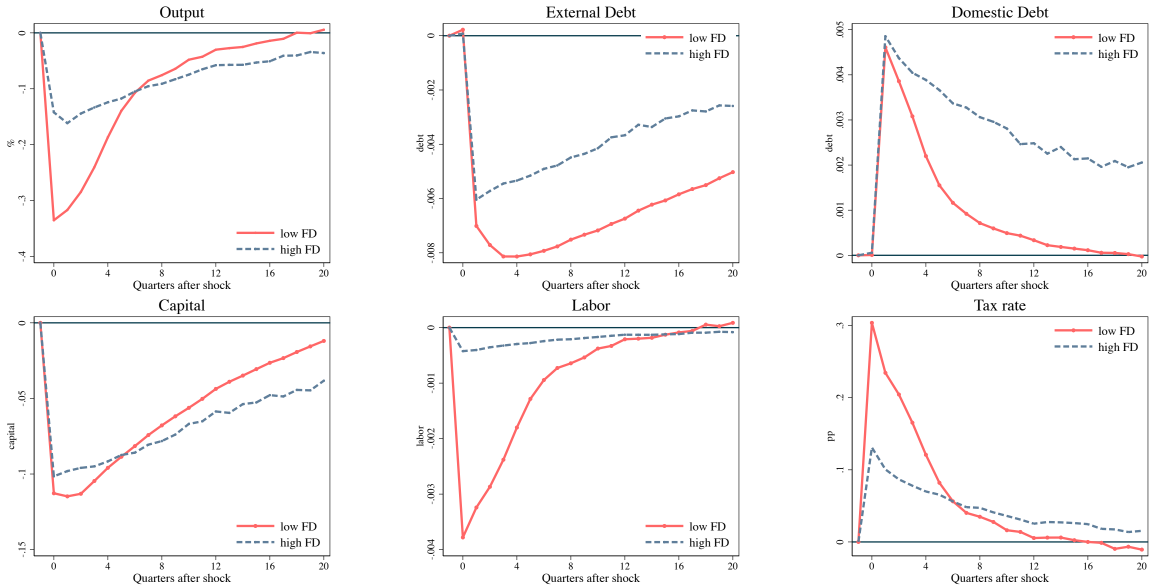
5.4 Financial Development and Domestic Debt

Now, I analyze how the share of domestic debt and the vulnerability to interest rate shocks depend on the level of financial development of the economy. In the model, the level of financial development of a country is controlled by the parameter θ in the collateral constraint (9). The higher θ is the lower the frictions in the financial sector. Specifically, higher θ is associated with a lower fraction of assets that banks can divert. This can be interpreted as improving the monitoring technology that depositors have over banks' behavior, which is linked to the level of development of domestic financial markets.

To see the implications of the model regarding the level of financial development, I perform the following quantitative exercise. First, I divide countries between low and high financial development depending on whether their average level of financial development is above or below the median level in the sample. Then, I vary the parameter θ so as to match the average level of deposits to GDP in each of the groups: low and high financial development. Importantly, in this exercise, the only parameter changing is the value of θ and the rest of the parameters are kept fixed at their level as in Table 4. The average share of domestic debt for countries that are less financially developed in the data is 0.36, whereas in countries that are more financially developed this share is 0.68. Moreover, the difference in the means across these two groups is significantly different from zero. The re-calibrated parameter θ is 0.36 for low financially developed and 0.65 for high financially developed (in the baseline economy it is 0.46).

Table 6 compares the levels of financial development and shares of domestic debt in the data and the ones generated by the model. The model captures the differences in the share of domestic debt between high and low-financially-developed countries. In particular, it captures quantitatively well the higher share of domestic debt in relatively high financially developed economies, and the drop in domestic debt in lower financially developed economies, although it overstates such drop relative to the data. Note that none of the domestic shares in this exercise are targeted moments. There are two main mechanisms in the model that generate this result. First, when banks face less stringent constraints, that is countries are more financially developed, the interest rate that the government

Figure 9: Response to a Positive Interest Rate Shock: Model



Notes: The dashed blue lines in this figure show the response of each of the variables for the economy with a high level of financial development (high θ). The solid red lines show the case when the economy is low financially developed (low θ).

has to pay on domestic debt decreases. Lower constraints imply lower returns to capital due to lower marginal product of capital, and thus lower interest rates on domestic debt. Similarly, when constraints are more relaxed, the crowding-out cost from issuing domestic debt is lower. Therefore, domestic debt becomes more attractive relative to the case where financial constraints are tighter, and governments issue more domestic debt.

The response to a shock that increases the interest rate for the low- and high-financially developed economies is shown in Figure 9. After the shock, in both economies, there is a decrease in external debt, due to its increase in cost, an increase in domestic debt, and an increase in taxes, to substitute for the decrease in external debt. However, the magnitude of these changes is different for low and high-financially developed economies, which in turn affects the magnitude of the drop in total output.

The main difference between the two economies is that, in low financially developed countries, the government mostly substitutes external debt for higher taxes, whereas in high financially developed countries, the government substitutes into higher domestic debt instead of taxes. To understand these decisions, it is useful to consider the effect of each of these policies on capital through the bank aggregate budget constraint when the collateral constraint is binding, that is, $K' = (1+\theta)N - qB'$. An increase in domestic debt, B', has a direct negative effect on capital that depends on the price of domestic debt. In particular, the effect of increasing debt on capital is given by $\frac{\partial q}{\partial B'}B' + q$. On the other hand, increasing taxes have an effect on today's aggregate net worth of banks, N. An increase in taxes translates into a decrease in total labor, which in turn decreases the return on capital and

Table 7: Private Credit From Banks and Domestic Debt

	(1)	(2)	(3)
Domestic debt to GDP	-0.140***	-0.145***	-0.156***
	(0.026)	(0.041)	(0.043)
Total debt to GDP	-0.122***	-0.053***	-0.073***
	(0.031)	(0.029)	(0.033)
Financial development	1.083***	0.636***	0.666***
	(0.038)	(0.075)	(0.078)
Log GDP per capita	5.259***	20.990***	8.092
	(0.853)	(4.351)	(5.586)
Country Effects	No	Yes	Yes
Year Effects	No	No	Yes
Observations	272	272	272

Standard errors in parentheses. * p<0.1, ** p<0.05, *** p<0.01

therefore, the bank's net worth. The effect on capital is then, $(1+\theta)\sigma\frac{\partial R_K}{\partial\tau}$. Notice that, for a given $\frac{\partial R_K}{\partial\tau}$, this effect will be larger the higher θ is, that is, the more financially developed the economy is. The reason is that higher financial development implies that a higher fraction of banks' net worth can be invested in capital.

Empirical Evidence on Main Mechanisms. There are two main mechanisms in the model that explain the patterns observed in the data. First, the model assumes that default in domestic debt implies an endogenous cost arising from a decrease in domestic banks' net worth. And second, domestic debt crowds out investment in the private sector through the collateral constraint affecting banks' investment decisions. Here, I discuss and provide evidence on these mechanisms.

After default, as standard in the literature on sovereign default, an exogenous drop in productivity will prevail for a given amount of periods. This exogenous cost is modeled as exactly the same for domestic and external defaults. However, the model also features an additional endogenous default cost when defaulting on domestic debt, that is not present when defaulting on external debt. This happens because domestic banks' net worth depends on the repayment of such debt, and therefore default would imply a decrease in banks' net worth. Empirically, we observe such an effect. Gennaioli et al. (2018) use a panel of data on banks for different European countries to analyze the relationship between government default risk, banks holding of domestic debt, and their loan-to-assets ratio. Baskaya and Kalemli-Ozcan (2016) estimate the impact of default risk on domestic banks holding government debt using a natural experiment in Turkey. They find that increases in default risk imply a significant reduction in bank credit supply and in net worth. Key to this mechanism is also the introduction of selective default in the model. This is assumed following what is observed in the data, as there are indeed episodes of defaults where governments discriminate between domestic and external creditors.

The second important mechanism in the model is the crowding out effect of domestic debt.

Becker and Ivashina (2018) show, for the Eurozone countries, that increases in banks' government bond holdings were associated with a decrease in their corporate lending. I show a similar result using aggregate data on the sample of emerging countries studied here. Using data on total private credit from banks, and total domestic government debt in a given country, consider the following regression

$$\frac{\text{Private Credit}_{it}}{GDP_{it}} = \beta \frac{\text{Domestic Debt}_{it}}{GDP_{it}} + \Gamma X_{it} + \alpha_i + \delta_t \varepsilon_{it}, \tag{29}$$

where the coefficient β is an indicator of the *crowding out effect* that domestic debt has on the amount of credit that banks are able to provide. The controls included in X in this regression are log GDP per capita, level of financial development, and total debt. Table 7 reports the results and shows that when there is a high domestic debt to GDP ratio, the amount of private credit from domestic banks tends to significantly decrease, indicating a crowding out effect of domestic debt.

5.5 Patterns of Discriminatory Default

Here, I explain what are the patterns of default implied by the model, and I compare them with the ones observed in the data. When do governments default on external debt and when in domestic debt? There are three main points regarding the patterns of discriminatory default in the data. First, governments tend to default more often on external debt alone. Second, the output drop after a default only on external debt is lower than after a default on both types of debt. And third, before defaulting, the patterns of output are already different between defaults only on external debt and defaults on both types of debt. In particular, when output is moderately below trend countries default only on external debt but when it is greatly below trend they tend to default on both types of debt. I refer to this pattern as *pecking order of default*.

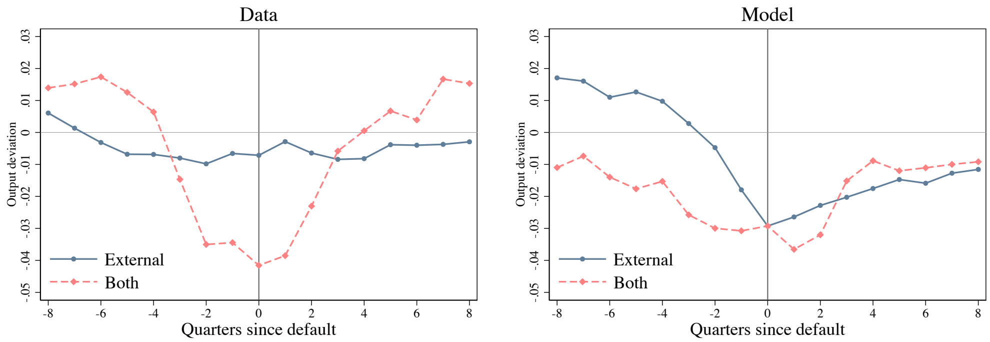
The model is able to generate the same patterns around default that we observe in the data. First, regarding the frequency of each type of default, note that one crucial assumption in the model is that governments can discriminate when they default between external and domestic creditors. As it has been shown in Section 2, this is consistent with what we observe in the data, based on

Table 8: Pecking Order of Default

	Data		Model	
	External only	Both	External only	Both
Frequency of default	74%	26%	67%	33%
Output deviation from trend:				
Before default	-0.8%	-1.9%	-0.3%	-2.5%
Default period	-0.7%	-4.2%	-2.9%	-2.9%
After default	-0.6%	-2.2%	-2.3%	-3.4%

Note: Deviations from trend are measured using the Hodrick-Prescott filter on quarterly series for real GDP. The periods before and after default correspond to the average over the previous and following year of default, respectively. The data used here covers the period 1960 to 2007.

Figure 10: Output Deviation at the Time of Default: Data and Model

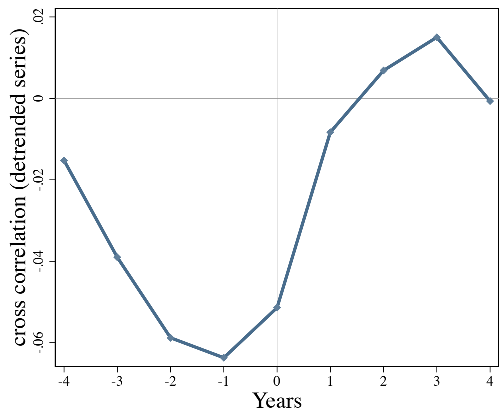


Notes: In the data, output deviation is measured as deviations of the quarterly real GDP from its HP-filtered series (smoothing parameter set to 1600)

the evidence, for instance, in Reinhart and Rogoff (2011). Table 8 shows the frequency of each type of default. As in the data, the model generates a higher fraction of external defaults than of defaults on both types of debt, and it matches well the frequencies even though these are non-targeted moments. Second, Table 8 also presents the output patterns around a default event, which shows that the circumstances under which governments default only on external debt and when they default on both, domestic and external, are very different. In particular, in the periods preceding defaults, output is below trend, but it is much lower when the country defaults on both debts than when it only defaults on external debt. This can also be seen in Figure 10, which shows the average evolution of output around the time of default across all the events of default in the data (left panel) and in the simulations of the model (right panel). In the data, the year before defaulting on both types of debt, output was already almost 2% below trend, and in the year of default it fell more than 4% below trend, whereas for defaults on external debt, only the fall in output is much lower: 0.8% and 0.7% respectively. Similarly, in the model output is already very low in the year before defaulting on both types of debt, and less so when defaulting only on external debt.

In order to understand why the model generates these patterns it is important to see what are the cyclical properties of domestic debt. In the model, higher productivity today increases the expected returns to capital, and therefore, given that the price of debt must be such that banks are indifferent between investing in bonds or in capital, the interest rates that the government has to offer are higher when productivity is high. Moreover, higher productivity implies that the costs from the crowding effect of issuing debt are larger, because a high z today implies a high expected marginal product of capital tomorrow, so that an additional unit of investment would be turned into relatively high output, and therefore consumption, tomorrow. These two effects, that is, relatively higher interest rates on domestic debt and higher costs from issuing domestic debt, makes this type of debt is less attractive to the government when productivity is high. This results in a countercyclical share of domestic debt. Importantly, the same patterns hold in the data. This is shown in Figure 11 which displays the cross-correlation between the detrended series of GDP and

Figure 11: Correlation GDP_{t+h} and Domestic Debt_t in Data



Notes: This figure uses data on annual series of real GDP and share of domestic debt for the countries in the sample used in this paper (see Section 2). The series are detrended using the Hodrick-Prescott filter with a smoothing parameter set to 100.

that of the share of domestic debt. In the sample of countries considered in this paper, periods when GDP is below trend are associated with higher shares of domestic debt.

The countercyclicality of domestic debt explains the pecking order of default. When productivity is not too low, like in the period before defaulting on external debt (see Figure 10), the government has incentives to issue more external debt than domestic because it is relatively cheaper and it does not crowd out capital at a time when investment in capital is very valuable. Then, a drop in productivity will induce defaulting only on external debt because defaulting on domestic debt is more costly, and the share of domestic debt is low. On the other hand, if productivity is already very low, such as in the period before defaulting on both domestic and external debt, the price of external debt becomes more expensive due to a relatively higher probability of defaulting on external than domestic debt. Moreover, both the crowding cost and the interest rates from issuing domestic debt are lower than when productivity is high. Therefore, the government starts issuing a higher share of domestic debt. Then, when there is a sharp decrease in productivity the government has to default on both types of debt, because, given the low shares of external debt, defaulting on foreign creditors is not enough to solve the financing problems of the government.

6 Discussion

In this section, I discuss two main assumptions of the model and show quantitatively that the main results of the paper continue to hold if we modify those assumptions. The first assumption is the abstraction from real exchange rate changes in the model. The second is the presence of working capital through which firms are directly affected by fluctuations in the international interest rate.

Table 9: Parameters and Moments to Match: Model With Exchange Rates

Parameter		Moment	Data	Model
Discount factor	$\beta = 0.979$	Default probability	4.51	0.96
Banks survival rate	$\sigma = 0.93$	Deposits to GDP	0.32	0.33
Collateral constraint	$\theta = 0.485$	Share of domestic debt	0.54	0.53
Banks initial net worth	$\bar{n} = 0.70$	Returns on equity	12.5	16.9
Gov. expenditures	$G = 0.036$	Government expenditures to GDP	0.14	0.11
Disutility of working	$\xi = 2.30$	Hours worked	0.22	0.22
Productivity standard dev.	$\sigma_z = 0.009$	Volatility of GDP	3.08	3.24
Productivity cost of default	$\zeta_0 = -0.178$	Debt to GDP	32.5	29.9
Productivity cost of default	$\zeta_1 = 0.188$	Spread	2.45	1.67

6.1 The Role of Real Exchange Rates

A large literature has explored the effects of monetary policy on exchange rates. This literature finds that a shock that increases the U.S. interest rate is associated with a significant appreciation of the U.S. dollar.[22] The decision of the government between domestic and external is affected by changes in the exchange rate. Usually, external credit is issued in foreign currency, so an appreciation of the dollar makes the repayment of dollar-denominated debt more expensive. Moreover, if changes in the international interest rate are associated with changes in the real exchange rate, then the vulnerability of emerging countries to an increase in the international interest rate will also be affected by the change in the exchange rate.

To explore the role of exchange rates both in determining the share of domestic and external debt, and in the effects of an increase in the international interest rate, I introduce real exchange rates in the model. Let e_t be the real exchange rate that expresses units of local goods in terms of units of foreign goods. Then, the government budget constraint when the economy is in non-autarky times is

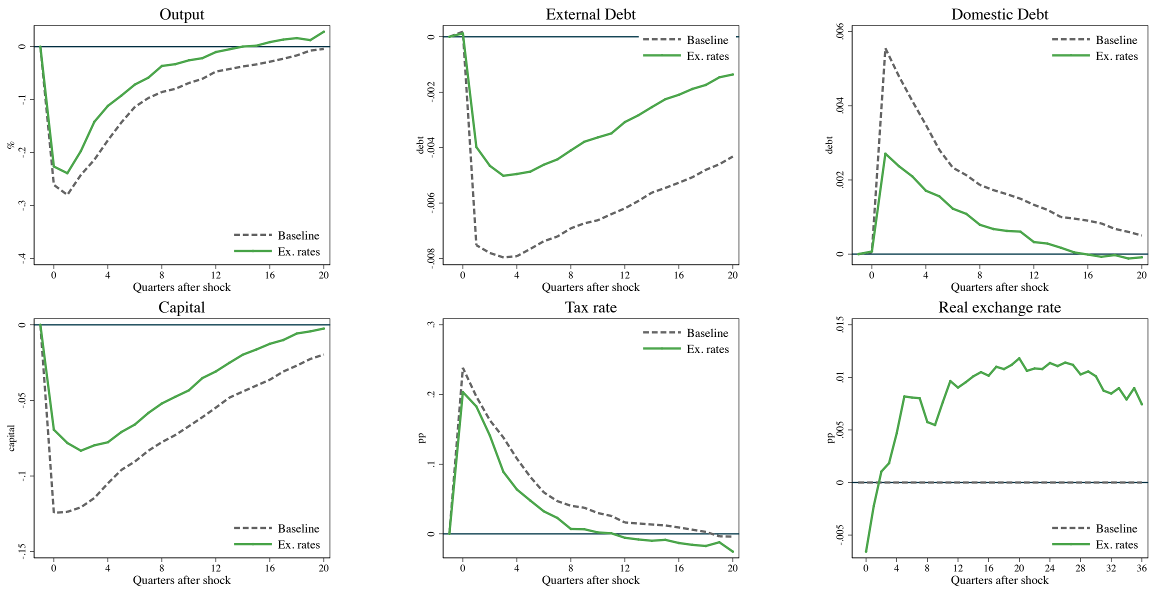
$$G + \delta_t \lambda B_t + \delta_t^* \lambda e_t B_t^* = \tau_t w_t L_t + \delta_t q_t \left(B_{t+1} + (1-\lambda) B_t\right) + \delta_t^* e_t q_{t+1}^* \left(B_{t+1}^* + (1-\lambda) B_t^*\right). \tag{30}$$

An increase in the real exchange rate, e_t, implies that the cost of repaying the debt increases, as more units of local goods are required to repay a given amount of foreign goods. This can be seen in the left-hand side of the government budget constraint (30). In periods of high exchange rates, then, keeping everything else the same, the government will have more incentives to issue domestic debt relative to external debt.

Exchange rates are affected by shocks to the exchange rate, ϵ_t^e, but also by shocks to the international interest rate, ϵ_t^r. This allows to capture the idea that the exchange rate will be affected by a shock increase in the international interest rate. In particular, the joint stochastic

[22] See, for instance, Eichenbaum and Evans (1995), Faust and Rogers (2003).

Figure 12: Response to a Positive Interest Rate Shock: Model With Exchange Rates



Notes: The solid green lines in this figure show the response of each of the variables for the economy that includes real exchange rate fluctuations, and the dashed lines correspond to the baseline economy without real exchange rate fluctuations.

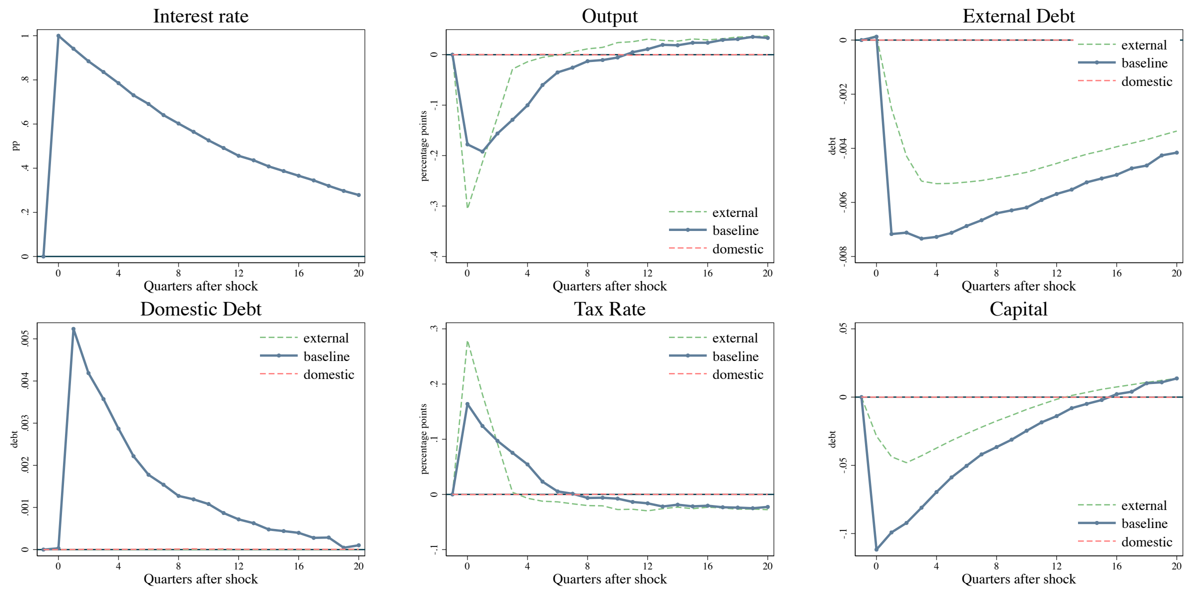
process for international interest rates and the exchange rate is

$$\begin{bmatrix} r^*_{t+1} \\ e_{t+1} \end{bmatrix} = A + B \begin{bmatrix} r^*_t \\ e_t \end{bmatrix} + \Sigma \begin{bmatrix} \epsilon^r_{t+1} \\ \epsilon^e_{t+1} \end{bmatrix}, \tag{31}$$

where Σ is the variance-covariance matrix, and A and B are matrices containing the constants and the auto-regressive coefficients of the system. I estimate this system of equations using the data for the countries in the sample and solve the model using by discretizing the vector of interest rates and exchange rates, following the methodology proposed in Terry and Knotek II (2011). This allows us to capture the joint process for r^*_t and e_t. Then, the parameters of the model are set using the same strategy as in the baseline economy without exchange rates. Appendix E.6 shows the details of the estimation and the corresponding results. The estimation shows that on impact after a shock that increases the international interest rate, there is a decrease of around 0.8% in the real exchange rate of the emerging countries relative to the United States. However, after the initial negative impact, relative exchange rates increase up to 2%.

Next, I show that the effect of an increase in the international rates—which now has also a direct effect on the real exchange rates— is similar to the economy in which there are no fluctuations in the real exchange rate. The model with exchange rates is then calibrated to match the same moments as in the baseline economy. The parameters used and the model fit are shown in Table 9, and Figure 12 shows the response to an increase in the interest rate of 1 percentage point annualized. In this case, the real exchange rate also responds to the shock to the interest rate, as specified by

Figure 13: Response to a Positive Interest Rate Shock: No Working Capital



Notes: The solid blue lines in this figure show the response of each of the variables in the baseline economy, that is, when the government has access to both domestic and external borrowing. The dashed green lines show the case when the government can only borrow externally, and the dashed red lines show the case when the government can only borrow domestically.

the estimated equation (31). The patterns are the same as in the baseline, with a similar decrease in output. To compensate for the higher cost of borrowing externally, the government substitutes external debt by increasing taxes and domestic debt. External debt in Figure 12 is expressed in international goods units. Therefore, the increase in the real exchange rate causes external debt in local goods units to decrease even further.

6.2 The Role of Working Capital

The model assumes that the international interest rate has not only an effect through changes in the cost of borrowing for the government, but it assumes that firms working capital is paid using international frictionless intra-period borrowing. The objective of this assumption is to introduce a direct effect on private agents in the domestic economy. Here, I show that dropping this assumption by setting the amount of working capital to zero, that is $\kappa = 0$, implies no change in the mechanisms of the model, although it decreases the magnitude of the output drop after an increase in interest rates.

Figure 13 shows the response to an increase in the international interest rate in such a case. All parameters are kept the same as in the baseline calibration in 4 except for κ which is set to zero. I show the response in three different scenarios: when the government has access to both domestic and external debt (solid dark line), when the government only has access to external debt (dashed green line), and when the government only has access to domestic debt (dashed red line).

This figure shows the same patterns as those in the main analysis in Figure 8, although the drop in output after the increase in interest rate is naturally smaller than in the case with working capital. In particular, in the scenario with both types of debt, an increase in the interest rate of 1 percentage point implies a decrease in output of 0.2 percent and 0.3 percent in the case with only external debt. Clearly, when there is no working capital and the government has only access to domestic debt, changes in the international interest rate have no effect on the domestic economy.

7 Conclusion

The key motivation in this paper is the evidence that the aggregate fluctuations and default rates in emerging market economies are not driven solely by internal shocks, as is typically assumed, but also by external shocks, such as movements in world interest rates. I argue that an omission in the existing literature is the link between financial development, the share of domestic debt, and the vulnerability of developing economies to fluctuations in world interest rates. I have shown that empirically, countries that are less financially developed, as measured by their deposit-to-output ratio, have low shares of domestic government debt to the externally issued government debt. Such countries are also more vulnerable to fluctuations in world interest rates.

I developed a model consistent with these features. By embedding a financial intermediation sector into an otherwise standard model of sovereign default, I have a way of naturally model financial development as a strengthening of the ability to enforce contracts as measured by a parameter governing the tightness of the resulting collateral constraint. As this ability to enforce contracts increases so does the share of domestic debt and, through the equilibrium, the vulnerability to external shocks decreases. The model naturally also produces other key features of the data: the pecking order of default and the countercyclicality of the domestic debt share.

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Appendix A Banks Problem

A.1 Proof of Lemma 1

Consider the recursive formulation of the bank problem. To write the problem of the bank it is sufficient to have as an individual state variable its net worth, n, at the beginning of the period. Then, the value of a bank is

$$V^b(n; S) = \beta \max_{k',b',d'} \mathbb{E}\left[\sigma V^b(n'; S') + (1-\sigma)n'\right] \tag{32}$$

subject to the budget constraint

$$k' + q(S)b' - q^D(S)d' \leq n \tag{33}$$

collateral constraint,

$$q^D(S)d' \leq \theta n \tag{34}$$

and, evolution of net worth

$$n' = R_K(S')k' + \delta(S')\left[\lambda + (1-\lambda)q(S')\right]b' - d' \tag{35}$$

We can rewrite the banks problem (32) by substituting in the evolution of net worth (35)

$$\begin{aligned} V^b(n; S) = \max_{k',b',d'} \quad & \beta\sigma\mathbb{E}\left\{V^b(R^K(S')k' + \delta(S')\left[\lambda + (1-\lambda)q(S')\right]b' - d'; S')\right\} \\ & + \beta(1-\sigma)\mathbb{E}\left\{R^K(S')k' + \delta(S')\left[\lambda + (1-\lambda)q(S')\right]b' - d'\right\} \end{aligned}$$

subject to

$$k' + q(S)b' - q^D(S)d' \leq n$$

$$q^D(S)d' \leq \theta n$$

Let $\rho(S)$ be the Lagrange multiplier on the budget constraint (33), and let $\mu(S)$ be the Lagrange multiplier on the collateral constraint (34). Then,

$$\text{FOC}(k'): \qquad \beta\mathbb{E}\left\{\left[1-\sigma+\sigma V_n^b(n'; S')\right]R_K(S')\right\} - \rho(S) = 0 \tag{36}$$

$$\text{FOC}(b'): \qquad \beta\mathbb{E}\left\{\left[1-\sigma+\sigma V_n^b(n'; S')\right]\delta(S')\left[\lambda + (1-\lambda)q(S')\right]\right\} - q(S)\rho(S) = 0 \tag{37}$$

$$\text{FOC}(d'): \qquad -\beta\mathbb{E}\left[1-\sigma+\sigma V_n^b(n';S')\right]+q^D(S)\rho(S)-q^D(S)\mu(S)=0, \tag{38}$$

where, V_n^b denotes the derivative of the value function with respect to net worth, n.

Now, we guess that the value function is linear in net worth, that is, $V^b(n;S)=n\nu(S)$. Then, optimality conditions are

$$\text{FOC}(k'): \qquad \rho(S)=\beta\mathbb{E}\left\{\left[1-\sigma+\sigma\nu(S')\right]R^K(S')\right\} \tag{39}$$

$$\text{FOC}(b'): \qquad q(S)\rho(S)=\beta\mathbb{E}\left\{\left[1-\sigma+\sigma\nu(S')\right]\left[\lambda+(1-\lambda)q(S')\right]\delta(S')\right\} \tag{40}$$

$$\text{FOC}(d'): \qquad q^D(S)\left[\rho(S)-\mu(S)\right]=\beta\mathbb{E}\left\{1-\sigma+\sigma\nu(S')\right\}, \tag{41}$$

Substituting the guess into the value function, we get

$$\begin{aligned} n\nu(S)=&\beta\mathbb{E}\left\{\left[1-\sigma+\sigma\nu(S')\right]R^K(S')\right\}k'+\beta\mathbb{E}\left\{\left[1-\sigma+\sigma\nu(S')\right]\left[\lambda+(1-\lambda)q(S')\right]\delta(S')\right\}b' \\ &-\beta\mathbb{E}\left[1-\sigma+\sigma\nu(S')\right]d', \end{aligned}$$

and, using optimality conditions (39)-(41), we can rewrite it as

$$\begin{aligned} n\nu(S)=&\rho(S)\left[k'+q(S)b'-q^D(S)d'\right]+\mu(S)q^D(S)d' \\ =&\rho(S)n+\mu(S)q^D(S)d' \end{aligned} \tag{42}$$

Then the guess is verified: if the constraint binds, $q^D(S)d'=\theta n$, so

$$\nu(S)=\rho(S)+\theta\mu(S), \tag{43}$$

where, $\rho(S)=\beta\mathbb{E}\left\{[1-\sigma+\sigma n'\nu(S')]R^K(S')\right\}$, and

$$\mu(S)=\beta\mathbb{E}\left\{\left[1-\sigma+\sigma\nu(S')\right]\left[R^K(S')-\frac{1}{q^D(S)}\right]\right\}, \tag{44}$$

so,

$$\nu(S)=\mathbb{E}\left\{\beta\left[1-\sigma+\sigma\nu(S')\right]\left[(1+\theta)R_K(S')-\theta/q^D\right]\right\}. \tag{45}$$

If the constraint does not bind, $\mu(S)=0$, and $\nu(S)=\rho(S)$.

Moreover, from the first order condition we get the pricing equation for domestic bonds. Combining (39) and (40):

$$q(A';S)=\frac{\mathbb{E}\left\{[1-\sigma+\sigma\nu']\left[\lambda+(1-\lambda)q(A'';S)\right]\right\}}{\mathbb{E}\left\{[1-\sigma+\sigma\nu']R_K'\right\}} \tag{46}$$

Appendix B Aggregate Budget Constraint

Here we derive the country-level budget constraint under the case where the government repays its debt and thus, it is in the no autarky state. Derivation of the autarky states country-level budget constraint follow the same steps. To get the country's aggregate budget constraint (15) we start with the households budget constraint:

$$C + q^D D' + \sigma \bar{n} = (1-\tau) wL + D + X. \tag{47}$$

From the firm's first order condition, we can substitute $w = zF_L(K, L)$. Moreover, from the banks net worth, the dividends that are transferred to the households are this period returns for the fraction σ of bankers that do not survive to next period, so $X = \sigma \left[R^K K + (\lambda + (1-\lambda)q) B - D \right]$, which we can write in terms of N using the aggregate net worth definition as, $X = \frac{\sigma}{1-\sigma} N - \frac{\sigma^2}{1-\sigma} \bar{n}$. So, substituting w and X into (47) we get

$$C + q^D D' + \frac{\sigma}{1-\sigma} \bar{n} = (1-\tau) zF_L(K, L) L + D + \frac{\sigma}{1-\sigma} N. \tag{48}$$

Now, using banks aggregate net worth definition,

$$N = (1-\sigma) \left[R^K K + (\lambda + (1-\lambda)q) B - D \right] + \sigma \bar{n}, \tag{49}$$

we can substitute for D in (48), so

$$C + q^D D' = zF(K, L) L - T + R^K K + (\lambda + (1-\lambda)q) B - N, \tag{50}$$

where $T = \tau wL$, and substituting $R^K = zF_K(K, L) + 1 - \delta_K$ from firms first order condition, we get,

$$C + q^D D' = zF(K, L) - T + (\lambda + (1-\lambda)q) B - N, \tag{51}$$

where we used the fact that $zF(K, L) = zF_L(K, L) L + zF_K(K, L) K$.

Now, from the government budget constraint under repayment we have,

$$G + \lambda B + \lambda B^* = T + q \left(B' - (1-\lambda) B \right) + q \left(B^{*\prime} - (1-\lambda) B^* \right),$$

so we can substitute T into (51),

$$C + N + q^D D' = zF(K, L) + (1-\delta^K) K - G - \lambda B^* + qB' + q \left(B^{*\prime} - (1-\lambda) B^* \right). \tag{52}$$

Finally, using the aggregate bank budget constraint,

$$N + q^D D' = K' + qB',$$

so, substituting $N + q^D D'$ into (52) we get

$$C + K' - (1 - \delta_K)K + G + \lambda B^* = zF(K, L) + q\left(B^{*\prime} - (1 - \lambda)B^*\right), \tag{53}$$

which is equation (15).

Appendix C Government Problem

Consider the government of the problem deciding between domestic and external debt in the simpler case with one-period bonds, no working capital, no capital adjustment costs, and no stochastic international interest rate. Let the aggregate state of the economy be $S_t = (A_t; z_t)$, and A_t denote the assets, that is, $A_t = (K_t, D_t, B_t, B_t^*)$. The problem of the government if it repays its debt this period is:

$$V^R(S) = \max_{B', B'^*, L, C, K_{t+1}} C + v(L) + \beta \mathbb{E}\left\{\tilde{\delta}(S')V^R(S') + \Gamma^D(S')\right\} \tag{54}$$

where, Γ^D are the collected continuation values for the government in terms of default, and $\tilde{\delta}(S_t) = \delta(S_t)\delta^*(S_t)$. The government budget constraint under repayment is

$$q(A', z)B' + q^*(A', z)B^{*\prime} = G + B + B^* - \left(zF_L + v'(L)\right) L \tag{55}$$

the banks budget constraint is

$$K' + q(A', z)B' = (1 + \beta\theta)\left[\sigma\left(zF_K K + B - D\right) + (1 - \sigma)\bar{n}\right], \tag{56}$$

where, F_K is simplified notation for $\partial F(K, L)/\partial K$; the country's resource constraint is

$$C + G + K' + B^* = zF(K, L) + q(A', z)B^{*\prime}; \tag{57}$$

and, the pricing equations are

$$q^*(A', z) = \frac{\mathbb{E}\delta^*(S')}{R^*}, \tag{58}$$

for the external debt, and,

$$q(A', z) = \frac{\mathbb{E}\left[m(S')\delta(S')\right]}{\mathbb{E}\left[m(S')z'F_K(S')\right]}, \tag{59}$$

for domestic debt, where $m(S) = \beta\mathbb{E}(1 - \sigma + \sigma\nu(S'))$ and ν is the marginal value of net worth at the bank, defined in (16).

Then, the problem of the government is to solve (54) subject to (55)-(59). We can rewrite this problem as

$$\begin{aligned} V^R(S) = \max_{B', B^{*\prime}} & zF(K, L) - B^{*\prime} - G - K'(S, B') + q^*(A', z)B^{*\prime} + v(L) \\ & + \beta\mathbb{E}\left\{\tilde{\delta}(S')V^R(S') + \Gamma^D(S')\right\} \end{aligned} \tag{60}$$

subject to

$$\mu(S): \qquad G + B + B^* - q(A', z)B' - q^*(A', z)B^{*\prime} - \hat{T}(L; S)L \leq 0 \tag{61}$$

$$\rho(S): \quad K' + q(A', z)B' - \sigma\left(zF_K(K, L)K + B - D\right) - \beta D' - (1-\sigma)\bar{n} \leq 0 \tag{62}$$

$$\eta(S): \quad \beta D' - \theta\sigma\left(zF_K(K, L)K + B - D\right) - \theta(1-\sigma)\bar{n} \leq 0 \tag{63}$$

and, to the pricing equations, where the function $K'(S, B')$ is defined by the banks budget constraint, and the function $\hat{T}(S)$ is the tax revenues per unit of labor. Let $\mu(S)$ be the Lagrange multiplier on the government budget constraint, $\rho(S)$ on the aggregate bank budget constraint, and $\eta(S)$ on the aggregate bank collateral constraint. Then, taking first order condition with respect to external borrowing we get:

$$\begin{aligned} q^* + \frac{\partial q^*}{\partial B^{*\prime}} B^{*\prime} + \mu(S)\left(q^* + \frac{\partial q^*}{\partial B^{*\prime}} B^{*\prime} + \frac{\partial q}{\partial B^{*\prime}} B'\right) - \rho(S)\frac{\partial q}{\partial B^{*\prime}} B' \\ = -\beta\frac{\partial \mathbb{E}\left\{\tilde{\delta}(S')V^R(S'))\right\}}{\partial B^{*\prime}} + \Delta^*(S). \end{aligned} \tag{64}$$

To simplify the exposition of the model forces, we focus on the continuation value under repayment, so that we can ignore the extra terms involving default, Γ^D, or changes in default probability, which we collect with the variable, $\Delta^*(S)$, and set to 0. Now, taking first order condition with respect to domestic debt we get:

$$\begin{aligned} \frac{\partial q^*}{\partial B'} B^{*\prime} + \mu(S)\left(q + \frac{\partial q}{\partial B'} B' + \frac{\partial q^*}{\partial B'} B^{*\prime}\right) - \rho(S)\left(q + \frac{\partial q}{\partial B'} B'\right) \\ = -\beta\frac{\partial \mathbb{E}\left\{\tilde{\delta}(S')V^R(S')\right\}}{\partial B'} + \Delta(S), \end{aligned} \tag{65}$$

where, similarly to equation (64), $\Delta(S)$ captures the terms involving changes in default probability and the default continuation values, after a change in B_{t+1}. We can further derive (64) and (65), to obtain

$$q^* + \frac{\partial q^*}{\partial B^{*\prime}} B^{*\prime} + \mu(S)\left(q^* + \frac{\partial q^*}{\partial B^{*\prime}} B^{*\prime} + \frac{\partial q}{\partial B^{*\prime}} B'\right) = \beta\mathbb{E}\left[\tilde{\delta}(S')\left(1 + \mu(S')\right)\right] + \rho(S)\frac{\partial q}{\partial B^{*\prime}} B' \tag{66}$$

$$\begin{aligned} \frac{\partial q^*}{\partial B'} B^{*\prime} + \mu(S)\left(q + \frac{\partial q}{\partial B'} B' + \frac{\partial q^*}{\partial B'} B^{*\prime}\right) \\ = \beta\mathbb{E}\left[\tilde{\delta}(S')\left(\mu(S') - \rho(S')\,\sigma - \eta(S')\theta\sigma\right] + \rho(S)\left(q + \frac{\partial q}{\partial B'} B'\right). \end{aligned} \tag{67}$$

To complete the problem of the government we need to specify the first order conditions with respect to labor, L, capital, K', and deposits D', which give us three additional conditions:

$$\mu(S) = \frac{\left(\rho(S) + \theta\eta(S)\right)\sigma z F_{KL} K + \hat{T}(L; S)}{\hat{T}_L(L; S)L - \hat{T}(L; S)}, \tag{68}$$

$$
\begin{aligned}
\rho(S)+\eta(S)=&\sigma\mathbb{E}\left[\big(\rho(S')+\theta\eta(S')\big)-\mu(S')\hat{T}_K(L';S')L'\right] \\
&-\frac{1}{\beta}\left[\frac{\partial q^*}{\partial D'}B^{*\prime}-\mu(S)\left(\frac{\partial q^*}{\partial D'}B^{*\prime}+\frac{\partial q}{\partial D'}B'\right)\right],
\end{aligned}
\tag{69}
$$

and,

$$
\begin{aligned}
\rho(S)=&\frac{\partial q^*}{\partial K'}B^{*\prime}-1+\mu(S)\left(\frac{\partial q^*}{\partial K'}B^{*\prime}+\frac{\partial q}{\partial K'}B'\right) \\
&+\beta\mathbb{E}\left[z'F'_K+\mu(S')\hat{T}_K(L';S')L'+\big[\rho(S')+\theta\eta(S')\big]\sigma z'\left(F'_{KK}K'+F'_K\right)\right]
\end{aligned}
\tag{70}
$$

Appendix D Computational Algorithm

The model is solved as the limit of a finite horizon model with a large enough number of periods, T. Starting from the last period T, I solve the model backwards taking as given the value functions and bond prices one period ahead until convergence. I use a grid search method to find the optimal amount of borrowing $B', B^{*\prime}$ over a grid $\mathcal{B}$ of a total of 100 points. Moreover, in order to smooth the decision of the government, which is necessary computationally in the case of long-term debt, I follow Dvorkin et al. (2021) and introduce i.i.d. shocks distributed as Extreme Value Type I into the government decision on borrowing.

Notation. Aggregate state is $S = (B^*, B, K, D, z, R^*, a)$. Define for each autarky state:

- Normal times: $S^n = (B^*, B, K, D, z, R^*)$
- Domestic autarky: $S^d = (B^*, K, D, z, R^*)$
- External autarky: $S^e = (B, K, D, z, R^*)$
- Autarky in both markets: $S^b = (K, D, z)$

Let $V^{a,R}(S^a)$ be the value of repaying in autarky state $a = \{n, d, e\}$, that is, normal times, domestic autarky, and external autarky respectively; and $V^b(S^b)$ the value when in autarky in both (no repayment decision)

Discretize Space State. The state space is discretized as follows:

- Aggregate endogenous state variables: B^*, B, K, D
- Productivity, z, process: AR1 discretized using Tauchen (1986) method
- International interest rate, R^*, process: AR1 discretized using Tauchen (1986) method

Solve Problem in the Last Period $t = T$

Here, I show how to solve the value functions and price functions in the last period. I assume that in the last period there is no new borrowing and no investment in capital. First, I show how to compute the repayment values in each of the autarky states: no-autarky, domestic autarky, external autarky, and full autarky. Given the repayment values, we can compute the default decisions, and therefore the prices.

Repayment Values.

1. Enter the last period in no-autarky state (normal times). The value of repaying both debts is

$$V_T^{n,R}(S^n) = zK^\alpha L(S^n)^{1-\alpha} + (1-\delta_K)K - G - B^* + \psi\frac{L(S^n)^{1+\phi}}{1+\phi}$$

where, $L(S^n)$ solves

$$G + B + B^* = z(1-\alpha)K^\alpha L(S^n)^{1-\alpha} + \psi L(S^n)^{1+\phi}$$

2. Enter the last period in domestic autarky state. The value of repaying external debt is

$$V_T^{d,R}(S^d) = zK^\alpha L(S^d)^{1-\alpha} + (1-\delta_K)K - G - B^* + \psi\frac{L(S^d)^{1+\phi}}{1+\phi}$$

where, $L(S^d)$ solves

$$G + B^* = h(z)(1-\alpha)K^\alpha L(S^d)^{1-\alpha} + \psi L(S^d)^{1+\phi}$$

3. Enter the last period in external autarky state. The value of repaying domestic debt is

$$V_T^{e,R}(S^e) = zK^\alpha L(S^e)^{1-\alpha} + (1-\delta_K)K - G + \psi\frac{L(S^e)^{1+\phi}}{1+\phi}$$

where, $L(S^e)$ solves

$$G + B = h(z)(1-\alpha)K^\alpha L(S^e)^{1-\alpha} + \psi L(S^e)^{1+\phi}$$

4. Enter the last period in autarky in both markets. The value of full autarky is

$$V_T^{b}(S^b) = zK^\alpha L(S^b)^{1-\alpha} + (1-\delta)K - G + \psi\frac{L(S^b)^{1+\phi}}{1+\phi}$$

where, $L(S^b)$ solves

$$G = h(z)(1-\alpha)K^\alpha L(S^b)^{1-\alpha} + \psi L(S^b)^{1+\phi} - \xi^b$$

where ξ^b is a fixed cost of defaulting on both. Note that this is only included in the last period because absent this cost the government always wants to default on both rather than defaulting only on domestic or external.

Default Decisions and Value Function.

1. If currently in normal times:

   (a) Default on external only if $V_T^{e,R}(S^e) > V_T^{n,R}(S^n)$, and then $\delta_T(S^n) = 1$, $\delta_T^*(S^n) = 0$

(b) Default on domestic only if $V_T^{d,R}(S^d) > V_T^{n,R}(S^n)$, and then $\delta_T(S^n) = 0$, $\delta_T^*(S^n) = 1$

(c) Default on both if $V_T^b(S^b) > V_T^{n,R}(S^n)$, and then $\delta_T(S^n) = 0$, $\delta_T^*(S^n) = 0$

(d) Otherwise, and then $\delta_T(S^n) = 1$, $\delta_T^*(S^n) = 1$

and therefore value is $V_T(S^n) = \max\{V_T^{n,R}(S^n), V_T^{d,R}(S^d), V_T^{e,R}(S^e), V_T^b(S^b)\}$

2. If currently in domestic autarky default if $V_T^b(S^b) > V_T^{d,R}(S^d)$, and then $\delta_{d,T}(S^d) = 0$; otherwise $\delta_{d,T}(S^d) = 1$

   and therefore value is $V_T^d(S^d) = \max\{V_T^{d,R}(S^d), V_T^b(S^b)\}$

3. If currently in external autarky default if $V_T^b(S^b) > V_T^{e,R}(S^e)$, and then $\delta_{e,T}^*(S^e) = 0$; otherwise $\delta_{e,T}^*(S^e) = 1$

   and therefore value is $V_T^e(S^e) = \max\{V_T^{e,R}(S^e), V_T^b(S^b)\}$

Bond Prices. Given repayment decisions, $(\delta_T^*, \delta_T, \delta_{e,T}^*, \delta_{d,T})$, compute prices that creditors will offer to the government at $t = T - 1$

$$q_T^*(B^{*\prime}, B', K', D'; z, R^*) = \frac{\mathbb{E}_{z',R^{*\prime}}\left(\delta_T^*(B^{*\prime}, B', K', D', z', R^{*\prime})\Big| z, R^*\right)}{R^*}$$

$$q_{d,T}^*(B^{*\prime}, K', D'; z, R^*) = \frac{\mathbb{E}_{z',R^{*\prime}}\left(\gamma\delta_T^*(B^{*\prime}, 0, K', D', z', R^{*\prime}) + (1-\gamma)\delta_{e,T}^*(B^{*\prime}, K', D', z', R^{*\prime})\Big| z, R^*\right)}{R^*}$$

$$q_T(B^{*\prime}, B', K', D'; z, R^*) = \frac{\mathbb{E}_{z',R^{*\prime}}\left(\delta_T(B^{*\prime}, B', K', D', z', R^{*\prime})\Big| z, R^*\right)}{\mathbb{E}_{z',R^{*\prime}}\left(F_K(B^{*\prime}, B', K', D', z', R^{*\prime})\Big| z, R^*)\right)}$$

$$q_{e,T}(B^{*\prime}, K', D'; z, R^*) = \frac{\mathbb{E}_{z',R^{*\prime}}\left(\gamma\delta_T(0, B', K', D', z', R^{*\prime}) + (1-\gamma)\delta_{d,T}(B', K', D', z', R^{*\prime})\Big| z, R^*\right)}{\mathbb{E}_{z',R^{*\prime}}\left(\gamma F_K(0, B', K', D', z', R^{*\prime}) + (1-\gamma)\gamma F_K^d(B', K', D', z', R^{*\prime})\Big| z, R^*)\right)}$$

where, $F_K(S^n) = z\alpha K^{\alpha-1}L(S^n)^{1-\alpha}$ and $F_K^d(S^d) = h(z)\alpha K^{\alpha-1}L(S^d)^{1-\alpha}$. I set the marginal value of a unit in the bank in the last period, ν_T, to be 1, as it is immediately transfer to the household.

Iterate Backwards: $t = T - 1$

Taking as given the value functions and price functions from last period, T, we can iterate backwards to compute the value functions and price functions at $t = T - 1$.

Value Functions Under Repayment.

1. **Normal times** (access to both credit markets)

Let $\tilde{V}_t^{n,R}(B^{*\prime}, B'; S^n)$ be the value of repaying existing debt and issuing new debt $(B^{*\prime}, B')$:

$$\begin{aligned}
\tilde{V}_t^{n,R}(B^{*\prime}, B'; S^n) =& zK^\alpha L(B^{*\prime}, B'; S^n)^{1-\alpha} - K'(B^{*\prime}, B'; S^n) + (1-\delta_K)K - G \\
&+ q^*_{n,t+1}(B^{*\prime}, B'; S^n)\left[B^{*\prime} - (1-\lambda)B^*\right] - \lambda B^* \\
&- \frac{\kappa r^*}{1+r^*} z(1-\alpha)K^\alpha L(B^{*\prime}, B'; S^n)^{1-\alpha} + \psi \frac{L(B^{*\prime}, B'; S^n)^{1+\phi}}{1+\phi} \\
&+ \beta \mathbb{E}_{z', R^{*\prime}}\left[V_{t+1}\left(B^{*\prime}, B', K'(B^{*\prime}, B'; S^n), D'(B^{*\prime}, B'; S^n), z', R^{*\prime}\right)|z, R^*\right]
\end{aligned}$$

where functions $L(B^{*\prime}, B'; S^n), K'(B^{*\prime}, B'; S^n), D'(B^{*\prime}, B'; S^n)$ solve the competitive equilibrium conditions for given values $(B^{*\prime}, B'; S^n)$. That is, these functions solve the following system of equations.

- Collateral constraint

$$\beta D'(B^{*\prime}, B'; S^n) = \theta N$$

  where aggregate net worth N is defined as

$$N = \sigma\left[\alpha z K^\alpha L(B^{*\prime}, B'; S^n)^{1-\alpha} + (1-\delta_K)K + B(\lambda + (1-\lambda)B) - D\right] + (1-\sigma)\bar{n}$$

- Bank aggregate budget constraint

$$K'(B^{*\prime}, B'; S^n) = (1+\theta)N - \hat{q}_{n,t+1}(B^{*\prime}, B'; S^n)B'$$

- Government budget constraint

$$G + \lambda B + \lambda B^* = T + \hat{q}(B^{*\prime}, B'; S^n)\left[B' - (1-\lambda)B\right] + \hat{q}^*_{n,t+1}(B^{*\prime}, B'; S^n)\left[B^{*\prime} - (1-\lambda)B^*\right]$$

  where T are tax revenues

$$T = z(1-\alpha)K^\alpha L(B^{*\prime}, B'; S^n)^{1-\alpha} + \psi L(B^{*\prime}, B'; S^n)^{1+\phi}$$

Solution method: Nelder-Mead algorithm to find (L, K', D') that solve

$$\begin{cases}
K'(B^{*\prime}, B'; S^n) - (1+\theta)N - \hat{q}_{n,t+1}(B^{*\prime}, B'; S^n)B' & = 0 \\
G + \lambda B + \lambda B^* - T - \hat{q}(B^{*\prime}, B'; S^n)\left[B' - (1-\lambda)B\right] - \hat{q}^*_{n,t+1}(B^{*\prime}, B'; S^n)\left[B^{*\prime} - (1-\lambda)B^*\right] & = 0 \\
\beta D'(B^{*\prime}, B'; S^n) - \theta\left\{\sigma\left[\alpha z K^\alpha L(B^{*\prime}, B'; S^n)^{1-\alpha} + (1-\delta)K + B(\lambda + (1-\lambda)B) - D\right] + (1-\sigma)\bar{n}\right\} & = 0
\end{cases}$$

where, $\hat{q}_{n,T}(B^{*\prime}, B'; S^n)$ is the value q_T piecewise linearly interpolated at the points (K', D'), and similarly for $q^*_{n,T}$.

Then, value of repaying both debts is

$$\begin{aligned} V_t^{n,R}(S^n) =& \mathbb{E}\max\left\{\tilde{V}_t^{n,R}(B^{*\prime}, B'; S^n) + \xi_B\right\} \\ =& \eta \log \sum_{(B',B^{*\prime})\in\mathcal{B}} \exp\left(\tilde{V}_t^{n,R}(B^{*\prime}, B'; S^n)\right)^{1/\eta} \end{aligned}$$

where η is the scale parameter of the Extreme Value Type I shocks, ξ_B, which I assume is equal to 0.0001. Note also that given the assumed distribution for the borrowing shocks, we can compute the probability of the government choosing a bond portfolio of $(B^{*\prime}, B')$ as

$$\pi(B^{*\prime}, B'; S^n) = \frac{\exp\left(\tilde{V}_t^{n,R}(B^{*\prime}, B'; S^n)\right)^{1/\eta}}{\sum_{\hat{B}^{*\prime},\hat{B}'} \exp\left(\tilde{V}_t^{n,R}(\hat{B}^{*\prime}, \hat{B}'; S^n)\right)^{1/\eta}}$$

2. **Domestic autarky** (no access to domestic borrowing)

Let $\tilde{V}_t^{d,R}(B^{*\prime}; S^d)$ be the value of repaying existing debt and issuing new debt $B^{*\prime}$:

$$\begin{aligned} \tilde{V}_t^{d,R}(B^{*\prime}; S^d) =& h(z)K^\alpha L(B^{*\prime}; S^d)^{1-\alpha} - K'(B^{*\prime}; S^d) + (1-\delta_K)K - G + \psi\frac{L(B^{*\prime}; S^d)^{1+\phi}}{1+\phi} \\ &+ q^*_{d,t+1}(B^{*\prime}; S^d)\left[B^{*\prime} - (1-\lambda)B^*\right] - \lambda B^* \\ &+ \beta\mathbb{E}_{z',R^{*\prime}}\Big\{\gamma V_{t+1}\left(B^{*\prime}, 0, K'(B^{*\prime}; S^d), D'(B^{*\prime}; S^d), z', R^{*\prime}\right) + \\ &+ (1-\gamma)V^d_{t+1}\left(B^{*\prime}, K'(B^{*\prime}; S^d), D'(B^{*\prime}; S^d), z', R^{*\prime}\right)\Big| z, R^*\Big\}, \end{aligned}$$

where functions $L(B^{*\prime}; S^d), K'(B^{*\prime}; S^d), D'(B^{*\prime}; S^d)$ solve the competitive equilibrium conditions for given $\left(B^{*\prime}; S^d\right)$, that is summarize by the following system of equations:

– Collateral constraint

$$\beta D'(B^{*\prime}; S^d) = \theta N$$

where aggregate net worth N is defined as

$$N = \sigma\left[\alpha h(z)K^\alpha L(B^{*\prime}; S^d)^{1-\alpha} + (1-\delta_K)K - D\right] + (1-\sigma)\bar{n}$$

– Bank aggregate budget constraint

$$K'(B^{*\prime}; S^d) = (1+\theta)N$$

– Government budget constraint

$$G + \lambda B^* = T + \hat{q}^*_{d,t+1}(B^{*\prime}; S^d)\left[B^{*\prime} - (1-\lambda)B^*\right]$$

where T are tax revenues

$$T = h(z)(1-\alpha)K^{\alpha}L(B^{*\prime};S^d)^{1-\alpha} + \psi L(B^{*\prime};S^d)^{1+\phi}$$

Solution method: Nelder-Mead algorithm to find (L', K') that solve

$$\begin{cases} K'(B^{*\prime};S^d) - (1+\theta)\left\{\sigma\left[\alpha h(z)K^{\alpha}L(B^{*\prime};S^d)^{1-\alpha} + (1-\delta_K)K - D\right] + (1-\sigma)\bar{n}\right\} & = 0 \\ G + \lambda B^* - T - \hat{q}^*_{d,t+1}(B^{*\prime};S^d)\left[B^{*\prime} - (1-\lambda)B^*\right] & = 0 \end{cases}$$

where, $q^*_{d,t+1}(B^{*\prime}, S^d)$ is the value of $q^*_{d,T}$ piecewise linearly interpolated at the points (K', D') and

$$D' = \frac{\theta}{\beta}\left\{\sigma\left[\alpha h(z)K^{\alpha}L(B^{*\prime};S^d)^{1-\alpha} + (1-\delta_K)K - D\right] + (1-\sigma)\bar{n}\right\}$$

Then, value of repayment is

$$\begin{aligned} V_t^{d,R}(S^d) =& \mathbb{E}\max\left\{\tilde{V}_t^{d,R}(B^{*\prime};S^d) + \xi_B\right\} \\ =& \eta\log\sum_{B^{*\prime}\in\mathcal{B}^d}\exp\left(\tilde{V}_t^{d,R}(B^{*\prime};S^d)\right)^{1/\eta} \end{aligned}$$

3. **External autarky** (no access to external borrowing)

Let $\tilde{V}_t^{e,R}(B';S^e)$ be the value of repaying existing debt and issuing new debt B':

$$\begin{aligned} \tilde{V}_t^{e,R}(B';S^e) =& h(z)K^{\alpha}L(B';S^e)^{1-\alpha} - K'(B';S^e) + (1-\delta)K - G \\ &+ \psi\frac{L(B';S^e)^{1+\phi}}{1+\phi} \\ &+ \beta\mathbb{E}_{z',R^{*\prime}}\Big\{\gamma V_{t+1}\left(0, B^{*\prime}, K'(B';S^e), D'(B';S^e), z', R^{*\prime}\right) \\ &+ (1-\gamma)V^e_{t+1}\left(B', K'(B';S^e), D'(B';S^e), z', R^{*\prime}\right)\Big|z, R^*\Big\} \end{aligned}$$

where functions $L(B';S^e), K'(B';S^e), D'(B';S^e)$ solve the competitive equilibrium conditions for given$(B';S^e)$, that is summarize by the following system of equations:

– Collateral constraint

$$\beta D'(B^{e\prime};S^e) = \theta N$$

where aggregate net worth N is defined as

$$N = \sigma\left[\alpha h(z)K^{\alpha}L(B';S^e)^{1-\alpha} + (1-\delta_K)K + B(\lambda + (1-\lambda)B) - D\right] + (1-\sigma)\bar{n}$$

– Bank aggregate budget constraint

$$K'(B^{e\prime}; S^e) = (1+\theta)N + \hat{q}_{e,t+1}(B'; S^e)B'$$

– Government budget constraint

$$G + \lambda B = T + \hat{q}_{e,t+1}(B'; S^e)\left[B' - (1-\lambda)B\right]$$

where T are tax revenues

$$T = h(z)(1-\alpha)K^\alpha L(B'; S^e)^{1-\alpha} + \psi L(B'; S^e)^{1+\phi}$$

<u>Solution method:</u> Nelder-Mead algorithm to find (L, K') that solve

$$\begin{cases} K'(B'; S^e) - (1+\theta)N - \hat{q}_{e,t+1}(B'; S^e)B' & = 0 \\ G + \lambda B - T - \hat{q}(B'; S^e)\left[B' - (1-\lambda)B\right] & = 0 \\ \beta D'(B'; S^e) - \theta\left\{\sigma\left[\alpha h(z)K^\alpha L(B'; S^e)^{1-\alpha} + (1-\delta_K)K + B(\lambda + (1-\lambda)B) - D\right] + (1-\sigma)\bar{n}\right\} & = 0 \end{cases}$$

where, $\hat{q}_{e,t+1}(B'; S^n)B'$ is the value $q_{e,T}$ piecewise linearly interpolated at the points (K', D'); and

$$D' = \theta\left\{\sigma\left[\alpha h(z)K^\alpha L^{1-\alpha} + (1-\delta_K)K + B - D\right] + (1-\sigma)\bar{n}\right\}$$

Then, value of repayment is

$$\begin{aligned} V_t^{e,R}(S^e) =& \mathbb{E}\max\left\{\tilde{V}_t^{e,R}(B'; S^e) + \xi_B\right\} \\ =& \eta \log \sum_{(B')\in\mathcal{B}^\rceil} \exp\left(\tilde{V}_t^{e,R}(B'; S^e)\right)^{1/\eta} \end{aligned}$$

4. **Autarky in both markets**: no debt issuance or default decision

$$\begin{aligned} V_t^b(S^b) =& h(z)K^\alpha L(S^b)^{1-\alpha} - K'(S^b) + (1-\delta)K - G \\ &+ \psi\frac{L(S^b)^{1+\phi}}{1+\phi} \\ &+ \beta\mathbb{E}_{z',R^{*\prime}}\left\{\gamma V_{t+1}\left(0, 0, K'(S^b), D'(S^b), z', R^{*\prime}\right) + \right. \\ &\left. + (1-\gamma)V_{t+1}^b\left(K'(S^b), D'(S^b), z', R^{*\prime}\right)\big| z, R^*\right\} \end{aligned}$$

where functions $L(S^b), K'(S^b), D'(S^b)$ solve the competitive equilibrium conditions for givenS^b, that is summarize by the following system of equations:

– Collateral constraint

$$\beta D'(S^b) = \theta N$$

where aggregate net worth N is defined as

$$N = \sigma \left[\alpha z K^\alpha L(S^b)^{1-\alpha} + (1-\delta_K)K - D\right] + (1-\sigma)\bar{n}$$

– Bank aggregate budget constraint

$$K'(S^b) = (1+\theta)N$$

– Government budget constraint

$$G = h(z)(1-\alpha)K^\alpha L(S^b)^{1-\alpha} + \psi L(S^b)^{1+\phi}$$

Default Decisions and Value Function.

1. If currently in normal times:

   (a) Default on external only if $V_t^{e,R}(S^e) > V_t^{n,R}(S^n)$, and then $\delta_t(S^n) = 1$, $\delta_t^*(S^n) = 0$

   (b) Default on domestic only if $V_t^{d,R}(S^d) > V_t^{n,T}(S^n)$, and then $\delta_t(S^n) = 0$, $\delta_t^*(S^n) = 1$

   (c) Default on both if $V_t^b(S^b) > V_t^{n,R}(S^n)$, and then $\delta_t(S^n) = 0$, $\delta_t^*(S^n) = 0$

   (d) Otherwise, , and then $\delta_t(S^n) = 1$, $\delta_t^*(S^n) = 1$

   and therefore value is $V_t(S^n) = \max\{V_t^{n,R}(S^n), V_t^{d,R}(S^d), V_t^{e,R}(S^e), V_t^b(S^b)\}$

2. If currently in domestic autarky default if $V_t^b(S^b) > V_t^{d,R}(S^d)$, and then $\delta_{d,t}(S^d) = 0$; otherwise $\delta_{d,t}(S^d) = 1$

   and therefore value is $V_t^d(S^d) = \max\{V_T^{d,R}(S^d), V_T^b(S^b)\}$

3. If currently in external autarky default if $V_t^b(S^b) > V_t^{e,R}(S^e)$, and then $\delta_{e,t}^*(S^e) = 0$; otherwise $\delta_{e,t}^*(S^e) = 1$

   and therefore value is $V_t^e(S^e) = \max\{V_t^{e,R}(S^e), V_t^b(S^b)\}$

Bond Prices. Given repayment decisions, $(\delta_t^*, \delta_t, \delta_{e,t}^*, \delta_{d,t})$, compute prices that creditors will offer to the government at $t = T-1$

$$q_t^*(B^{*\prime}, B', K', D'; z, R^*) = \frac{1}{R_t^*}\mathbb{E}_{z',R^{*\prime}}\left\{\delta_t^*(B^{*\prime}, B', K', D', z', R^{*\prime})\left[\lambda + (1-\lambda)q_T^*(S')\right] \Big| z, R^*\right\}$$

$$q_{d,t}^*(B^{*\prime},K',D';z,R^*) = \frac{1}{R_t^*}\mathbb{E}_{z',R^{*\prime}}\Big(\gamma\delta_t^*(B^{*\prime},0,K',D',z',R^{*\prime})\left[\lambda+(1-\lambda)q_T^*(S')\right]$$
$$+(1-\gamma)\delta_{e,t}^*(B^{*\prime},K',D',z',R^{*\prime})\left[\lambda+(1-\lambda)q_{d,T}^*(S')\right]\Big|z,R^*\Big)$$

$$q_t(B^{*\prime},B',K',D';z,R^*) = \frac{\mathbb{E}_{z',R^{*\prime}}\Big(\nu_T(S')\delta_T(S')\left[\lambda+(1-\lambda)q_T(S')\right]\Big|z,R^*\Big)}{\mathbb{E}_{z',R^{*\prime}}\Big(\nu_T(S')F_K(B^{*\prime},B',K',D',z',R^{*\prime})\Big|z,R^*\Big)\Big)}$$

$$q_{e,t}(B^{*\prime},K',D';z,R^*) = \frac{1}{\mathbb{E}_{z',R^{*\prime}}\left(\gamma\nu_T(S')F_K(0,B',K',D',z',R^{*\prime})+(1-\gamma)\nu_{e,T}(S')F_K^d(S^{e\prime})\right)}$$
$$\times\Bigg\{\mathbb{E}_{z',R^{*\prime}}\Big(\gamma\nu_T(S')\delta_t(0,B',K',D',z',R^{*\prime})\left[\lambda+(1-\lambda)q_T(S')\right]$$
$$+(1-\gamma)\nu_{e,T}(S')\delta_{d,t}(B',K',D',z',R^{*\prime})\left[\lambda+(1-\lambda)q_{d,T}(S')\right]\Big|z,R^*\Big)\Bigg\}$$

where, $F_K(S^n) = z\alpha K^{\alpha-1}L(S^n)^{1-\alpha}$ and $F_K^d(S^d) = h(z)\alpha K^{\alpha-1}L(S^d)^{1-\alpha}$.

Value of the Bank. Finally, we need to compute the marginal value of a unit in the bank ν, which is given by (16). In particular,

$$\nu_t(S) = \beta\mathbb{E}_{z',R^{*\prime}}\sum_{(B^{*\prime},B')\in\mathcal{B}}\pi(B^{*\prime},B';S)\Big[F_K(B^{*\prime},B',K',D',z',R^{*\prime})$$
$$+\theta\nu_T(B^{*\prime},B',K',D',z',R^{*\prime})\Big|z,R^*\Big],$$

and similarly for the other autarky states.

Iterate Backwards and Check Convergence

Check value function convergence: $d = \max\left\{|V_t - V_{t-1}|,|V_t^e - V_{t-1}^e|,|V_t^d - V_{t-1}^d|,|V_t^b - V_{t-1}^b|\right\}$, and bond price schedule convergence, $d^q = \max\Big\{|q_t - q_{t-1}|,|q_{e,t}-q_{e,t-1}|,|q_t^* - q_{t-1}^*|,|q_{d,t}^* - q_{d,t-1}^*|\Big\}$, update them and iterate until convergence.

Simulate Equilibrium Results

1. Start from a given initial state $S_0 = (B_0^*, B_0, K_0, D_0, z_0, R_0^*, a_0)$
2. Using value functions, policy functions in equilibrium, and extreme value shocks for the determination of borrowing, find new assets, $A_1 \equiv (B_1^*, B_1, K_1, D_1)$; given repayment decision $\delta(S_0), \delta^*(S_0)$, update (intra-period) autarky state
3. Realization of new shocks: productivity, z_1, international interest rate, R_1^*, and, if intra-period autarky-state $\tilde{a} = \{d,e,b\}$, re-enter credit markets with probability γ, and updated autarky state, a_1 correspondingly.

4. Start with new state $S_1 = (B_1^*, B_1, K_1, D_1, z_1, R_1^*, a_1)$, and iterate for $T = 50000$ periods.

Appendix E Additional Empirical Evidence

E.1 Domestic Debt Definition

The definition used in the empirical evidence for domestic vs. external debt is by the jurisdiction where the debt is issued. This data is taken from Reinhart and Rogoff (2011). The definition in the model is based on who holds the debt: domestic vs. foreign investors. However, in the data the two definitions align in most cases. Using data from Arslanalp and Tsuda (2014), I compare these two definitions for the years in which we have information on both series and for the countries in the sample (for most countries, this time period is 1999-2010). Figure 14 compares the share of domestic debt using each definition, and it can be seen that the correlation is high for the two series. Moreover, Table 10 compares the main statistics of the series for the overlapping periods, and we can see that they are remarkably similar.

Figure 14: Domestic debt share by definition

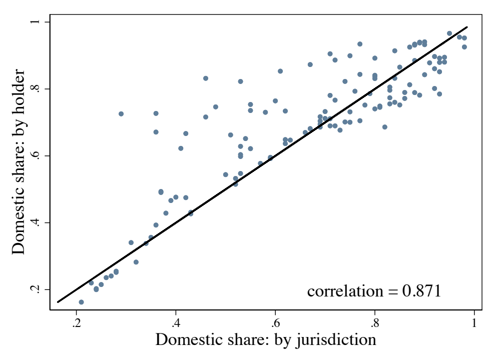


Table 10: Domestic debt definition

	By holder	By jurisdiction
Mean	0.547	0.549
Median	0.577	0.570
P25	0.341	0.360
P75	0.752	0.780

E.2 Descriptive Statistics: Different Time Periods

Table 11 shows the same descriptive statistics as in the main text in Table 1, but compares two different periods in time: the period 1969-1996 is the one used in the main analysis as is the relevant one for analyzing the monetary shocks to the U.S. interest rate, and the period 1960-2007 for which data is available.

Table 11: Descriptive Statistics

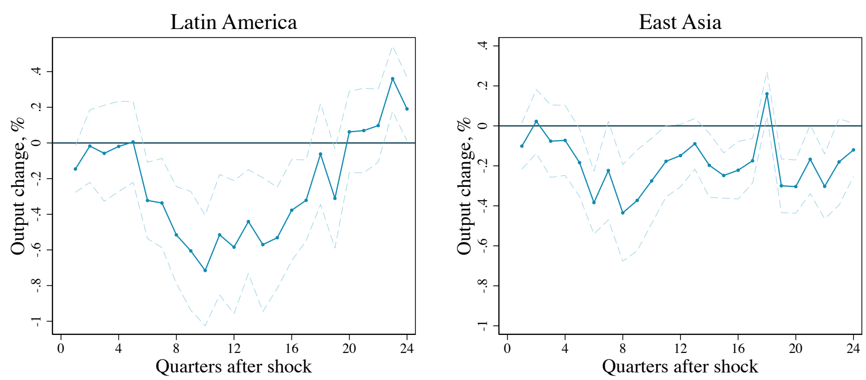
	1969-1996				1960-2007			
	Liquid liabilities	Domestic debt share	Total defaults	External defaults	Liquid liabilities	Domestic debt share	Total defaults	External defaults
Argentina	0.172	0.449	2	0	0.184	0.306	3	0
Brazil	0.172	0.848	2	0	0.275	0.787	4	2
Chile	0.267	0.489	1	1	0.340	0.583	4	4
China	0.636	0.359	0	0	0.849	0.443	0	0
Colombia	0.189	0.398	0	0	0.189	0.406	0	0
Ecuador	0.177	0.418	1	1	0.194	0.219	2	1
India	0.297	0.761	2	2	0.389	0.797	2	2
Indonesia	0.194	0.034	1	1	0.261	0.126	4	4
Malaysia	0.632	0.762	0	0	0.868	0.759	0	0
Mexico	0.222	0.562	1	0	0.223	0.460	2	1
Peru	0.186	0.186	6	5	0.245	0.179	6	5
South Africa	0.514	0.958	3	3	0.483	0.937	3	3
Thailand	0.474	0.752	0	0	0.665	0.773	0	0
Turkey	0.195	0.526	2	2	0.239	0.553	2	2
Total	0.312	0.540	22	16	0.370	0.581	31	23

Sources: World Bank Financial Structure and Development, and Reinhart and Rogoff (2011).
Notes: Last row shows the average liquid liabilities to GDP and domestic debt share for the countries in the sample. *Total defaults* refers to the number of years a given country defaulted on either type of debt, and *External defaults* refers to the number of years a country defaulted only on external debt.

E.3 Effect of Interest Rates on Emerging Countries: Robustness Exercises

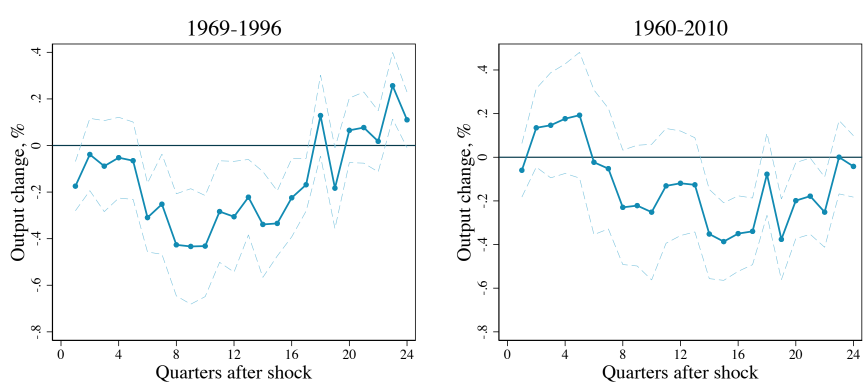
Sample of Countries. In the baseline sample there are 7 countries from Latin America and 4 East Asian, as well as Turkey and South Africa. Here, I show that the negative output response after a shock to the U.S. interest rates is present regardless of the sub-sample of countries that we consider. The left panel of Figure 15 shows the estimated response of real GDP when only countries in Latin America are considered in equation (2), and the right panel shows the response for East Asian countries. Real GDP drops around 0.6 percent in Latin America, and around 0.4 percent in East Asia.

Figure 15: Output response by sample of countries



Volcker Period. Another potential concern about the results on the effect of interest rate shocks on emerging countries is whether results are partially driven by the Volcker shock. The beginning of 1980s was a period characterized by large increases in the U.S. interest rates which was followed by several crisis in developing countries. To see the effect of this period in the main results, I run regression (2) taking out the years of the Volcker shock: 1980-1985. Figure 16 shows the results for the two sample periods considered in the paper. The results do not change if we exclude the Volcker shock years.

Figure 16: Output response excluding Volcker shock periods



E.4 Financial Development and External Shocks: Robustness

In this section, I show robustness exercises regarding the results on the difference in the vulnerability of emerging countries to interest rate shocks by their level of financial development. To this end, regression (3) is estimated in a number of different sub-samples including different time period, countries, and excluding the Volcker shock periods. Table 12 shows the results for all sub-samples. The main result prevails: the coefficient γ_h is positive and significantly so for many quarters after the shock. This indicates that a higher level of financial development, measured as liquid liabilities to GDP, mitigates the drop in output in emerging countries from a shock that increases U.S. interest rates.

Table 12: Output response and financial development: Robustness

	Interaction coefficient, γ_h					
Quarters after shock:	4	6	8	10	12	16
All countries 1960-2007	0.015***	0.026***	0.023**	0.019	0.010	-0.004
	(0.006)	(0.008)	(0.009)	(0.008)	(0.009)	(0.008)
Latin America 1969-1996	0.024	0.037	0.055	0.082*	0.095*	-0.008
	(0.036)	(0.033)	(0.041)	(0.047)	(0.051)	(0.043)
Latin America 1960-2007	0.051	0.102***	0.117***	0.101**	0.065**	0.007
	(0.034)	(0.035)	(0.042)	(0.041)	(0.044)	(0.041)
East Asia 1969-1996	0.010	0.014*	0.020*	0.013	0.021*	-0.001
	(0.007)	(0.008)	(0.011)	(0.010)	(0.011)	(0.008)
East Asia 1960-2007	0.019***	0.030***	0.024**	0.014	0.009	-0.009
	(0.007)	(0.011)	(0.012)	(0.010)	(0.011)	(0.011)
All countries 1969-1996	0.008	0.015	0.030*	0.024	0.036*	-0.006
excluding Volcker shock	(0.012)	(0.013)	(0.017)	(0.018)	(0.021)	(0.013)
All countries 1960-2007	0.015*	0.027***	0.031**	0.020	0.013	0.002
excluding Volcker shock	(0.009)	(0.010)	(0.011)	(0.013)	(0.012)	(0.013)

Driscoll-Kraay standard errors in parentheses. *p<0.1, **p<0.05, ***p<0.01

E.5 Financial Development and Domestic Debt: 1960-2007

Here, I consider the same regression as in (1) but focus on the broader sample period from 1960 to 2007. Results are reported in Table 13, which shows that higher financial development is associated with a larger share of domestic debt. The magnitudes are also similar to the baseline sample 1969-1996.

Table 13: Domestic debt and financial development

	(1)	(2)	(3)
Financial development	0.332***	0.420***	0.189***
	(0.045)	(0.070)	(0.076)
Total debt to GDP	-0.156***	-0.168***	-0.103***
	(0.046)	(0.032)	(0.035)
Log GDP per capita	0.432*	-16.497***	-1.409
	(1.316)	(3.236)	(5.130)
Country Effects	No	Yes	Yes
Year Effects	No	No	Yes
Observations	516	516	516

Standard errors in parentheses. * p<0.1, ** p<0.05, *** p<0.01

E.6 U.S. Interest Rate and Real Exchange Rate

Here I show the results for the estimation of the bivariate process for the U.S. interest rate and the real exchange rate between the United States and each of the emerging countries considered in the sample, as specified in equation (31). I estimate the process for each country separately and then take averages of each of the coefficients. The estimation is done using the average quarterly U.S. Federal funds rate, and the average quarterly log real exchange rates. The estimated coefficients are

$$\begin{bmatrix} r^*_{t+1} \\ e_{t+1} \end{bmatrix} = \begin{bmatrix} 0.0011 \\ -0.0198 \end{bmatrix} + \begin{bmatrix} 0.933 & 0.000 \\ 1.237 & 0.967 \end{bmatrix} \begin{bmatrix} r^*_t \\ e_t \end{bmatrix} + \begin{bmatrix} 0.00001 & 0.00000 \\ -0.00002 & 0.01076 \end{bmatrix} \begin{bmatrix} \epsilon^r_{t+1} \\ \epsilon^e_{t+1} \end{bmatrix}. \quad (71)$$

Figure 17 plots the resulting impulse response of a shock that increases the annualized interest rate by 100 basis points.

Figure 17: Bivariate VAR process for r^* and e